ROBUST FAULT DIAGNOSIS OF NONLINEAR SYSTEMS BASED ON AN UNKNOWN INPUT EXTENDED KALMAN OBSERVER

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Abstract: Unknown input observer is one of the most important strategies for robust fault diagnosis of linear systems. Inspired by the unknown input Kalman filter, we extend it to nonlinear cases as the EKF does. Using this as a nonlinear observer, we prove its convergence under some mild conditions. As a result, a robust FDI strategy for nonlinear systems is obtained. Simulation studies on a three-tank system "DTS200" demonstrate the effectiveness of the proposed approach.

Keywords: Fault diagnosis, Kalman filters, Disturbance rejection, Convergence analysis, Lyapunov methods.

1. INTRODUCTION

With increasing demand on safety in industrial processes, fault detection and isolation (FDI) has received much more attention in the past three decades, in which one important strategy is based on analytical models. The most frequently used FDI approaches based on analytical models include observers, parity space, Kalman filter, parameter estimation, and so on (Venkatasubramanian et al., 2003). Due to the universal existence of nonlinearity and model uncertainty in practice, robust FDI of nonlinear systems is of great significance. Since model uncertainties are unexpected dynamics of the system as well as faults, they constitute a source of false alarms which corrupt the performance of the FDI system to such an extent that it may even become totally useless. As is well-known, nonlinearity is a fundamental problem in mathematics, which causes many difficulties in the construction of FDI systems such as the design of observers.

The robustness of FDI systems is one of the important research topics, in which unknown input observer scheme is very famous (Chen and Patton, 1999). The basic idea behind unknown input observer is to design a fault diagnosis observer which is decoupled with the unknown disturbance or structured model uncertainty. However, unknown input observer are mostly designed for linear systems, few nonlinear cases have been considered (Frank, 1990). It is difficult to extend the results of linear systems with unknown input to general nonlinear cases, since the design of nonlinear observers is very difficult, especially in the case with unknown input. Another solution to robust FDI of nonlinear systems was proposed by Demetriou and Polycarpou (1998) using an online approximator. But it didn't consider the trade-off between the robustness to disturbances and the sensitivity to faults.

The EKF is very famous and has been widely used as an estimator for nonlinear systems (Anderson and Moore, 1979). The convergence of the EKF used as an observer for nonlinear deterministic discrete-time systems was first discussed by Boutayeb et al. (1997), and the results were improved later using Linear Matrix Inequality (LMI) technology (Boutayeb and Aubry, 1999). Based on some other assumptions, Guo and Zhu (2002) also analyzed the convergence of the EKF by formulating another Lyapunov function, and they proposed to use neural network to reduce the initial state estimation error, which could improve the convergent rate of the EKF.

On the other hand, unbiased minimum-variance linear state estimator for linear stochastic systems with unknown inputs has also been investigated (Kitanidis, 1987), which was called unknown input Kalman filter (UIKF). It was also analyzed by Darouach et al. (1995) from the view of singular systems, and Keller et al. (1998) gave a brief algorithm of Kitanidis's (1987) results, from which the relation to the classical Kalman filter were explicit.

In this paper, inspired by the results of UIKF, we extend the UIKF to nonlinear cases as the EKF does. The extension is intuitive and the convergence of this new state estimator as a nonlinear observer is analyzed. Then a new strategy of robust fault diagnosis for nonlinear systems based on this nonlinear estimator is proposed. Simulation studies on a three-tank system "DTS200" demonstrate that the proposed unknown input EKF is effective as an estimator for a class of nonlinear systems with structured unknown disturbance, and the results of fault diagnosis are satisfactory.

2. BRIEF REVIEW OF UNKNOWN INPUT KALMAN FILTER

The unknown input Kalman filter (UIKF) was first discussed by Kitanidis (1987). Based on the disturbance decoupled condition which is the same as that of the general unknown input observer, an optimal filter gain, which makes the state estimation covariance minimized, is obtained by solving an optimization problem with constraints using Lagrange method. The UIKF is a simple modification to the classical Kalman filter, which is explicitly demonstrated by Keller et al. (1998). The gain of the filter and the filtered state estimation variance are modified from the original form. Via the results of the optimal state estimation of singular systems, Darouach et al. (1995) derived another equivalent form of the UIKF.

