ROBUST CONTROL OF PVTOL AIRCRAFT USING SATURATIONS FUNCTIONS

S. Salazar-Cruz R. Lozano[1]

Heudiasyc-UTC UMR 6599 Centre de Recherches de Royallieu B.P. 20529 60205 Compiegne France Tel.: + 33 (0)3 44 23 44 23 ; fax: +33 (0)3 44 23 44 77

Abstract: We present a new approach based on Lyapunov analysis to control the PVTOL which can lead to further developments in nonlinear systems. The controller we propose is robust in the sense that it remains stable in spite of small coupling between the rolling moment and the lateral acceleration of the aircraft. The controller performance is shown in a real application.

Keywords: Robust control, PVTOL, nonlinear control, bounded input systems

1. INTRODUCTION

A major application area of automatic control techniques is aircraft guidance and control. In the last few years modelling and control of helicopters has received a lot of attention. Avila and Brogliato proposed a detailed model for a helicopter in (Avila-Vilchis and Brogliato, 2000). Marconi et al. proposed a control algorithm for helicopter landing on oscillating platforms (Marconi *et al.*, 2002). In particular the problem of vertical landing and taking off of special airplanes represents an important challenge for the control community. This is known as the PVTOL problem where PVTOL stands for Planar Vertical Take-off and Landing. The PVTOL can represent either the longitudinal model of a helicopter or a simplified model of a bi-rotor plane capable of vertical take-off and landing. Such a system represents a nonlinear dynamical problem and some nonlinear controllers have been proposed in the last decade by (Sepulchre *et al.*, 1997), (Fliess *et al.*, 1995), (Olfati-Saber, 2000) and (Tanaka *et al.*, 2004). An algorithm to control the PVTOL based on a approximate I-O linearization procedure was proposed in (Hauser *et al.*, 1992). Their algorithm achieves bounded tracking and asymptotic stability. A non-linear small gain theorem was proposed in (Teel, 1996) which can be used to obtain a control law to stabilize a PVTOL. He has proved the stability of a controller based on nested saturations. An extension of the algorithm proposed by (Hauser *et al.*, 1992) was presented in (Martin *et al.*, 1996). They were able to find a flat output of the system which was used for tracking control of the PVTOL in presence of unmodelled dynamics. In (Lin *et al.*, 1999) an optimal control was used for robust control of PVTOL and simulations have shown the robustness of the proposed algorithm. In this paper we present a simple control algorithm for the PVTOL whose convergence analysis is relatively simple as compared to other controllers proposed in the literature. We present a new approach based on Lyapunov analysis to control the PVTOL which can lead to further developments in nonlinear systems. The controller we propose is robust in the sense that it remains stable in spite of small coupling between the rolling moment and the lateral acceleration of the aircraft. It is proved that state of the PVTOL converges to a neighborhood of the origin. The size of such a neighborhood is determined by the size of the unmodeled dynamics. The control

[1] Corresponding author rlozano@hds.utc.fr

algorithm has been successfully tested in a real-time platform application which uses a four-rotors mini-rotorcraft.

2. MODELING

The PVTOL aircraft, as shown in Figure 1, has a minimum number of states and inputs but retains many of the features that must be considered when designing control laws for a real aircraft. Here θ is the roll angle that the aircraft makes with the horizon. The control inputs u_1 and u_2 are respectively the thrust (directed out the bottom of the aircraft) and the angular acceleration (rolling moment). The ϵ_0 parameter is a small coefficient which characterizes the coupling between the rolling moment and the lateral acceleration of the aircraft. We have the following dynamic

