FORMATED NAVIGATION OF MOBILE ROBOTS USING DISTRIBUTED LEADER-FOLLOWER CONTROL

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Abstract: This paper presents a formated navigation based on the leader-follower approach. In this approach, the control law of the follower robot includes the states of the leader robot. This is a drawback for implementing the control laws on real mobile robots. To overcome this drawback, this paper proposes a new follower's control law, called the self-made follower input, in which the state of the leader robot is estimated by the relative equation between the leader and the follower robots in the discrete-time domain. On the other hand, a control law of the leader robot is designed with a chained system to track a given reference path. The effectiveness of the proposed techniques is demonstrated in numerical simulation.

Keywords: Mobile robots, tracking systems, navigation, nonlinear control, discretization.

1. INTRODUCTION

Navigation of multiple mobile robots has been one of great attentions in robotic society over the past decade because multiple robotic systems contain potentials for accomplishing tasks that are more than single robotic systems can manage. Multiple robots may be required to make a formation for accomplishing such tasks. To realize the formated navigation, the following approaches have been proposed: leader-follower graphs (Desai, et al., 2001; Das, et al., 2002; Tanner, et al., 2004), reactive behaviors (Balch and Arkin,1998; Burridge, et al., 1999) and virtual structure (Tan and Lewis, 1997). The formated navigation considered in this paper is based on the leader-follower technique. In the leader-follower technique, multiple robots are classified into leader and follower robots. The follower robots follow the leader robot with keeping the a specified distance and a relative angle. A control law of the follower robot is designed by the dynamic inversion to keep the specified distance and the relative angle (Das, et al., 2002). A problem of this method is that the control law of the follower robot includes the states of the leader robot. Then, a communication tool between the leader and the follower robots is required to implement the follower's control law on real mobile robots. Otherwise, the navigation system is centralized (Das, et al., 2002; Tanner, et al., 2004). When the number of mobile robots is increased, however, this centralized system is not convenient because the navigation system becomes complicated and the computational burden is increased.

To overcome this drawback, this paper proposes a new follower's control law, called the self-made follower input, in which the control law of the follower robot is given by itself. The state of the leader robot is estimated by the relative equation between the leader and the follower robots in the discrete-time domain. On the other hand, the control objective of the leader robot is to track a specified path under a nonholonomic constraint of a wheeled mobile robot. To do this, the equation of motion of the leader robot is transformed into a chained system (Samson, 1995) and a control law is then designed (Sordalen and Egeland, 1995). The effectiveness of the proposed techniques is demonstrated in numerical simulation.

2. PATH TRACKING OF LEADER ROBOT

2.1 Equations of motion of mobile robot.

Let us consider a mobile robot which moves on the two-dimensional plane. The pair of $(x(t), y(t))$ represents the position of the mobile robot by the Cartesian coordinates where t is time. Its direction angle is $\theta(t)$, $(-\pi \leq \theta(t) < \pi$ [rad]), which is measured from the x-axis. The linear and the angular velocities are respectively $v(t)$ and $\omega(t)$. The equation of motion of the mobile robot is given by

$$\begin{cases} \dot{x}(t) = v(t)\cos\theta(t) \\ \dot{y}(t) = v(t)\sin\theta(t) \\ \dot{\theta}(t) = \omega(t) \end{cases} \tag{1}$$

$v(t)$ and $\omega(t)$ are the inputs to the mobile robot.

In the leader-follower technique considered in this paper, the role of the leader robot R_l is to track a reference path as shown in Fig. 1. The reference path is drawn by the bold-solid-line. The direction of the reference path is shown by the arrow. Let P be the point on the path in which the distance between the robot and the path is the minimum. Define new Cartesian coordinates $(\tilde{x}, \tilde{y})$ on the reference path. The origin of the coordinates is located at P. The $\tilde{x}$-axis is defined as the direction of the path and its angle from x-axis is denoted as $\theta_t(t)$. The relative angle of $\theta(t)$ to $\theta_t(t)$ is defined as