In this section, we outline the formulation of Keller et al. (1998), based on which we will extend the results to nonlinear case in the next section. Some results of Darouach et al.(1995) will also be used for the convergence analysis in the next section due to the equivalence.

Consider the linear discrete-time stochastic system with unknown disturbance as follows,

$$\begin{cases} x_{k+1} = F_k x_k + B_k u_k + E_k d_k + w_k \\ y_k = H x_k + v_k \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the state vector; $y_k \in \mathbb{R}^m$ is the observation vector; $u_k \in \mathbb{R}^r$ is the known input vector; $d_k \in \mathbb{R}^q$ is the unknown input vector which represents the unknown disturbances or model uncertainties; the system and measurement noises (w_k and v_k) are zero mean uncorrelated random Gaussian sequences with variance matrix Q_k and R_k, respectively. F_k, B_k, E_k and H are known matrices with proper dimensions.

The optimal linear filter for system (1) is given by,

$$\hat{x}_{k+1|k} = F_k \hat{x}_{k|k} + B_k u_k \tag{2}$$

$$P_{k+1|k} = F_k P_{k|k} F_k^T + Q_k \tag{3}$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + L_{k+1}(y_{k+1} - H\hat{x}_{k+1|k}) \tag{4}$$

$$P_{k+1|k+1} = (I - K_{k+1}H)P_{k+1|k} + \eta_{k+1}\Pi_{k+1}V_{k+1}\Pi_{k+1}^T\eta_k^T \tag{5}$$

with

$$L_{k+1} = K_{k+1} + \eta_{k+1}\Pi_{k+1} \tag{6}$$

$$K_{k+1} = P_{k+1|k} H^T V_{k+1}^{-1} \tag{7}$$

$$\eta_{k+1} = (I - K_{k+1}H)E_k \tag{8}$$

$$\Pi_{k+1} = [(HE_k)^T V_{k+1}^{-1} (HE_k)]^{-1} (HE_k)^T V_{k+1}^{-1} \tag{9}$$

$$V_{k+1} = HP_{k+1|k}H^T + R_{k+1} \tag{10}$$

From above it is obvious to notice the relation of the UIKF to the classical Kalman filter. K_{k+1} in (7) and the first part of (5) are the gain matrix and the filtered state estimation covariance in the classical Kalman filter, respectively. Therefore to design an UIKF, one only need to modify these two matrices by adding a term respectively, which is constructed by η_{k+1} and Π_{k+1}, and compensates the effect of unknown input, to those matrices of the classical Kalman filter.

In the UIKF, the gain matrix L_{k+1} satisfies the decoupling condition which is the same as that in the general unknown input observer (Kitanidis, 1987):

$$L_{k+1}HE_k = E_k \tag{11}$$

And equation (11) has a solution if and only if (Chen and Patton, 1999)

$$\text{rank}(HE_k) = \text{rank}(E_k) = q \tag{12}$$

3. EXTENSION TO NONLINEAR CASE AND CONVERGENCE ANALYSIS

Consider the following nonlinear discrete-time system with unknown disturbance,

$$\begin{cases} x_{k+1} = f(x_k, u_k) + E(x_k)d_k \\ y_k = Hx_k \end{cases} \tag{13}$$

where f and E are assumed to be smooth. For simplicity we investigate deterministic cases here, and the measurement equation is also linear and constant, which is general in practice, such as process control. Notice that the disturbance distribution matrix $E(x_k)$ in (13) is assumed to be related to current state x_k, which is different from that in (1) and is more general in parctice. And we assume that $E(x_k)$ satisfies the condition (11) for all feasible x_k, which ensure the total decoupling of disturbances.

