ADAPTIVE COORDINATED DECENTRALIZED CONTROL OF LARGE-SCALE SYSTEMS WITH MULTIVARIABLE SUBSYSTEMS

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Abstract: Exact decentralized output-feedback Lyapunov-based designs of direct MRAC for linear interconnected systems with MIMO subsystems are introduced. The design process uses a coordinated decentralized structure of adaptive control with reference model coordination. We develop decentralized MRAC on the base of *a priory* information about only the local subsystems gain frequency matrices without additional *a priory* knowledge about the full system gain frequency matrix. To achieve a better adaptation performance we propose proportional, integral time-delayed adaptation laws.

Keywords: Decentralized adaptive control, Coordination, Robust control.

1. INTRODUCTION

An increasing number of control problems for composite interconnected systems requires the use of an adaptive decentralized control structure with physically distributed controllers. These problems are found in various application areas, such as large-scale computer networks, power systems, automotive systems, web handling systems etc. In the case of plants with single-input single-output (SISO) interconnected subsystems, these problems are extensively studied and many important results have been obtained. The design has been based mainly on the following approaches: the traditional certainty equivalence approach or Morse's "dynamic certainty equivalence approach" (see, e.g., (Ioannou, 1986),(Gavel and Šiljak, 1989), (Ortega, 1996), (Mirkin, 1999), (Narendra and Oleng', 2002), (Mirkin, 2003)), and the nonlinear design tool based on recursive backstepping (see, e.g., (Jiang, 2000), (Krishmanurthy and Khorrami, 2003)).

However, for the conventional DMRAC scheme, these results do not appear to be easily applied to the case of composite systems with multi-input multi-output (MIMO) subsystems. One reason for this is that the type of *a priori* information pertaining to the overall MIMO plant structure is not as apparent in the case of MIMO subsystems as it is in the SISO case.

One of main difficulties of this case, even without the requirement of a decentralized control structure, is the generalization of the high frequency gain sign condition, since we deal with matrix gains instead of scalar gains. Current MIMO adaptive control algorithms for systems without a decentralized control structure require some a priori knowledge or constraints on the high-frequency gain matrix K_p of the overall plant (controlled object) $W(s)$. Most available results assume that either the high frequency matrix of the overall plant is known fully or partially, see e.g. (Sastry and Bodson, 1989), or satisfies some positive definiteness condition, see e.g. (Ioannou and Sun, 1996). To reduce the amount of *a priory* requirements to execute the design, some recent solutions based on Morse's fac-

[1] This work was supported by the Israel Science Foundation under Grant 38/03.

torizations of K_p were suggested. See, e.g. the recent monograph (Tao, 2003) for a summary of the state of the art.

We postulate that the controller is decentralized which creates information constraints. These pose control design difficulties in addition to those caused by the centralized case. The adaptive control problem becomes more complex in the decentralized case, and is yet an *unsolved problem*. To the authors' knowledge, results for composite systems with MIMO subsystems have not been reported in the literature.

We propose to design the decentralized controller without *a priory* knowledge about the full matrix K_p. Instead we assume some *a priory* knowledge about the subsystem matrices K_{pi}. The reason is that the transfer function of the overall controller has a *block-diagonal form*, and is not a full matrix as in the centralized case. Our concept of reference model coordination (Mirkin, 2003; Mirkin and Gutman, 2003) and recent advances in output feedback design for MIMO centralized systems, e.g. (Costa *et al.*, 2003; Tao, 2003) provide tools to overcome the difficulties caused by the lack of *a priori* centralized information.

We develop an adaptive decentralized control parametrization for the class of composite linear systems with MIMO subsystems which admits output decentralized model reference adaptive designs with zero asymptotical errors. Thus this paper generalizes the results in (Mirkin and Gutman, 2003) to a class of linear large-scale systems with MIMO subsystems.

2. PROBLEM FORMULATION

We consider a class of large-scale systems, which are composed of M multi-input multi-output (MIMO) subsystems described by equations of the form

$$\begin{aligned} \dot{x}_i(t) =& A_i x_i(t) + B_i u_i(t) + \sum_{j=1}^{M} A_{ij} x_j(t) \\ y_i(t) =& C_i x_i(t),\ i = 1, 2, \ldots, M \end{aligned} \tag{1}$$

where for the i-th subsystem $x_i \in \mathbb{R}^{n_i}$ is the state vector, $u_i(t) \in \mathbb{R}^{m_i}$ is the control input and $y_i(t) \in \mathbb{R}^{m_i}$ is the output. The constant matrices $A_i \in \mathbb{R}^{n_i \times n_i}$, $B_i \in \mathbb{R}^{n_i \times m_i}$, $C_i \in \mathbb{R}^{m_i \times n_i}$, $A_{ij} = B_i A^*_{ij} \in \mathbb{R}^{m_i \times n_j}$ have unknown elements and $\sum_{j=1}^{M} n_i = n$.