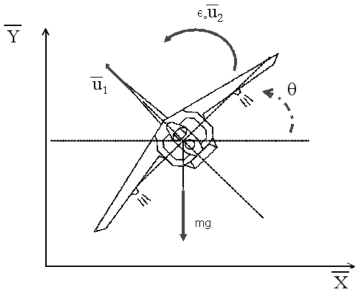

Fig. 1. The PVTOL aircraft

model:

$$
\begin{aligned}
m\ddot{\overline{x}} &= -\overline{u}_1 \sin\theta + \epsilon_0 \overline{u}_2 \cos\theta \\
m\ddot{\overline{y}} &= -\overline{u}_1 \cos\theta + \epsilon_0 \overline{u}_2 \sin\theta - mg \\
J\ddot{\theta} &= \overline{u}_2
\end{aligned}
\tag{1}
$$

where mg represents the gravitational force exerted on the aircraft center of mass and J is the inertial moment mass about the axis through the center of mass of the aircraft and along the fuselage. Dividing (1) by mg and using $x = \overline{x}/g$, $y = \overline{y}/g$, $u_1 = \overline{u}_1/mg$, $u_2 = \overline{u}_2/J$ and $\epsilon = \epsilon_0 J/mg$, the dynamics becomes

$$
\begin{aligned}
\ddot{x} &= -\sin\theta u_1 + \epsilon \cos\theta u_2 \\
\ddot{y} &= \cos\theta u_1 + \epsilon \sin\theta u_2 - 1 \\
\ddot{\theta} &= u_2
\end{aligned}
\tag{2}
$$

3. CONTROL LAW SYNTHESIS

The PVTOL usually operate in a region such that the angle θ is bounded by

$$|\theta| \leq \frac{\pi}{10} \tag{3}$$

We first stabilize the PVTOL altitude $y(t)$ using the following linearizing control strategy

$$u_1 = \frac{r+1}{\cos\theta} \tag{4}$$

and

$$r = -a_1\dot{y} - a_2(y - y_d) \tag{5}$$

where y_d is the desired altitude and $a_i > 0$, for $i = 1, 2$ such that the polynomial $s^2 + a_1 s + a_2$ is stable. We obtain from (2) and (4)

$$
\begin{aligned}
\ddot{x} &= -(r+1)\tan\theta + \epsilon u_2 \cos\theta \\
\ddot{y} &= -a_1\dot{y} - a_2(y - y_d) + \epsilon u_2 \sin\theta \\
\ddot{\theta} &= u_2
\end{aligned}
\tag{6}
$$

Now, we are assuming that input u_2 is bounded by a positive constant as

$$|u_2| \leq a \tag{7}$$

We have BIBO (Bounded Input Bounded Output) stability for sub-system $y(t)$. Considering $y_d = 0$ and defining

$$z \triangleq \dot{y} + c_2 y \tag{8}$$

we have

$$\dot{z} + c_1 z = \epsilon u_2 \sin\theta \tag{9}$$

where

$$
\begin{aligned}
c_1 + c_2 &= a_1 \\
c_1 c_2 &= a_2
\end{aligned}
\tag{10}
$$

For $z(t)$, we have $z(t) = z(0)e^{-c_1 t} + \int_0^t e^{-c_1(t-\tau)} \epsilon u_2 \sin\theta d\tau$, and for a time arbitrarily large

$$|z(t)| \leq \frac{|\epsilon|\, a}{c_1}\overline{\theta} \tag{11}$$

from $|\sin\theta| \leq |\theta| \leq \overline{\theta} \leq \frac{\pi}{10}$, where $\overline{\theta}$ will be defined later. For $\dot{y}$, we have $y(t) = y(0)e^{-c_2 t} + \int_0^t e^{-c_2(t-\tau)} z(t) d\tau$, and for a time arbitrarily large

$$|y(t)| \leq \frac{|\epsilon|\, a}{c_1 c_2}\overline{\theta} \tag{12}$$

Then, for a time arbitrarily large (from (8))

$$\left|\dot{y}(t)\right| \leq 2\frac{|\epsilon|\, a}{c_1}\overline{\theta} \tag{13}$$

and for $r(t)$ from (5) and considering a time arbitrarily large (from (10)), we have

$$
\begin{aligned}
|r| &\leq a_1 \max\left|\dot{y}\right| + a_2 \max|y| \\
&= c_{\overline{\theta}} |\epsilon|\, a
\end{aligned}
\tag{14}
$$

where

$$c_{\overline{\theta}} = 2\frac{a_1}{c_1}\overline{\theta} + \overline{\theta} \tag{15}$$