Then like the EKF, we extend the UIKF to (13) as an nonlinear observer.

$$\hat{x}_{k+1|k} = f(\hat{x}_{k|k}, u_k) \quad (14)$$

$$P_{k+1|k} = F_k P_{k|k} F_k^T + Q_k \quad (15)$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + L_{k+1}(y_{k+1} - H\hat{x}_{k+1|k}) \quad (16)$$

$$P_{k+1|k+1} = (I - K_{k+1}H)P_{k+1|k} + \eta_{k+1}\Pi_{k+1}V_{k+1}\Pi_{k+1}^T\eta_k^T \quad (17)$$

with

$$L_{k+1} = K_{k+1} + \eta_{k+1}\Pi_{k+1} \quad (18)$$

$$K_{k+1} = P_{k+1|k}H^T V_{k+1}^{-1} \quad (19)$$

$$\eta_{k+1} = (I - K_{k+1}H)\hat{E}_k \quad (20)$$

$$\Pi_{k+1} = [(H\hat{E}_k)^T V_{k+1}^{-1}(H\hat{E}_k)]^{-1}(H\hat{E}_k)^T V_{k+1}^{-1} \quad (21)$$

$$V_{k+1} = HP_{k+1|k}H^T + R_{k+1} \quad (22)$$

where

$$F_k = \partial f / \partial x \big|_{(\hat{x}_{k|k}, u_k)} \quad (23)$$

$$\hat{E}_k = E(\hat{x}_{k|k}) \quad (24)$$

Compared to the linear case, the unknown input extended Kalman observer (UIEKO) is different from the UIKF in the calculation of the predicted state and the matrices F_k and E_k. Similar to the case that the EKF used as an observer, $Q_k \geq 0$ and $R_{k+1} > 0$ are arbitrarily chosen to ensure convergence.

Next we shall discuss the convergence of the UIEKO proposed above by following the work of Guo and Zhu (2002).

Lemma 1. For the UIEKO formulated by (14)-(24), we have the following equivalent relations:

$$P_{k+1|k+1}^{-1} = P_{k+1|k}^{-1} + H^T R_k^{-1} H - P_{k+1|k}^{-1}\hat{E}_k(\hat{E}_k^T P_{k+1|k}^{-1}\hat{E}_k)^{-1}\hat{E}_k^T P_{k+1|k}^{-1} \quad (25)$$

$$L_{k+1} = P_{k+1|k+1}H^T R_{k+1}^{-1} \quad (26)$$

Similar relations are also existed in the classical Kalman filter. Here for briefness we omit the detailed proof, and only point out that these relations can be obtained from the results of Darouach et al.(1995) and Keller et al. (1998). These two works both proposed the optimal filter for linear systems with unknown input from different views, but the results are equivalent due to the equivalence of the optimum. The formulation of (14)-(24) comes from the result of Keller et al. (1998), while the relations (25)-(26) can be found in Darouach et al.(1995).

Lemma 2. Given any symmetric positive definite matrix $P \in \mathbb{R}^{n\times n}$ and any full column rank matrix $E \in \mathbb{R}^{n\times m}$ ($m \leq n$), we have the following conclusions:
(1) $M := P - PE(E^T PE)^{-1}E^T P$ is semi-positive definite;
(2) $\|M\| \leq \|P\|$.

Proof. It is obvious that E has a singular decomposition as follows,

$$E = U^T \begin{bmatrix} \Sigma \\ 0 \end{bmatrix} V \quad (27)$$

where $\Sigma \in \mathbb{R}^{m\times m}$ is a diagonal matrix, and U, V are orthogonal matrices with proper dimensions. Let $\bar{P} = UPU^T = \begin{bmatrix} \bar{P}_{11} & \bar{P}_{12} \\ \bar{P}_{21} & \bar{P}_{22} \end{bmatrix}$ ($\bar{P}_{11} \in \mathbb{R}^{m\times m}$), then,

$$\begin{aligned} M &= P - PU^T \begin{bmatrix} \Sigma \\ 0 \end{bmatrix} V \\ &\quad \cdot (V^T \begin{bmatrix} \Sigma & 0 \end{bmatrix} \bar{P} \begin{bmatrix} \Sigma \\ 0 \end{bmatrix} V)^{-1} V^T \begin{bmatrix} \Sigma & 0 \end{bmatrix} UP \\ &= P - PU^T \begin{bmatrix} \Sigma \\ 0 \end{bmatrix} \Sigma^{-1}\bar{P}_{11}^{-1}\Sigma^{-1} \begin{bmatrix} \Sigma & 0 \end{bmatrix} UP \\ &= U^T(\bar{P} - \bar{P}\begin{bmatrix} \bar{P}_{11}^{-1} & 0 \\ 0 & 0 \end{bmatrix}\bar{P})U \\ &= U^T \begin{bmatrix} 0 & 0 \\ 0 & \bar{P}_{22} - \bar{P}_{21}\bar{P}_{11}^{-1}\bar{P}_{12} \end{bmatrix} U \end{aligned} \quad (28)$$