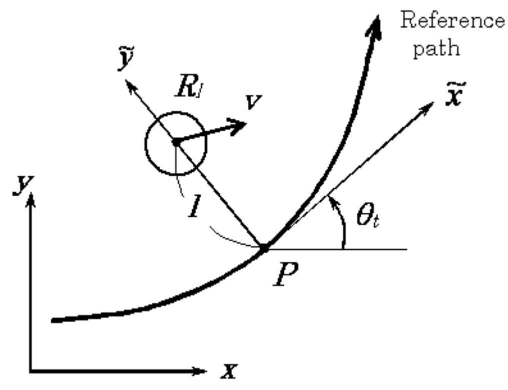

Fig. 1. Relative position and relative orientation of mobile robot to given reference path.

$$\theta_p(t) \triangleq \theta(t) - \theta_t(t). \tag{2}$$

Then, the position of the leader robot R_l is represented by $(0, l(t))$ on the $(\tilde{x}, \tilde{y})$ coordinates. Furthermore, let $s(t)$ be the traced length of point P where $s(0) = 0$. A positive $\dot{s}(t)$ means that point P moves to the direction of the path. Denoting the curvature of the reference path as $c(s)$, it is given by

$$c(s) = \frac{d\theta_t}{ds}. \tag{3}$$

Using Eq. (3), the derivative of $\theta_t(t)$ is given by

$$\dot{\theta}_t(t) = \frac{d\theta_t}{ds}\frac{ds}{dt} = c(s)\dot{s}(t). \tag{4}$$

The inverse of the absolute value of $c(s)$ means the radius of the curvature. The center of the curvature is located at $(\tilde{x}, \tilde{y}) = (0, 1/c(s))$. When $lc(s) \neq 1$, the following relation holds:

$$\frac{\dot{s}}{v\cos\theta_p} = \frac{1/c(s)}{1/c(s) - l} = \frac{1}{1 - lc(s)} \tag{5}$$

Using the above equations, Eq. (1) can be rewritten by the variables $s(t)$, $l(t)$ and $\theta_p(t)$ instead of $x(t)$, $y(t)$ and $\theta(t)$

$$\begin{cases} \dot{s}(t) = v(t)\frac{\cos\theta_p(t)}{1 - l(t)c(s)} \\ \dot{l}(t) = v(t)\sin\theta_p(t) \\ \dot{\theta}_p(t) = \omega(t) - v(t)\frac{c(s)\cos\theta_p(t)}{1 - l(t)c(s)} \end{cases} \tag{6}$$

Then, the purpose of the leader robot is that $l(t)$ and $\theta_p(t)$ asymptotically converge to zero while $s(t)$ is increased.

2.2 Transform to chained system.

In this section, Eq. (6) is transformed into a chained system. The chained system used in this paper is referred from Samson (1995) and Sastry (1999). The process of the transformation is based on the theories of nonlinear systems which are related to Lie derivatives, the involution and Frobenius' theorem (Sastry, 1999) Since this paper just uses the chained system for tracking control of the leader robot, the rest of this section shows the variable transformation into the chained system.

To transform to a chained system, the following variables are defined (Samson, 1995):

$$\begin{cases} z_1 \triangleq s \\ z_2 \triangleq (1-lc)\tan\theta_p \\ z_3 \triangleq l \end{cases} \tag{7}$$

$$\begin{cases} u_1 \triangleq \frac{\cos\theta_p}{1-lc} v \\ u_2 \triangleq -\left(\frac{lc'\sin\theta_p}{1-lc} + \frac{c(\sin^2\theta_p+1)}{\cos\theta_p}\right) v \\ \qquad + \frac{1-lc}{\cos^2\theta_p}\omega \end{cases} \tag{8}$$

where c' is the derivative of $c(s)$ with respect to s. Differentiating Eq. (7) by t, it is confirmed that the following chained relation is obtained.

$$\begin{cases} \dot{z}_1 = u_1 \\ \dot{z}_2 = u_2 \\ \dot{z}_3 = z_2 u_1 \end{cases} \tag{9}$$

When $\theta_p = \pm\pi/2$ and/or $lc = 1$, Eq. (8) is singular. That is, v and ω can not be obtained from the control law u_1 and u_2, which will be given in the following section. If the leader robot is trapped in such singular situations, v and ω are fixed at the values just before becoming singular until the robot escapes from the singularity. It is just a short time because the singular situation is restricted in $\theta_p = \pm\pi/2$ and/or $lc = 1$.