Then, $y(t)$ and $r(t)$ will be bounded by a small bound by choosing a, a_1, a_2 and $\overline{\theta}$ in suitable form value. We propose to use u_2 for controlling ($\dot{\theta}$, θ, $\dot{x}$, x). The control algorithm is obtained step by step in the following subsections.

3.1 Boundedness of $\dot{\theta}$

In order to establish a bound for $\dot{\theta}$, we define u_2 as

$$u_2 \triangleq -\sigma_a(\dot{\theta} + \sigma_b(z_1)) \tag{16}$$

where

$$\sigma_\eta(s) = \begin{cases} \eta & if\ s > \eta \\ s & if\ \text{-}\eta \le s \le \eta \\ -\eta & if\ s < -\eta \end{cases} \tag{17}$$

and z_1 will be defined later. Let us propose

$$V_1 = \frac{1}{2}\dot{\theta}^2 \tag{18}$$

differentiating, we have

$$\dot{V}_1 = -\dot{\theta}\sigma_a(\dot{\theta} + \sigma_b(z_1)) \tag{19}$$

Note that if $\left|\dot{\theta}\right| > b + \delta$ for some $b > 0$ and $\delta > 0$ arbitrarily small, then $\dot{V}_1 < 0$. After some finite time T_1, we will have

$$\left|\dot{\theta}\right| \le b + \delta \tag{20}$$

Let us assume that b verifies

$$a \ge 2b + \delta \tag{21}$$

Then, from (2) and (16) we obtain for $t \ge T_1$

$$\ddot{\theta} = -\dot{\theta} - \sigma_b(z_1) \tag{22}$$

3.2 Boundedness of θ

First we establish a bound for θ, which will be essential in avoiding fast displacement on x and y. Defining z_1 as

$$z_1 \triangleq z_2 + \sigma_c(z_3) \tag{23}$$

for some z_3 to be defined later and z_2 as

$$z_2 = \theta + \dot{\theta} \tag{24}$$

From (22) and (24), we have

$$\dot{z}_2 = -\sigma_b(z_2 + \sigma_c(z_3)) \tag{25}$$

Let us define

$$V_2 = \frac{1}{2}z_2^2 \tag{26}$$

then, the derivative is

$$\dot{V}_2 = -z_2\sigma_b(z_2 + \sigma_c(z_3)) \tag{27}$$

Note that, if $|z_2| > c+\delta$ for some small positive δ and $c > 0$, then $\dot{V}_2 < 0$. Therefore, it follows that after some finite time $T_2 \ge T_1$

$$|z_2| \le c + \delta \tag{28}$$

From (24) we obtain for $t \ge T_2$

$$\theta = \theta(T_2)e^{-(t-T_2)} + \int_{T_2}^{t} e^{-(t-\tau)}z_2(\tau)d\tau \tag{29}$$

There exists a finite time T_3 such that for $t \ge T_3 > T_2$

$$|\theta| \le \overline{\theta} \triangleq c + 2\delta \tag{30}$$

if

$$c + 2\delta \le \frac{\pi}{10} \tag{31}$$

Assume that b and c satisfy

$$b \ge 2c + \delta \tag{32}$$

Then, in view of (28), (25) reduces to

$$\dot{z}_2 = -z_2 - \sigma_c(z_3) \tag{33}$$

3.3 Boundedness of $\dot{x}$

Define z_3 and z_4 as

$$z_3 \triangleq z_4 + \sigma_d(z_5) \tag{34}$$

$$z_4 \triangleq z_2 + \theta - \dot{x} \tag{35}$$

then, from (6), (24) and (33) we have

$$\begin{aligned}\dot{z_4} = &-\sigma_c(z_4 + \sigma_d(z_5)) - \theta + \\ &(r+1)\tan\theta - \epsilon\cos\theta u_2\end{aligned} \tag{36}$$