Since

$$\begin{aligned} \bar{P}^{-1} &= \begin{bmatrix} \bar{P}_{11} & \bar{P}_{12} \\ \bar{P}_{21} & \bar{P}_{22} \end{bmatrix}^{-1} \\ &= \begin{bmatrix} (\bar{P}_{11} - \bar{P}_{12}\bar{P}_{22}^{-1}\bar{P}_{21})^{-1} & -(\bar{P}_{11} - \bar{P}_{12}\bar{P}_{22}^{-1}\bar{P}_{21})^{-1}\bar{P}_{12}\bar{P}_{22}^{-1} \\ -(\bar{P}_{22} - \bar{P}_{21}\bar{P}_{11}^{-1}\bar{P}_{12})^{-1}\bar{P}_{21}\bar{P}_{11}^{-1} & (\bar{P}_{22} - \bar{P}_{21}\bar{P}_{11}^{-1}\bar{P}_{12})^{-1} \end{bmatrix} \end{aligned} \quad (29)$$

Then $\bar{P}_{22} - \bar{P}_{21}\bar{P}_{11}^{-1}\bar{P}_{12} > 0$, i.e. M is semi-positive definite. Besides, it's obvious that $\lambda_{\min}((\bar{P}_{22} - \bar{P}_{21}\bar{P}_{11}^{-1}\bar{P}_{12})^{-1}) \geq \lambda_{\min}(\bar{P}^{-1})$ which means $\|M\| \leq \|P\|$. □

Let $\Delta f(x,u) := \partial f / \partial x\big|_{(x,u)}$ (i.e. $F_k = \Delta f(\hat{x}_{k|k}, u_k)$) and $\Delta \mathrm{E}(x,d) := \partial(E(x)d)/\partial x\big|_{(x,d)}$.

Assumption 3.
$\|\Delta f\| := \sup\{\|\Delta f(x,u)\|, \text{for all feasible } x \text{ and } u\}$ and $\|\Delta \mathrm{E}\| := \sup\{\|\Delta \mathrm{E}(x,d)\|, \text{for all feasible } x \text{ and } d\}$ exist and are finite.

Assumption 4. The state estimation error covariance $P_{k|k}$ of the UIEKO (14)-(24) is uniformly bounded, i.e. $\forall k > 0$, there exist $0 < p, q < \infty$, such that $\|P_{k|k}\| \leq p$ and $\|P_{k|k}^{-1}\| \leq q$.

Theorem 5. Consider system (13) and its UIEKO (14)-(24). Suppose Assumption*s* 3 and 4 hold. Let ς ($0 < \varsigma < 1$) be arbitrary positive constant. If for each $k > 0$ the following inequality satisfies,

$$\|(F_k P_{k|k} F_k^T + Q_k)^{-1}\| \leq \frac{1 - \varsigma}{(\|\Delta f\| + \|\Delta \mathrm{E}\|) \cdot \|P_{k|k}\|} \quad (30)$$

then the UIEKO (14)-(24) is asymptotic convergent.

Proof. Let $\tilde{x}_k = x_k - \hat{x}_{k|k}$ and select Lyapunov function $V_{k+1} = \|\tilde{x}_{k+1}\| / \|P_{k+1|k+1}\|$. Then by the smoothness of f and E, there exist two points $\hat{x}_{k|k} + \xi_k^f$ and $\hat{x}_{k|k} + \xi_k^E \in \mathbb{R}^n$ between x_k and $\hat{x}_{k|k}$, such that:

$$
\begin{aligned}
&x_{k+1}-\hat{x}_{k+1|k}\\
&=f(x_k,u_k)+E(x_k)d_k-f(\hat{x}_{k|k},u_k)\\
&=\Delta f(\hat{x}_{k|k}+\xi_k^f,u_k)\tilde{x}_k+E(x_k)d_k-E(\hat{x}_k)d_k+E(\hat{x}_k)d_k\\
&=\left(\Delta f(\hat{x}_{k|k}+\xi_k^f,u_k)+\Delta \mathrm{E}(\hat{x}_{k|k}+\xi_k^E,d_k)\right)\tilde{x}_k+\hat{E}_k d_k \qquad (31)
\end{aligned}
$$