We make the following assumptions about the i-th isolated subsystem transfer function $W_i(s) = C_i(sI - A_i)^{-1}B_i$: $(A1)$ the observability index ν_i of $W_i(s)$ is known; $(A2)$ the transmission zeros of $W_i(s)$ have negative real parts (minimum phase plants); $(A3)$ the plant has full rank and vector relative degree 1. For the high frequency gain matrix $K_{pi} = \lim_{s\to\infty} sW_i(s)$ we assume that $(A4)$ the signs of the high frequency gain matrix K_{pi} leading principal minors are known.

Remark 1. Assumptions $(A1) - (A4)$ are independent of the decentralized MRAC problem. They are well understood in centralized adaptive control literature and various techniques for their relaxation are known, see, e.g. (Ioannou and Sun, 1996; Tao, 2003),

Our control objective is to achieve that y_i *asymptotically exactly* follows the output $y_{ri} \in \mathbb{R}^{m_i}$ of a stable strictly positive real (SPR) reference model

$$y_{ri} = W_{ri}(s) r_i. \tag{2}$$

The reference signal $r_i(t)$ is assumed piecewise continuous and uniformly bounded.

3. PROPOSED CONTROLLER STRUCTURE

Motivated by the similarities with the SISO case (Mirkin and Gutman, 2003), we will use the decentralized adaptive control scheme with *reference model coordination* to achieve the control objective - *asymptotic exactly tracking*. The control law for the ith local MIMO subsystem u_i is chosen to be of the form

$$u_i(t) = u_{li}(t) + u_{ci}(t) \tag{3}$$

where the part of the control law $u_{li}(t)$ is based only on the local signals of the ith subsystem, and the component $u_{ci}(t)$ is the coordinated component which is based on the reference signals of the all other subsystems. Exchange of the reference signals between subsystems can be easily implemented in real-life control systems.

Remark 2. For the case of systems with MIMO interconnected subsystems, the main difficulties and the main difference from the case with SISO interconnected subsystems is the choice of a suitable parametrization of the local and coordinated component of the control law (3), in order to anticipate the effect of the cross-coupling. The component based on local signals, $u_{li}(t)$, will be modified in comparison with (Mirkin and Gutman, 2003) by adding a term based on local data. As the basic building block for the coordinated component $u_{ci}(t)$, we suggest a dynamical system (pre-filter) with adjustable parameters that describes how the reference signal $r_j(t)$ of the j-th reference model acts on the i-th control input. We consider here the case of *a priory* knowledge about K_{pi} based on the SDU decomposition of K_{pi}, e.g. (Costa *et al.*, 2003).

The part of the control law u_{li} which is based only on the local information is parameterized as follows

$$u_{li}(t) = \theta_{fi}^T(t)\omega_{fi}(t) + \theta_i^T(t)\omega_i(t) \tag{4}$$

where $\theta_{li}(t) = [\theta_{fi}^T(t)\ \theta_i^T(t)]^T \in \mathbb{R}^{2m_i(\nu_i+1)\times m_i}$, $\theta_{fi}(t) = [\theta_{ei}(t)\ \theta_{1i}^T(t)\ \theta_{2i}^T(t)\ \theta_{ui}(t)]^T \in \mathbb{R}^{2m_i\nu_i\times m_i}$, $\theta_{ei}, \theta_{ui} \in \mathbb{R}^{m_i\times m_i}$, $\theta_{1i}, \theta_{2i} \in \mathbb{R}^{m_i(\nu_i-1)\times m_i}$ and $\theta_i(t) = [\theta_{ri}(t)\ \ \theta_{r2i}(t)]^T \in \mathbb{R}^{2m_i\times m_i}$, $\theta_{ri}, \theta_{r2i} \in \mathbb{R}^{m_i\times m_i}$ are some time-varying parameter matrices,