Considering the following positive definite function

$$V_3 = \frac{1}{2}z_4^2 \tag{37}$$

differentiating V_3 and using (36), we have

$$\begin{aligned}\dot{V}_3 = &-z_4(-\sigma_c(z_4 + \sigma_d(z_5)) \\ &-\theta + (r+1)\tan\theta - \epsilon\cos\theta u_2)\end{aligned} \tag{38}$$

We will use the following inequalities which can be easily verified graphically for $|\theta| \le \frac{\pi}{10}$

$$|\tan\theta - \theta| \le \frac{2}{15}\theta^2 \tag{39}$$

$$|r\tan\theta| \le c_{\overline{\theta}}\,|\epsilon|\,a\left(|\theta| + \frac{2}{15}\theta^2\right)$$

$$|\epsilon\cos\theta u_2| \le |\epsilon|\,a\left(1 - \frac{2}{15}\theta^2\right) \tag{40}$$

then, the expression $-\theta + (r+1)\tan\theta - \epsilon\cos\theta u_2$ becomes bounded as follows

$$\begin{aligned} &|-\theta + (r+1)\tan\theta - \epsilon\cos\theta u_2| \\ &\le |\tan\theta - \theta| + |r\tan\theta| + |\epsilon\cos(\theta)u_2| \\ &\le \frac{2}{15}\left(1 + c_{\overline{\theta}}\,|\epsilon|\,a - |\epsilon|\,a\right)\theta^2 \\ &\quad + c_{\overline{\theta}}\,|\epsilon|\,a\,|\theta| + |\epsilon|\,a \\ &= \left(\sqrt{c_{\overline{\theta}|\epsilon|a}}\,|\theta| + \frac{1}{2\sqrt{c_{\overline{\theta}|\epsilon|a}}}c_{\overline{\theta}}\,|\epsilon|\,a\right)^2 + |\epsilon|\,a \\ &\quad -\frac{1}{4}\frac{a^2\,|\epsilon|^2\,c_{\overline{\theta}}^2}{c_{\overline{\theta}|\epsilon|a}} \\ &= M(\theta, \overline{\theta}, |\epsilon|\,, a)\end{aligned} \tag{41}$$

where

$$c_{\overline{\theta}|\epsilon|a} \triangleq \frac{2}{15}\left(1 + c_{\overline{\theta}}\,|\epsilon|\,a - |\epsilon|\,a\right) \tag{42}$$

and

$$M(\theta,\overline{\theta},|\epsilon|,a) \tag{43}$$
$$\triangleq \left(\sqrt{c_{\overline{\theta}|\epsilon|a}}\,|\theta| + \frac{1}{2\sqrt{c_{\overline{\theta}|\epsilon|a}}}c_{\overline{\theta}}\,|\epsilon|\,a\right)^2$$
$$+|\epsilon|\,a - \frac{1}{4}\frac{a^2\,|\epsilon|^2\,c_{\overline{\theta}}^2}{c_{\overline{\theta}|\epsilon|a}}$$

If the signal z_4 is bounded by (see (38))

$$|z_4| > d + M(\theta,\overline{\theta},|\epsilon|,a) + \delta \tag{44}$$

and

$$c \geq M(\theta,\overline{\theta},|\epsilon|,a) + \delta \tag{45}$$

then, for some δ arbitrarily small and $d > 0$; $\dot{V}_3 < 0$. Therefore, after $t > T_3$

$$|z_4| \leq d + M(\theta,\overline{\theta},|\epsilon|,a) + \delta \tag{46}$$

choosing

$$c \geq 2d + M(\theta,\overline{\theta},|\epsilon|,a) + \delta \tag{47}$$

Then, after time T_6 we are in the linear region of the saturation function and therefore (36) reduces to

$$\begin{aligned}\dot{z_4} &= -z_4 - \sigma_d(z_5) \\ &\quad -\theta + (r+1)\tan\theta - \epsilon\cos\theta u_2\end{aligned} \tag{48}$$