Then:

$$
\begin{aligned}
\tilde{x}_{k+1}&=x_{k+1}-\hat{x}_{k+1|k+1}\\
&=x_{k+1}-\hat{x}_{k+1|k}-L_{k+1}(y_{k+1}-H\hat{x}_{k+1|k})\\
&=(I-L_{k+1}H)(x_{k+1}-\hat{x}_{k+1|k})\\
&=(I-L_{k+1}H)\left(\Delta f(\hat{x}_{k|k}+\xi_k^f,u_k)+\Delta \mathrm{E}(\hat{x}_{k|k}+\xi_k^E,d_k)\right)\tilde{x}_k\\
&\quad+(I-L_{k+1}H)\hat{E}_k \qquad (32)
\end{aligned}
$$

By condition (11) we know that $(I-L_{k+1}H)\hat{E}_k=0$. According to Lemma 1 we have:

$$
\begin{aligned}
&I-L_{k+1}H\\
&=P_{k+1|k+1}(P_{k+1|k+1}^{-1}-H^T R_{k+1}^{-1}H)\\
&=P_{k+1|k+1}(P_{k+1|k}^{-1}-P_{k+1|k}^{-1}\hat{E}_k(\hat{E}_k^T P_{k+1|k}^{-1}\hat{E}_k)^{-1}\hat{E}_k^T P_{k+1|k}^{-1}) \qquad (33)
\end{aligned}
$$

Then it follows from (32), (33) and Lemma 2 that:

$$
\begin{aligned}
V_{k+1}&=\|\tilde{x}_{k+1}\|/\|P_{k+1|k+1}\|\\
&\le \left\|P_{k+1|k}^{-1}-P_{k+1|k}^{-1}\hat{E}_k(\hat{E}_k^T P_{k+1|k}^{-1}\hat{E}_k)^{-1}\hat{E}_k^T P_{k+1|k}^{-1}\right\|\\
&\qquad\cdot(\|\Delta f\|+\|\Delta \mathrm{E}\|)\|\tilde{x}_k\|\\
&\le \|P_{k+1|k}^{-1}\|(\|\Delta f\|+\|\Delta \mathrm{E}\|)\|P_{k|k}\|V_k\\
&\le \|(F_k P_{k|k}F_k^T+Q_k)^{-1}\|(\|\Delta f\|+\|\Delta \mathrm{E}\|)\|P_{k|k}\|V_k \qquad (34)
\end{aligned}
$$

If (30) is satisfied, it gives:

$$
V_{k+1}\le(1-\varsigma)V_k \qquad (35)
$$

i.e. V_k is an exponential decreasing series. Then according to Assumption 4:

$$
\begin{aligned}
\|\tilde{x}_k\|&=\|P_{k|k}\|V_k\le(1-\varsigma)^k\|\tilde{x}_0\|\cdot(\|P_{k|k}\|/\|P_{0|0}\|)\\
&\le pq(1-\varsigma)^k\|\tilde{x}_0\| \qquad (36)
\end{aligned}
$$

Since $p,q,\|\tilde{x}_0\|<\infty$, $\lim_{k\to\infty}\|\tilde{x}_k\|=0$. □

Remark 6. $\|\Delta f\|$ and $\|\Delta \mathrm{E}\|$ reflect the magnitude of the linearization error of $f(\cdot)$ and the approximation error of E_k, respectively. It is obvious that the smaller $\|\Delta f\|$ and $\|\Delta \mathrm{E}\|$ are, the easier it is to ensure the convergence of the UIEKO, which is reasonable intuitively. For linear case described in section 2, where $\|\Delta \mathrm{E}\|=0$ and $\|\Delta f\|$ is replaced by $\|F_k\|$, the convergence can be analyzed by Riccati equation.

Remark 7. Since we construct an observer for a nonlinear deterministic system in this section, the matrix Q_k and R_{k+1} don't denote any real physical parameters as they do in stochastic cases. So they can be arbitrarily chosen. Here, from (30) it is easy to see that Q_k can be chosen larger to enlarge the convergent region. Boutayeb et al. (1997 & 1999) and Guo and Zhu (2002) all proposed proper selections of Q_k and R_{k+1}.