$\omega_{fi}(t)$ and $\omega_i(t)$ are the local feedback and the local feedforward signals respectively and are given by the following equation ($i, j = 1 \ldots, M$)

$$\omega_{fi}(t) = \begin{bmatrix} I_{m_i} & 0 & 0 & 0 \\ 0 & \Phi_i(s) & 0 & 0 \\ 0 & 0 & \Phi_i(s) & 0 \\ 0 & 0 & 0 & I_{m_i} \end{bmatrix} \begin{bmatrix} e_i \\ y_i \\ u_i \\ u_i \end{bmatrix} = \begin{bmatrix} e_i(t) \\ x_{1i}(t) \\ x_{2i}(t) \\ u_i \end{bmatrix}$$

$$\omega_i(t) = \begin{bmatrix} I_{m_i} \\ W_{ri}(s) \end{bmatrix} [r_i](t), \quad \bar{\omega}_{fi}(t) = [e_i^T \ x_{1i}^T \ x_{2i}^T]^T \tag{5}$$

with

$$\Phi_i(s) = \frac{[I_{m_i} s^{\nu_i - 2}, \ldots, I_{m_i} s, I_{m_i}]^T}{\Lambda_i(s)} \tag{6}$$

$\Phi_i(s) \in \mathbb{R}^{m_i(\nu_i-1)\times m_i}$, $I_{m_i} \in \mathbb{R}^{m_i \times m_i}$ is an identity matrix and $\Lambda_i(s) = s^{\nu_i - 1} + \cdots + \lambda_{mi} s + \lambda_{0i}$ is a monic Hurwitz polynomial.

The coefficient matrix $\theta_{ui}(t)$ in $\theta_{fi}(t)$ has the specific upper triangular structure with zero diagonal element like in (Costa *et al.*, 2003) for the centralized case, i.e.

$$\theta_{ui}(t) = \begin{bmatrix} 0 & \theta_{ui}^{12}(t) & \theta_{ui}^{13}(t) & \ldots & \theta_{ui}^{1m_i}(t) \\ 0 & 0 & \theta_{ui}^{23}(t) & \ldots & \theta_{ui}^{2m_i}(t) \\ & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & 0 & \theta_{ui}^{m_i-1\ m_i}(t) \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{7}$$

This upper triangular matrix structure guarantees that the control component (4) is implementable without singularity, that is,

$$\begin{aligned} u_{li}^1(t) &= \bar{\theta}_{fi}^{1T} \bar{\omega}_{fi}(t) + \theta_i^{1T} \omega_i(t) + \sum_{k=2}^{m_i} \theta_{ui}^{1k} u_i^k(t) \\ u_{li}^2(t) &= \bar{\theta}_{fi}^{2T} \bar{\omega}_{fi}(t) + \theta_i^{2T} \omega_i(t) + \sum_{k=3}^{m_i} \theta_{ui}^{2k} u_i^k(t) \\ &\vdots \\ u_{li}^{m_i}(t) &= \bar{\theta}_{fi}^{m_i T} \bar{\omega}_{fi}(t) + \theta_i^{m_i T} \omega_i(t) + 0 \end{aligned} \tag{8}$$

where $\bar{\theta}_{fi}(t) = [\theta_{ei} \theta_{1i}^T \theta_{2i}^T]^T$ and $\bar{\omega}_{fi}(t) = [e_i^T \ x_{1i}^T \ x_{2i}^T]^T$

If we denote

$$\begin{aligned} \Theta_{li}^1(t) &= [\bar{\theta}_{fi}^{1T} \ \theta_i^{1T} \ \theta_{ui}^{12} \ \theta_{ui}^{13} \ \ldots \theta_{ui}^{1m_i}]^T \\ \Theta_{li}^2(t) &= [\bar{\theta}_{fi}^{2T} \ \theta_i^{2T} \ \theta_{ui}^{23} \ \theta_{ui}^{24} \ \ldots \theta_{ui}^{2m_i}]^T \\ &\vdots \\ \Theta_{li}^{m_i} &= [\bar{\theta}_{fi}^{m_i T} \ \theta_i^{m_i T}]^T \end{aligned} \tag{9}$$

and

$$\begin{aligned} \Omega_{li}^1(t) &= [\bar{\omega}_{fi}^T \ \omega_i^T \ u_i^2 \ u_i^3 \ldots u_i^{m_i-1} \ u_i^{m_i}]^T \\ \Omega_{li}^2(t) &= [\bar{\omega}_{fi}^T \ \omega_i^T \ u_i^3 \ldots u_i^{m_i-1} \ u_i^{m_i}]^T \\ &\vdots \\ \Omega_{li}^{m_i}(t) &= [\bar{\omega}_{fi}^T \ \omega_i^T \ ]^T \end{aligned} \tag{10}$$

we can rewrite the local control component (4) as

$$u_{li}(t) = \begin{bmatrix} \Theta_{li}^{1T}(t) \Omega_{li}^1(t) \\ \vdots \\ \Theta_{li}^{m_i T}(t) \Omega_{li}^{m_i}(t) \end{bmatrix} \tag{11}$$