3.4 Boundedness of x

We define

$$z_5 \triangleq z_4 + \theta - 2\dot{x} - x \tag{49}$$

then (from (6))

$$\begin{aligned}\dot{z}_5 &= \dot{z}_4 + \dot{\theta} - 2\ddot{x} - \dot{x} \\ &= -z_4 - \sigma_d(z_5) - \theta + (r+1)\tan\theta \\ &\quad -\epsilon\cos\theta u_2 + z_2 - \theta \\ &\quad -2\left(-\tan\theta(r+1) + \epsilon\cos\theta u_2\right) - \dot{x} \\ &= 3(r+1)\tan\theta - 3\epsilon\cos\theta u_2 \\ &\quad -3\theta - \sigma_d(z_5)\end{aligned}$$

where we have to use the fact that $z_2 = \dot{\theta} + \theta$ and $z_4 = z_2 + \theta - \dot{x}$ Now, let us define

$$V_4 = \frac{1}{2}z_5^2 \tag{50}$$

then

$$\begin{aligned}\dot{V}_4 &= z_5(3(r+1)\tan\theta \\ &\quad -3\theta - 3\epsilon\cos\theta u_2 - \sigma_d(z_5))\end{aligned} \tag{51}$$

There exists a finite time $T_7 > T_6$, such that if

$$|z_5| > 3M(\theta,\overline{\theta},|\epsilon|,a) + \delta \tag{52}$$

and

$$d \geq 3M(\theta,\overline{\theta},|\epsilon|,a) + \delta \tag{53}$$

then, $\dot{V}_4 < 0$. Therefore, after finite time $T_8 > T_7$ we have (see (43))

$$|z_5| \leq 3M(\theta,\overline{\theta},|\epsilon|,a) + \delta \tag{54}$$

and

$$\dot{z}_5 = 3(r+1)\tan\theta - 3\epsilon\cos\theta u_2 - 3\theta - z_5 \tag{55}$$

Boundedness of x follows from (54), (46) and (49). The constraints on the parameters are

$$\begin{aligned}&a \geq 2b + \delta \\ &b \geq 2c + \delta \\ &c \geq 2d + M(\theta,\overline{\theta},|\epsilon|,a) + \delta \\ &d \geq 3M(\theta,\overline{\theta},|\epsilon|,a) + \delta \\ &\overline{\theta} = c + 2\delta\end{aligned} \tag{56}$$

3.5 Convergence of θ, $\dot{\theta}$, x and $\dot{x}$

Choosing small constants c and δ for satisfying (56) et from (54), it follows that for a time large enough

$$|z_5| \leq 3M(\theta,|\epsilon|,a) + \delta \tag{57}$$

From (48), we have

$$|z_4| \leq 4M(\theta,|\epsilon|,a) + 2\delta \tag{58}$$

and from (34)

$$|z_3| \leq 7M(\theta,|\epsilon|,a) + 3\delta \tag{59}$$

and (from (33))

$$|z_2| \leq 7M(\theta,|\epsilon|,a) + 4\delta \tag{60}$$

from (24), finally we have

$$|\theta| \leq 7M(\theta,|\epsilon|,a) + 5\delta \tag{61}$$

or equivalently

$$|\theta| \leq 7\left(c_{\theta|\epsilon|a}\theta^2 + c_\theta\,|\epsilon|\,a\,|\theta| + |\epsilon|\,a\right) + 5\delta \tag{62}$$

which hold it

$$\begin{aligned}|\theta| \leq \frac{1}{14c_{\overline{\theta}|\epsilon|a}}&(1 - 7c_{\overline{\theta}}\,|\epsilon|\,a \\ &-\sqrt{\left(1 - 7c_{\overline{\theta}}\,|\epsilon|\,a\right)^2 - 196\,|\epsilon|\,ac_{\overline{\theta}|\epsilon|a}})\end{aligned} \tag{63}$$