2.3 Control law of leader robot.

This section presents a control law for Eq. (9) that the leader robot is tracked to a reference path. The purpose of the leader robot is that $z_2(t)$ and $z_3(t)$ asymptotically converge to zero, while $z_1(t)$ is increased. To do this, the inputs of leader robot in the chained system u_1 and u_2 are given by (Sordalen and Egeland, 1995)

$$\begin{cases} u_1 = v_0 \\ u_2 = -k_2 z_2 - (k_3/v_0) z_3 \end{cases} \tag{10}$$

where v_0 is the nominal linear velocity of the leader robot on the reference path. Substituting Eq. (10) into

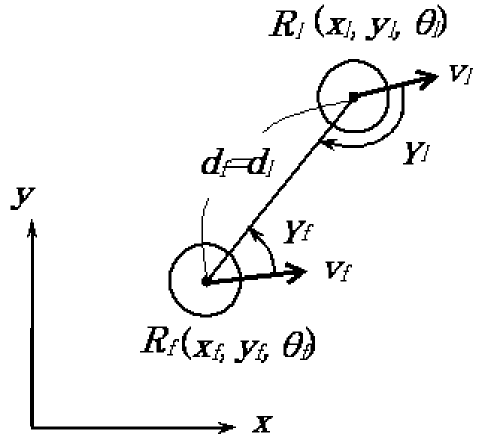

Fig. 2. Relative position and relative orientation between a leader and a follower robots.

Eq. (9), the closed-loop system is

$$\begin{cases} \dot{z}_1 = v_0 \\ \begin{pmatrix} \dot{z}_2 \\ \dot{z}_3 \end{pmatrix} = \begin{pmatrix} -k_2 & -k_3/v_0 \\ v_0 & 0 \end{pmatrix} \begin{pmatrix} z_2 \\ z_3 \end{pmatrix} \end{cases} \tag{11}$$

Thus, selecting k_2 and k_3 such that matrix of Eq. (11) is Hurwitz, $z_2(t)$ and $z_3(t)$ asymptotically converge to zero while $z_1(t)$ is proportionally increased.

3. FORMATED NAVIGATION BY LEADER-FOLLOWER TECHNIQUE

Let us consider a situation shown in Fig. 2. A leader and a follower robot are denoted as R_l and R_f, respectively. According to the notation defined in Section 2, the states and the inputs of R_l and R_f are newly denoted as (x_l, y_l, θ_l), (x_f, y_f, θ_f), (v_l, ω_l) and (v_f, ω_f), where the subscript 'l' and 'f' mean *leader* and *follower*, respectively. The relative distance between them is denoted as d_l $(= d_f)$ and the relative angles from their heading are denoted as γ_l and γ_f, respectively. Define the relative state and the input vectors of the leader and the follower robots as follows:

$$\xi_l \triangleq \begin{pmatrix} d_l \\ \gamma_l \end{pmatrix}, \ \xi_f \triangleq \begin{pmatrix} d_f \\ \gamma_f \end{pmatrix}, \ \rho_l \triangleq \begin{pmatrix} v_l \\ \omega_l \end{pmatrix}, \ \rho_f \triangleq \begin{pmatrix} v_f \\ \omega_f \end{pmatrix}$$

Then, the equations which express the relative relationships between the leader and the follower robots are given by

$$\dot{\xi}_l = A(\xi_l)\rho_l + b(\xi_f, v_f) \tag{12}$$

$$\dot{\xi}_f = A(\xi_f)\rho_f + b(\xi_l, v_l) \tag{13}$$

where

$$A(\xi) \triangleq \begin{pmatrix} -\cos\gamma & 0 \\ \frac{\sin\gamma}{d} & -1 \end{pmatrix}, \quad b(\xi, v) \triangleq \begin{pmatrix} -v\cos\gamma \\ \frac{v\sin\gamma}{d} \end{pmatrix}.$$