Remark 3. When comparing the control component based on local signals, $u_{li}(t)$, of subsystem i regarded as an isolated system, with the case of MIMO centralized adaptive control (Costa *et al.*, 2003), one notices the following differences: In $u_{li}(t)$ the additional term $\theta_{r2i}^* W_{ri}(s)[r_i](t)$ is present, and the tracking error $e_i(t) = y_i(t) - y_{ri}(t)$ is used in the local feedback signal vector $\omega_{fi}(t)$ instead of $y_i(t)$.

The coordinated control component, $u_{ci}(t)$, which is based on the reference signals of the all other subsystems is chosen as follows

$$\begin{aligned} u_{ci}(t) &= \sum_{j=1, j\neq i}^{M} \theta_{ij}^T(t) \omega_{ij}(t) \\ \omega_{ij}(t) &= P_{ij}(s)[x_{rj}](t) = \begin{bmatrix} I_{m_j} \\ \Phi_{ij}(s) \end{bmatrix} [x_{rj}](t) \end{aligned} \tag{12}$$

where $\theta_{ij}(t) = [\theta_{1ij}^T \ \theta_{2ij}^T]^T \in \mathbb{R}^{m_j \nu_i \times m_i}$, $\theta_{1ij}(t) \in \mathbb{R}^{m_j \times m_i}$, $\theta_{2ij}(t) \in \mathbb{R}^{m_j(\nu_i - 1)\times m_i}$ are some time-varying parameter matrices, and $\omega_{ij}(t)$ is the output of the dynamic system with the transfer function $P_{ij}(s)$

$$\Phi_{ij}(s) = \frac{[I_{m_j} s^{\nu_i - 2}, \ldots, I_{m_j} s, I_{m_j}]^T}{\Lambda_i(s)}. \tag{13}$$

4. ERROR EQUATION AND STABILITY ANALYSIS

To develop an adaptation law for the controller (3)-(12), we need to express the closed-loop system in terms of the tracking error $e_i(t) = y_i(t) - y_{ri}(t)$.

With the specification of $\Lambda_i(s)$, $\Phi_i(s)$ and $W_i(s)$ in the local control component (4) there exist some constant matrices $\theta_{ri}^* = K_{pi}^{-1}$, θ_{ei}^*, θ_{1i}^* and θ_{2i}^* (Sastry and Bodson, 1989; Ioannou and Sun, 1996) such that

$$\begin{aligned} I - \theta_{ei}^{*T} \Phi_i(s) - \theta_{1i}^{*T} \Phi_i(s) W_i(s) - \theta_{2i}^* W_i(s) \\ = \theta_{ri}^* W_{ri}^{-1}(s) W_i(s) \end{aligned} \tag{14}$$

Then from (1) and (14), for any u_i we have the following equation for the tracking error e_i, with

$$\begin{aligned} e_i = W_{ri}(s) K_{pi} \Big[& u_i - \theta_{ei}^* y_i - \theta_{1i}^{*T} x_{1i} \\ & - \theta_{2i}^{*T} x_{2i} - \theta_{ri}^* r_i + \sum_{j=1, j\neq i}^{M} A_{ij}^* x_j(t) \\ & - \sum_{j=1, j\neq i}^{M} \theta_{2i}^{*T} \Phi_i(s) [A_{ij}^* x_j](t) \Big] \end{aligned} \tag{15}$$