Choosing a such that

$$|\epsilon|\,a \leq \frac{28 + 15c_{\overline{\theta}} - 2\sqrt{2}\sqrt{83 + 120c_{\overline{\theta}}}}{7(8 - 8c_{\overline{\theta}} + 15c_{\overline{\theta}}^2)} \tag{64}$$

then exist a real solution for θ. Therefore, stabilization of θ in a neighborhood of the origin as in (63) is obtained in spite of the coupling between the rolling moment and the lateral acceleration of the PVTOL as long as ϵ is smaller that the upper bound (64). From (57)-(61) we have that $z_i(t)$ (for $i = 1, 2, ..., 5$) converge to a small neighborhood of the origin. From (24), (35), and (49), it follows that $\dot{\theta}$, x and $\dot{x}$ converge a small neighborhood of

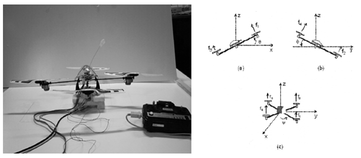

Fig. 2. The Quad-rotor mini rotorcraft and the configuration of the Quad-rotor (a)Pitch, (b)Roll and (c)Yaw control inputs

the origin. Finally, the control input u_2 is given by (16), (23), (24), (34), (35) and (49), i.e.

$$\begin{aligned} u_2 = &-\sigma_a(\dot{\theta} + \sigma_b(\theta + \dot{\theta} \\ &+\sigma_c(2\theta + \dot{\theta} - \dot{x} \\ &+\sigma_d(3\theta + \dot{\theta} - 3\dot{x} - x)))) \end{aligned} \tag{65}$$

4. REAL-TIME EXPERIMENTS

In this section we present the results obtained when applying the control strategy proposed in the previous section. We are using a minirotorcraft Draganfly III (see Figure 2). In this type of helicopter the front and the rear motors rotate counter clockwise while the other two motors rotate clockwise. Then the pitch movement is obtained by increasing the speed of the rear motor while reducing the speed of the front motor. The roll movement is obtained similarly using the lateral motors. The yaw movement is obtained by increasing the speed of the front and rear motor while decreasing the speed of the lateral motors. Note that when the yaw and roll angles are set to zero, the quad-rotor minirotorcraft reduces to the PVTOL system (see Figure 2). We are using a Futaba Skysport 4 radio for transmitting the control signals; these signals are refereed as throttle control input u_1 and pitch control input u_2. The radio joystick potentiometers are connect through of data acquisitions cards (Advantech PCL-818HG and PCL-726) to the PC and for real-time applications we are using MATLAB Simulink xPCtarget .The rotorcraft evolves freely in a 3D space without any flying stand. In order to measure the position (x, y, z) and the orientation (ψ, θ, ϕ) of the rotorcraft, we use the 3D tracker system (POLHEMUS) and we have built a Simulink S-function for connecting the POLHEMUS via RS232 to the xPCtarget. The controller parameters are selected using the following procedure. We first selected the gain concerning pitch angular velocity $\dot{\theta}$, this value is small due the on-board gyros, We next select the controller gain concerning the pitch displacement θ. We wish the pitch error converges to zero fast but without undesirable oscillations. The controller gain concerning $\dot{x}$ and the amplitude of the saturation function are selected in such a way that the mini-aircraft reduces its speed in the x-axis fast enough. To complete the tuning of the pitch control parameters we choose the gains concerning the x displacement in order to obtain a satisfactory performance. The computation of the control input requires the knowledge of the various angular and linear velocities. We obtained the angular velocity by means of gyro Murata ENV-05F-03. The linear velocity is not available and is estimated by using the following reduced-order observer