4. ROBUST FAULT DETECTION AND ISOLATION (FDI) STRATEGY

Robust FDI strategy of nonlinear systems using the UIEKO (14)-(24) is just like what is generally used in the unknown input observer scheme as before (Chen and Patton, 1999). In fact, both unknown disturbances and faults can be described in the form $E(x_k)d_k$, where d_k is the magnitude of unknown disturbances or the parameters of faults, and $E(\cdot)$ represents the distribution matrix of unknown disturbances or the type of faults. Assume that all unknown disturbances in the system are $E(x_k)d_k$, and faults are described by $F(x_k)\theta_k$, where $E(\cdot)\in\mathbb{R}^{n\times p}$, $F(\cdot)=\left[F^1(\cdot)\cdots F^s(\cdot)\right]\in\mathbb{R}^{n\times s}$ are assumed to be known, and $d_k\in\mathbb{R}^p$, $\theta_k:=[\theta_k^1\cdots\theta_k^s]^T\in\mathbb{R}^s$.

Assume that the unknown disturbances can be all decoupled, i.e. $E_k:=E(x_k)$ satisfies the decoupling conditions (11) and (12) for all feasible x_k. Then an UIEKO (called UIEKO_0) robust to all unknown disturbances can be constructed as proposed in section 3, and faults will be detected by comparing the residual of UIEKF_0 (γ_k^0) with a threshold ε^0.

One approach to fulfill the fault isolation task is to design a set of structured residuals. Each residual is designed to be sensitive to a subset of faults, whilst remaining robust to the remaining faults. The advantage of the structured residual set is that the diagnostic analysis is simplified to determining which residuals are non-zero. The threshold test may be performed separately for each residual, yielding a Boolean decision table, and the isolation task can be fulfilled using this table (Chen and Patton, 1999). Similar method was also used in Ge and Fang (1988).

A most commonly used scheme in designing the residual set is to make each residual sensitive to all but one fault. Design s observers each of which is robust to one fault, respectively besides disturbances. Call these observers UIEKO_*i* respectively, which is robust to the i^{th} fault with corresponding residual γ_k^i, i.e. UIEKO_*i* is designed for the system described by:

$$
\begin{cases}
x_{k+1}=f(x_k,u_k)+\left[E(x_k)\ \ F^i(x_k)\right]\cdot\begin{bmatrix}d_k\\ \theta_k^i\end{bmatrix}\\
y_k=Hx_k
\end{cases} \qquad (37)
$$

where $i=1,\cdots,s$. Thus the residuals satisfy:

$$
\begin{cases}
\gamma_k^1=R^1(F^2(x_k)\,\theta_k^2,\cdots,F^s(x_k)\,\theta_k^s)\\
\quad\vdots\\
\gamma_k^i=R^i(F^1(x_k)\,\theta_k^1,\cdots,F^{i-1}(x_k)\,\theta_k^{i-1},\\
\qquad\qquad F^{i+1}(x_k)\,\theta_k^{i+1},\cdots,F^s(x_k)\,\theta_k^s)\\
\quad\vdots\\
\gamma_k^s=R^s(F^1(x_k)\,\theta_k^1,\cdots,F^{s-1}(x_k)\,\theta_k^{s-1})
\end{cases} \qquad (38)
$$

where $R^i(\cdot)$ denotes the functional relation between faults and residuals obtained from UIEKO_*i* ($i=1,\cdots,s$). This is defined as a generalized residual set. The isolation task can be performed using simple threshold testing according to the following logic:

$$\left.\begin{array}{l}\gamma_k^i \le \varepsilon^i \\ \gamma_k^j > \varepsilon^j \ \forall j \ne i\end{array}\right\} \Rightarrow \text{The } i^{\text{th}} \text{ fault occurs} \qquad (39)$$

where ε^i ($i=1,\cdots,s$) are corresponding thresholds.