Denoting $A_{zij}^* = [\theta_{1i}^{*1T} A_{ij}^*, \theta_{1i}^{*2T} A_{ij}^*, \ldots, \theta_{1i}^{*(\nu_i - 1)T} A_{ij}^*]$, $A_{zij}^* \in \mathbb{R}^{m_i \times m_j(\nu_i - 1)}$, using (6) and (13) and doing some manipulations with the transfer functions we can write

$$\theta_{2i}^{*T} \Phi_i(s) A_{ij}^* = A_{zij}^* \Phi_{ij}(s) \tag{16}$$

In view of (16), (5) and (12)-(13) after substituting $x_j = e_{xj} + x_{rj}$ in the right part of (15) , the equation (15) can be rewritten as

$$e_i = W_{ri}(s)K_{pi}\Big[u_i - \bar{\theta}_i^{*T}\bar{\omega}_{fi}(t) - \theta_i^{*T}\omega_i(t) + \sum_{j=1,j\neq i}^{M}\big(A_{ij}^* e_{xj}(t) - A_{zij}^*\Phi_{ij}(s)[e_{xj}](t)\big)\Big] \quad (17)$$

where $\bar{\theta}_i^* = [\theta_{ei}^* \ \theta_{1i}^{*T} \ \theta_{2i}^{*T}]^T$, $\theta_i^* = [\theta_{ri}^* \ \theta_{ei}^*]^T$ and $\theta_{ij}^* = [-A_{ij}^{*T}, A_{zij}^{*T}]^T$

By using the high-frequency gain matrix decomposition $K_{pi} = S_i D_i U_i$ (Morse, 1993), where S_i is symmetric positive definite, D_i is diagonal, and U_i is unity upper triangular, and in view of $U_i u_i = u_i - (I_{m_i} - U_i u_i)$ we derive an error equation from the equation (17)

$$e_i = W_{ri}(s)S_i D_i\Big[u_i - \hat{\theta}_i^{*T}\bar{\omega}_{fi}(t) - \hat{\theta}_i^{*T}\omega_i(t) - \sum_{j=1,j\neq i}^{M}\hat{\theta}_{ij}^{*T}\omega_{ij}(t) + \sum_{j=1,j\neq i}^{M}\hat{\Phi}_{ij}(s)[e_{xj}](t)\Big]$$
$$\hat{\Phi}_{ij}(s) = U_i A_{ij}^* + U_i A_{zij}^*\Phi_{ij}(s) \quad (18)$$

where $\hat{\theta}_i^* = U_i\theta_i^*$, $\hat{\theta}_{ij}^* = U_i\theta_{ij}^*$ and matrix $\hat{\theta}_{ui}^* = I_{m_i} - U_i$ has the same specific upper triangular form as (7) with zero diagonal element but with unknown constant coefficients.

Then using the control law as given by (3), (4) and (11) the tracking error equation (18) can be written as

$$e_i = W_{mi}(s)S_i D_i\Big([\widetilde{\Theta}_{li}^{1T}\Omega_{li}^1 \ldots \widetilde{\Theta}_{li}^{m_i T}\Omega_{li}^{m_i}] + \sum_{j=1,j\neq i}^{M}[\widetilde{\Theta}_{ij}^T\omega_{ij} + \hat{\Phi}_{ij}(s)[e_{xj}](t)]\Big) \quad (19)$$

where $\widetilde{\Theta}_{li}^k(t) = \Theta_{li}^k(t) - \Theta_{li}^{*k}$ and $\widetilde{\Theta}_{ij}(t) = \Theta_{ij}(t) - \Theta_{ij}^*$ are the parameter errors, $(k = 1, 2, \ldots, m_i; i, j = 1, \ldots, M)$.

Let we define the augment state $X_i = [x_i^T, x_{1i}^T, x_{2i}^T]^T$ by combining the i-th subsystems state x_i of the plant (1) with the filter states x_{1i} and x_{2i} from (5). With $X_{mi} = [x_{mi}^T, x_{m1i}^T, x_{m2i}^T]^T$ we denote the state of the corresponding nonminimal realization $\hat{C}_i(sI - \hat{A}_i)^{-1}\hat{B}_i$ of $W_{mi}S_i$ that is SPR (Costa *et al.*, 2003). Let $(A_{\phi ij}, B_{\phi ij}, C_{\phi ij}, D_{\phi ij})$ be a minimal stable state space realization for the stable transfer matrix $\hat{\Phi}_{ij}(s)$ from (18). Then, the augment state error $\hat{e}_i = X_i(t) - X_{mi}(t)$ and the output error e_i in (19) satisfy