The purpose of the formated navigation is that a bunch of mobile robots navigates an environment with keeping a specified relative distance d^{rel} and a specified relative angle γ^{rel}. The leader robot is controlled to track a reference path as mentioned in the previous section. Therefore, the tracking control for the formation is done by the follower robot. Letting $\xi^{rel} \triangleq (d^{rel} \;\; \gamma^{rel})^T$ be the specified relative state of the follower robot ξ_f, Eq. (13) is controlled so that ξ_f converges to ξ^{rel}. When $A(\xi_f)$ is nonsingular, ρ_f is given by (Das, et al., 2002)

$$\rho_f = A^{-1}(\xi_f)[\mu_f - b(\xi_l, v_l)]. \tag{14}$$

μ_f is a new input vector to regulate ξ_f to ξ^{rel} in the sense of the linear differential equations. It is given by

$$\mu_f = \begin{pmatrix} k_d & 0 \\ 0 & k_\gamma \end{pmatrix} (\xi^{rel} - \xi_f) \tag{15}$$

where k_d and k_γ are positive constant values. Then, ξ_f asymptotically converges to ξ^{rel}.

When $\gamma_f = \pm\pi/2$, matrix $A(\xi_f)$ becomes singular. $\gamma_f = \pm\pi/2$ means that the leader robot is located in the left or right hand side of the follower one. As a counterplan to avoid this singularity, γ^{rel} should be given except $\gamma^{rel} = \pm\pi/2$. Nevertheless, the relative angle is near a singular situation during navigation, the follower input is fixed at the value just before becoming nonsingular until $A(\xi_f)$ becomes nonsingular.

4. MODIFIED LEADER-FOLLOWER TECHNIQUE IN DISCRETE-TIME

In the leader-follower technique described in Section 3, the follower input ρ_f given by Eq. (14) includes the relative leader state ξ_l and the velocity v_l. The follower robot has to know these variables to calculate the follower input. Then, a communication tool is required to implement the follower's control law on real mobile robots. Otherwise, the navigation system is centralized; that is, all states and inputs are corrected to a client computer and the follower input is calculated. As a matter of fact, Das et al. (2002) and Tanner et al. (2004) adopted a centralized control system to implement the leader-follower technique on real mobile robots. When the number of mobile robots is increased, however, this is not convenient because the control system becomes complicated and the computational burden is increased. To overcome this drawback, this paper proposes a technique that the follower input is generated by itself. It is given in the discrete-time because a control law is implemented on a digital computer of the mobile robots.

4.1 *Discretized control law.*

The relative equation (13) and the control law (14) of the follower robot based on the dynamic inversion are expressed in the discrete-time domain. Letting T_s be the sampling time, Eq. (13) is approximately discretized by

$$\frac{\xi_f[k+1] - \xi_f[k]}{T_s} = A(\xi_f[k])\rho_f[k] + b(\xi_l[k], v_l[k]) \tag{16}$$

where k means the sampling number and has the relation with time t; $t = kT_s$. The follower input is written as

$$\rho_f[k] = A^{-1}(\xi_f[k])[\mu_f[k] - b(\xi_l[k], v_l[k])] \tag{17}$$

where $\mu_f[k]$ is given by Eq. (15). Substituting Eqs. (15) and (17) into Eq. (16), the closed-loop system is

$$\begin{cases} d_f[k+1] = (1 - k_d T_s) d_f[k] + k_d T_s d^{rel} \\ \gamma_f[k+1] = (1 - k_\gamma T_s)\gamma_f[k] + k_\gamma T_s \gamma^{rel} \end{cases} \tag{18}$$

If k_d and k_γ are selected as

$$0 < k_d, \;\; k_\gamma < \frac{2}{T_s} \tag{19}$$

then, $d_f[k]$ and $\gamma_f[k]$ asymptotically converge to their references d^{rel} and γ^{rel}.

4.2 *Self-made follower input.*

The second term of Eq. (17) consists of the relative state and the relative velocity of the leader robot. To generate the follower input Eq. (17) by itself, this paper proposes a method in which the second term of Eq. (17) is estimated by using Eq. (16) as a predictor. The followings are assumed to be satisfied in the formated navigation considered in this paper.

(A1) The follower robot is able to measure the relative distance $d_f[k]$ and the relative angle $\gamma_f[k]$ at each sampling by distance sensors such as sonar, infrared sensor etc.