$$\begin{aligned} \dot{\widehat{x}}_1 &= \widehat{x}_2 + k_{11}(x_1 - \widehat{x}_1) \\ \dot{\widehat{x}}_2 &= -\sin\theta u_1 + k_{12}(x_1 - \widehat{x}_1) \\ \dot{\widehat{x}}_3 &= \widehat{x}_4 + k_{21}(x_3 - \widehat{x}_3) \\ \dot{\widehat{x}}_4 &= \cos\theta u_1 - 1 + k_{22}(x_3 - \widehat{x}_3) \end{aligned} \tag{66}$$

where $\widehat{x} \triangleq [\widehat{x}, \dot{\widehat{x}}, \widehat{y}, \dot{\widehat{y}}]^T$ and $k_{ij} > 0$ for $i, j = 1, 2$. and $x_1 = x$, $x_2 = \dot{x}$, $x_3 = y$, $x_4 = \dot{y}$ Then we have the error state (see (2) and (66)) is $\dot{\widetilde{x}} = A\widetilde{x} + g$ where

$$A = \begin{bmatrix} -k_{11} & 1 & 0 & 0 \\ -k_{12} & 0 & 0 & 0 \\ 0 & 0 & -k_{21} & 1 \\ 0 & 0 & -k_{22} & 0 \end{bmatrix}, g = \begin{bmatrix} 0 \\ \epsilon \cos\theta u_2 \\ 0 \\ \epsilon \sin\theta u_2 \end{bmatrix}$$

and

$$\begin{aligned} \widetilde{x} &\triangleq [\widetilde{x}_1, \widetilde{x}_2, \widetilde{x}_3, \widetilde{x}_4]^T \\ &= [x_1 - \widehat{x}_1, x_2 - \widehat{x}_2, x_3 - \widehat{x}_3, x_4 - \widehat{x}_4]^T \end{aligned} \tag{67}$$

we know that $\|g\| < |\epsilon|\, a$. We propose the following Lyapunov type function

$$V = \widetilde{x}^T P \widetilde{x} \tag{68}$$

where

$$P = \begin{bmatrix} \frac{1+k_{12}}{2k_{11}} & -\frac{1}{2} & 0 & 0 \\ -\frac{1}{2} & \frac{1+k_{11}^2+k_{12}}{2k_{11}k_{12}} & 0 & 0 \\ 0 & 0 & \frac{1+k_{22}}{2k_{21}} & -\frac{1}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1+k_{21}^2+k_{22}}{2k_{21}k_{22}} \end{bmatrix} \tag{69}$$

such that $PA + A^T P = -I$ then

$$\begin{aligned} \dot{V} &= \widetilde{x}^T(PA + A^T P)\widetilde{x} + 2\widetilde{x}^T P g \\ &\leq -\|\widetilde{x}\|^2 + 2\|\widetilde{x}\|\,\|P\|\,\|g\| \\ &\leq -\|\widetilde{x}\|^2 + 2\lambda_{\max}(P)\,|\epsilon|\, a\,\|\widetilde{x}\| \\ &= -\|\widetilde{x}\|^2 + 2\lambda_{\max}(P)\,|\epsilon|\, a\,\|\widetilde{x}\| \pm \gamma\|\widetilde{x}\|^2 \\ &= -(1-\gamma)\|\widetilde{x}\|^2 + 2\lambda_{\max}(P)\,|\epsilon|\, a\,\|\widetilde{x}\| \\ &\quad -\gamma\|\widetilde{x}\|^2 \end{aligned}$$

with $0 < \gamma < 1$. If $\|\widetilde{x}\| \geq \frac{2\lambda_{\max}(P)|\epsilon|a}{\gamma}$, then $\dot{V} \leq -(1-\gamma)\|\widetilde{x}\|^2$ If the perturbation satisfies

$$\|g\| \leq |\epsilon|\, a < \frac{1}{2\lambda_{\max}(P)}\sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}} \tag{70}$$