5. SIMULATIONS

The simulation model is the DTS200 three-tank system, which is a benchmark problem in process control engineering (Xie et al., 1999). Fig. 1 shows the layout of the setup.This setup consists of three plexiglas cylinders tank 1 (T1), tank 3 (T3) and tank 2 (T2) with the cross section A. These are connected serially with each other by cylindrical pipes with the cross section S_n. Located at tank 2 is the nominal outflow valve, it has a circular cross section S_n also. The outflowing liquid (usually distilled water) is collected in a reservoir, which supplies the pump1 and pump2. The necessary level measurements are carried out by three piezo-resistive differential pressure sensors and the reference pressure is the pressure of the atmosphere.

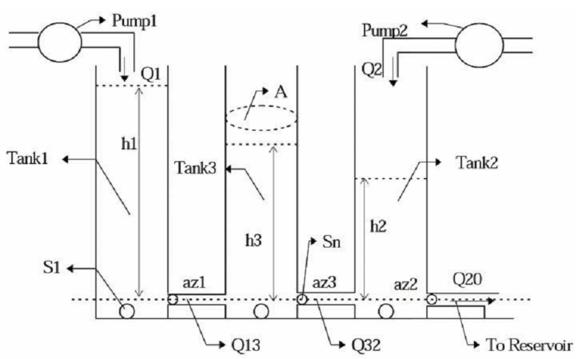

Fig. 1. The layout of DTS200 three-tank system

The dynamics are described as follows,

$$\begin{aligned} A\cdot dh_1/dt &= -Q_{13}+Q_1 \\ A\cdot dh_3/dt &= Q_{13}-Q_{32} \\ A\cdot dh_2/dt &= Q_{32}-Q_{20}+Q_2 \end{aligned} \qquad (40a)$$

where

$$\begin{aligned} Q_{13} &= az_1 S_n sign(h_1-h_3)\sqrt{2g|h_1-h_3|} \\ Q_{32} &= az_3 S_n sign(h_3-h_2)\sqrt{2g|h_3-h_2|} \\ Q_{20} &= az_2 S_n \sqrt{2gh_2} \end{aligned} \qquad (40b)$$

The state vector is the level of three tanks: $x=[h_1,h_2,h_3]^T$; the input vector is the controlled pump flow $u=[Q_1,Q_2]^T$; and the output vector is $y=[h_1,h_2]^T$. The actual parameters are $g=9.81\text{m/s}^2$, $az_1^0=0.5$, $az_3^0=0.45$, $az_2^0=0.6$, $A=0.0154\text{m}^2$, $S_n=5\times10^{-5}\text{m}^2$, $h_{\max}=62\pm1\text{cm}$, $Q_{1\max}=Q_{2\max}=100\text{ml/s}$. The levels of T1 and T2 are both controlled by PI controllers, respectively and the parameters are both $K_p=0.001$ (gain constant) and $T_I=5\text{s}$ (integral time constant). It is discretized by Euler method with sampling time $T=1\text{s}$.

We consider four types of faults here.

(1) Leakage in T1: $Q_{leak}^1 = az_1\pi r_1^2\sqrt{2gh_1}$.

(2) Leakage in T2: $Q_{leak}^2 = az_2\pi r_2^2\sqrt{2gh_2}$.

(3) Clogging between T1 and T3: $az_1=(1-\delta_1)az_1^0$.

(4) Clogging between T3 and T2: $az_3=(1-\delta_3)az_3^0$.

i.e. fault functions and the corresponding fault parameters are

$$F^1(x)=\left[-az_1\pi\sqrt{2gh_1}/A \quad 0 \quad 0\right]^T, \quad \theta^1=r_1^2 \qquad (41)$$

$$F^2(x)=\left[0 \quad -az_2\pi\sqrt{2gh_2}/A \quad 0\right]^T, \quad \theta^2=r_2^2 \qquad (42)$$

$$F^3(x)=\left[Q_{13}^0 \quad 0 \quad -Q_{13}^0\right]^T, \quad \theta^3=\delta_1 \qquad (43)$$

$$F^4(x)=\left[0 \quad -Q_{32}^0 \quad Q_{32}^0\right]^T, \quad \theta^4=\delta_3 \qquad (44)$$

where $r_1,r_2>0$; $0<\delta_1,\delta_3\le 1$; Q_{13}^0 and Q_{32}^0 are the values calculated in (40) by substituting az_1^0 and az_3^0 into it.