(A2) The leader robot moves on the reference path. Then, the linear velocity of the leader robot is constant; that is,

$$v_l[k] = v_l[k-1] (= v_0) \tag{20}$$

(A3) The change of the relative angle of the leader robot during one-sample time is *sufficiently small*.

As a simplified example that assumptions (A2) and (A3) hold, consider a situation in which the leader robot moves on the reference path and the relative distance is not changed during one sampling interval. Then, $l = 0$ and $\theta_p = 0$ in Eq. (6). In the continuous-time domain, the following relation holds.

$$\dot{\theta}_l = \dot{\theta}_t = c\dot{s} = cv_0 \tag{21}$$

The derivative of the relative angle γ_l is almost the same as that of the direction angle; that is, $\dot{\gamma} \simeq \dot{\theta}_l$. Discretizing Eq. (21), $\gamma_l[k]$ is approximately given by

$$\gamma_l[k] \simeq \gamma_l[k-1] + cv_0 T_s \tag{22}$$

Assumption (A3) means that $|cv_0 T_s|$ is *sufficiently small*. It depends on the shape of the reference path c, nominal linear velocity v_0 and the sampling time T_s. To be concrete, assumption (A3) indicates that the following relations hold:

$$\cos\gamma_l[k] \simeq \cos\gamma_l[k-1] - cv_0 T_s \sin\gamma_l[k-1] \tag{23}$$
$$\sin\gamma_l[k] \simeq \sin\gamma_l[k-1] + cv_0 T_s \cos\gamma_l[k-1] \tag{24}$$

Under assumptions (A2) and (A3), $b(\xi_l[k], v_l[k])$ is expressed as

$$\begin{aligned} b(\xi_l[k], v_l[k]) &= L[k]h[k]v_l[k] \\ &\simeq \{L[k]h[k-1] + \phi[k]\}\, v_l[k-1] \end{aligned} \tag{25}$$

where

$$L[k] \triangleq \begin{pmatrix} 1 & 0 \\ 0 & 1/d_f[k] \end{pmatrix}, \quad h[k] \triangleq \begin{pmatrix} -\cos\gamma_l[k] \\ \sin\gamma_l[k] \end{pmatrix}$$

Since the magnitude of the elements of $h[k-1]$ is equal to or less than one, $\phi[k]$ is expressed as

$$\phi[k] = \begin{pmatrix} \varepsilon \\ \varepsilon/d_f[k] \end{pmatrix}, \quad \varepsilon \le |cv_0 T_s| \tag{26}$$

Equation (16) is approximately rewritten as

$$\begin{aligned} \frac{\xi_f[k+1] - \xi_f[k]}{T_s} &= A(\xi_f[k])\rho_f[k] \\ &\quad + \{L[k]h[k-1] + \phi[k]\}\, v_l[k-1] \end{aligned} \tag{27}$$

Then, using $L[k]h[k-1]v_l[k-1]$ instead of $b(\xi_l[k], v_l[k])$ in Eq. (17), the follower input is given by

$$\rho_f[k] = A^{-1}(\xi_f[k]) \left(\mu_f[k] - L[k]h[k-1]v_l[k-1] \right). \tag{28}$$

The close-loop system is

$$\begin{cases} d_f[k+1] = (1 - k_d T_s) d_f[k] \\ \qquad + T_s(k_d d^{rel} + \varepsilon v_l[k-1]) \\ \gamma_f[k+1] = (1 - k_\gamma T_s)\gamma_f[k] \\ \qquad + T_s(k_\gamma \gamma^{rel} + \varepsilon v_l[k-1]/d_f[k]) \end{cases} \tag{29}$$

The stability condition of Eq. (29) is the same as that of Eq. (18) and is given by Eq. (19). $d_f[k]$ and $\gamma_f[k]$ approximately converge to

$$d_f \to d^{rel} + \frac{\varepsilon v_0}{k_d} \tag{30}$$

$$\gamma_f \to \gamma^{rel} + \frac{\varepsilon v_0}{k_\gamma d^{rel}}. \tag{31}$$

The tracking error; that is, the deviations from the references are bounded by the second terms of Eqs. (30) and (31). Thus, under assumptions (A2) and (A3), the follower input given by Eq. (28) is acceptable from the viewpoints of the stability and tracking error.