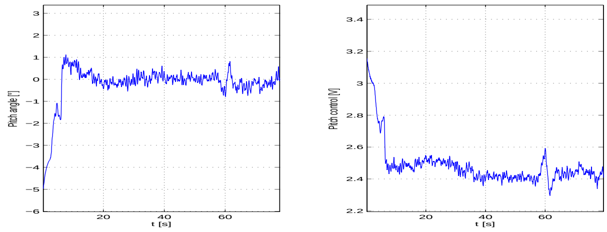

Fig. 3. Pitch angle and pitch control input of the rotorcraft

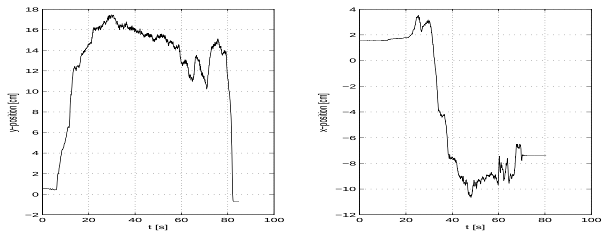

Fig. 4. y-axis and x-axis positions of the rotorcraft

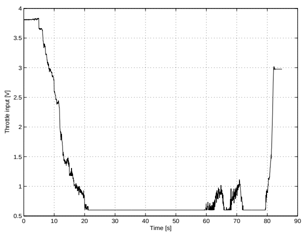

Fig. 5. Throttle control input of the rotorcraft

we have $\lambda_{\min}(P)\left\|\widetilde{x}\right\|^2 \le V \le \lambda_{\max}(P)\left\|\widetilde{x}\right\|^2$. Then, for all initial condition$\|\widetilde{x}_0\| \le \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}$ the solution $\widetilde{x}$ satisfies $\|\widetilde{x}\| \le ke^{-\alpha(t-t_0)}\left\|\widetilde{x}_0\right\|$ for $t_0 \le t < t_1$ and $\|\widetilde{x}\| \le \frac{2\lambda_{\max}(P)|\epsilon|a}{\gamma}\sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}$ for $t > t_1$ where $k = \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}$and $\alpha = \frac{1-\gamma}{2\lambda_{\max}(P)}$ so that the error $\widetilde{x}$ is uniformly ultimately bounded by choosing k_{ij} (Khalil, 1996). The choice of the values for a, b, c, d were carried satisfying the inequalities (56) and (64). However, these parameters have been tuned experimentally in sequence as they appear in the control input u_2. The control objective is to make the rotorcraft hover to reach the position $(x, y) = (0, 15)cm$ while $\theta = 0^o$. The initial conditions are $(x, \dot{x}, y, \dot{y}, \theta, \dot{\theta}) = (0, 0, 0, 0, -6^o, 0^o)$. The gain values used for the control law are

Phase	Control parameter	Value
Altitude	a_1	0.4
	a_2	0.04
Pitch control	a	0.25
	b	0.12
	c	0.6
	d	0.03
Sample	T	$\frac{1}{20}s$

Parameter	Value
k_{11}	4
k_{12}	2
k_{21}	4
k_{22}	2

Figures 3 − 5 show the performance of the controller when applied to the rotorcraft.

5. CONCLUSIONS

We have presented a simple robust control strategy for stabilizing the PVTOL. The Lyapunov convergence analysis has shown that the state of the PVTOL system converges to small neighborhood of the origin. The size of such a neighborhood reduces to zero as the unmodeled dynamics converge to zero. We have been able to successuly test the control strategy in a real application. We controlled the altitud, the orientation and the displacement of a radio-controlled electrical four-rotor mini-helicopter.

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