For simplicity we don't consider any other unknown disturbance here. Then we can design a general EKF as the fault detection observer. And as in section 4 proposed, four additional fault isolation observers are designed, each of which is robust to one specific type of faults, respectively. Fault isolation logic is obtained by (39).

In the following simulations, actual initial liquid levels are $h^0=[0.51 \quad 0.21 \quad 0.36]^T$ (unit: m), and the initial states of all five observers are taken to be $\hat{x}_{0|0}=[0.5 \quad 0.2 \quad 0.35]^T$ (unit: m). Assume that fault (1) occurs at 100s with $r_1=5\text{mm}$. Residuals of these five observers are illustrated in Fig. 2, and the FDI results are shown in Fig. 3.

From Fig. 2(a) it is obvious that after the fault occurs, the residual of EKF increases immediately, then fault is detected as is shown in Fig. 3(a). Fig. 2(b) demonstrates clearly the effectiveness of the disturbance (fault) decoupling by the UIEKO, whose $E(\cdot)$ match the real one. On the other hand, the residuals of other UIEKOs will depart from zero as well as EKF does, which are illustrated in Fig. 2(c)-(e). From Fig. 3(b)- (e), it can be seen that the fault isolation result is accurate, though there is some time delay comparing to fault detection. Notice that the residuals in Fig. 2 are large in the first few samples due to the error of initial estimation, which is easy to be excluded from faults.

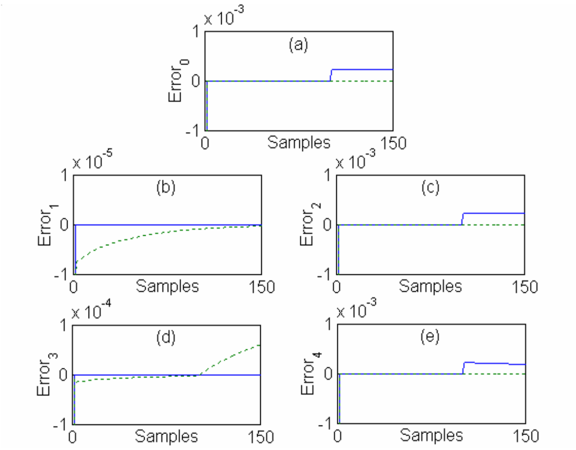

Fig. 2. Residuals of observers. (a) Residual of the fault detection observer, EKF. (b)-(e) Residual of the i^{th} fault isolation observer, UIEKO_*i* respectively ($i = 1,2,3,4$). (The solid and dash lines denote the first and second elements of the residual, respectively.)

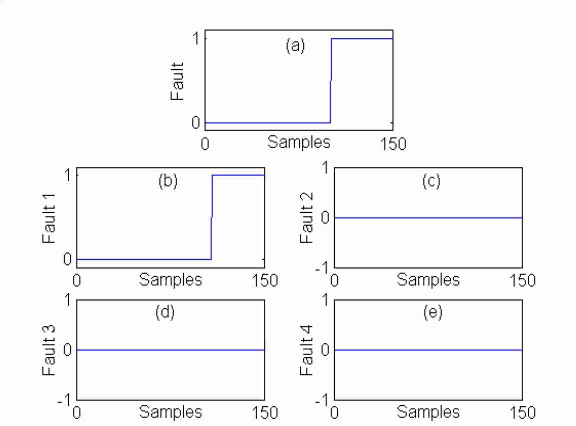

Fig. 3. Fault detection and isolation results. (a) Fault detection result. (b)-(e) Fault isolation results of each type of faults, respectively.

Due to page limitation, simulation results of fault (2)-(4) are omitted here.

6. CONCLUSIONS

Robust fault diagnosis of nonlinear systems is an important research area of FDI. However, there are few results in the past. One of the difficulties is to design a proper nonlinear observer. Inspired by the results of UIKF and based on the convergence analysis of the EKF as a nonlinear observer, we propose an algorithm of unknown input extended Kalman observer and analyze its convergence theoretically in this paper. As a result, the robust FDI strategy of nonlinear systems based on this nonlinear estimator is obtained. Simulations on the "DTS200" system show the good estimation performance of the UIEKO for the nonlinear system with some disturbances (faults), and the effectiveness of the FDI based on the UIEKO are also illustrated.

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