The rest of the problem is how $h[k-1]$ is obtained. In this paper, Eq. (16) is used as a predictor of $h[k-1]$. That is,

$$\begin{aligned} h[k-1] &= v_l^{-1}[k-1]L^{-1}[k-1] \\ &\quad \times \Big(\frac{\xi_f[k] - \xi_f[k-1]}{T_s} \\ &\qquad - A(\xi_f[k-1])\rho_f[k-1] \Big) \end{aligned} \tag{32}$$

Substituting Eq. (32) into Eq. (28), the follower input is given by

$$\begin{aligned} \rho_f[k] &= A^{-1}(\xi_f[k])\mu_f[k] \\ &- \{L^{-1}[k]A(\xi_f[k])\}^{-1} L^{-1}[k-1] \frac{\xi_f[k] - \xi_f[k-1]}{T_s} \\ &+ \{L^{-1}[k]A(\xi_f[k])\}^{-1} \\ &\times \{L^{-1}[k-1]A(\xi_f[k-1])\}\rho_f[k-1] \end{aligned} \tag{33}$$

Thus, the follower input $\rho_f[k]$ is obtained from the new input $\mu_f[k]$, the relative follower state $\xi_f[k]$ and the previous input $\rho_f[k-1]$, but not require the leader state ξ_l and the velocity v_l. In this paper, Eq. (33) is called self-made follower input. Equation (33) is valid when assumptions (A2) and (A3) are satisfied. Before the leader robot does not closely approach the reference path, the formation may be not formed by Eq. (33).

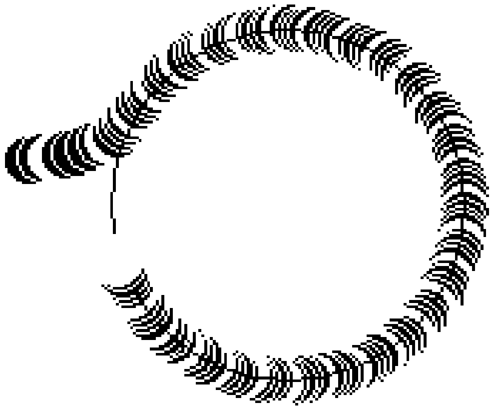

Fig. 3. Trace of leader robot where reference path is given by a circle.

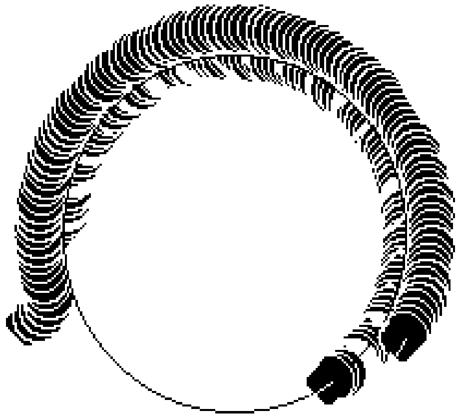

Fig. 4. Traces of leader and follower robots using proposed formated navigation.

5. SIMULATION OF FORMATED NAVIGATION

This section shows numerical simulations to verify the tracking control law of the leader robot in Section 2. Figure 3 shows the trace of the leader robot where the reference path was given by a circle. The the robot successfully tracked the reference path. The relative distance and the relative angle smoothly converged to zero.

Figure 4 shows the traces of leader and follower robots by using proposed formated navigation. The follower robot followed the leader one with keeping the specified relative distance and the relative angle.

6. CONCLUDING REMARKS

This paper has presented a formated navigation based on the leader-follower approach. In this approach, the control law of the follower robot included the states of the leader robot. This was a drawback for implementing the control laws on real mobile robots. To overcome this drawback, this paper proposed a new follower's control law, called the self-made follower input, in which the state of the leader robot was estimated by the relative equation between the leader and the follower robots in the discrete-time domain. On the other hand, a control law of the leader robot was designed with a chained system to track a given reference path. The effectiveness of the proposed techniques was demonstrated in numerical simulation.

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