$$\dot{\hat{e}}_i = \hat{A}_i\hat{e}_i(t) + \hat{B}_i D_i[\widetilde{\Theta}_{li}^{1T}\Omega_{li}^1 \ldots \widetilde{\Theta}_{li}^{m_i T}\Omega_{li}^{m_i}] + \sum_{j=1,j\neq i}^{M}\hat{B}_i D_i\Big(\widetilde{\Theta}_{ij}^T\omega_{ij} + D_{\phi ij}L^T\hat{e}_j(t) + C_{\phi ij}Z_{eij}(t)\Big)$$
$$\dot{Z}_{eij}(t) = A_{\phi ij}Z_{eij}(t) + B_{\phi ij}L^T\hat{e}_j(t)$$
$$z_{eij}(t) = C_{\phi ij}Z_{eij}(t) + D_{\phi ij}L^T\hat{e}_j(t)$$
$$e_i(t) = y_i(t) - y_{mi}(t) = \hat{C}_i\hat{e}_i(t) \quad (20)$$

where $L = [I \quad 0 \quad 0]^T$ and $0_{n_j\times l_j(\nu_i-1)}$ is a zero matrix.

Because $\hat{C}_i^T(sI - \hat{A}_i)^{-1}\hat{B}_i = W_{mi}(s)S_i$ is SPR, the triple $(\hat{A}_i, \hat{B}_i, \hat{C}_i)$ satisfies the following equations given by the matrix version of the KY Lemma (Narendra and Annaswamy, 1989, page 67)

$$\hat{A}_i^T P_i + P_i\hat{A}_i + Q_i = 0 \qquad P_i\hat{B}_i = \hat{C}_i^T \quad (21)$$

Since $A_{\phi ij}$ in (20) is stable, it also hold that $(i, j = 1, \ldots, M)$

$$A_{\phi ij}^T P_{zij} + P_{zij}A_{\phi ij} + Q_{zij} = 0 \quad (22)$$

We now choose the adaptation algorithms as $(i, j = 1, \ldots, M)$

$$\Theta_{li}^k(t) = -\int_0^t \eta_i^k(s)ds - \eta_i^k(t) - \eta_i^k(t - h_i)$$
$$\eta_i^k(t) = \gamma_i^k \text{sign}(d_i)\Omega_{li}^k e_i^k, (k = 1, \ldots, l_i)$$
$$\Theta_{ij}(t) = -\int_0^t \eta_{ij}(s)ds - \eta_{ij}(t) - \eta_{ij}(t - h_{ij})$$
$$\eta_{ij}^T(t) = \text{Sign}(D_i)\Gamma_{gi}e_i(t)\omega_{ij}^T(t) \quad (23)$$

where $\text{Sign}(D_i) = \text{diag}\{\text{sign}(d_i^1), \ldots, \text{sign}(d_i^{l_i})\}$, $\Gamma_i = \Gamma_i^T > 0$ and h_i and h_{ij} are some arbitrary design parameters to be chosen.

Remark 4. Although only the integral component of the adaptation algorithm is needed for stability and exact asymptotic tracking, the use of the proportional and the proportional delayed terms in the adaptation algorithm (23) makes it possible to achieve better adaptation performance than the traditional I and PI schemes. This adaptation algorithm includes the traditional I and PI schemes as a special case. The design parameters h_i and h_{ij} are chosen in the same way as the traditional gains γ_i^k and Γ_i in (23).

For the stability analysis we use the following Lyapunov-Krasovskii type functional

$$V = \sum_{i=1}^{M}\Big[V_{ei} + V_{\eta i} + \sum_{j=1,j\neq i}^{M}\Big(V_{\eta ij} + V_{zij}\Big)\Big]$$
$$V_{ei} = \hat{e}_i^T P_i\hat{e}_i, \qquad V_{zij} = Z_{eij}^T P_{zij}Z_{eij}$$
$$V_{\eta i} = \sum_{k=1}^{l_i}\left(\gamma_i^k\right)^{-1}|d_i|\Big(\tilde{\eta}_i^{kT}(t)\tilde{\eta}_i^k(t) + \int_{t-h_i}^t \eta_i^{kT}(s)\eta_i^k(s)ds\Big)$$
$$V_{\eta ij} = \sum_{j=1,j\neq i}^{M}\text{tr}\Big(\tilde{\eta}_{ij}(t)\Gamma_i^{-1}\bar{D}_i\tilde{\eta}_{ij}^T(t) + \int_{t-h_{ij}}^t \eta_{ij}(s)\Gamma_i^{-1}\bar{D}_i\eta_{ij}^T(s)ds\Big) \quad (24)$$

where $\tilde{\eta}_i = \tilde{\theta}_{li}(t) + \eta_i^* + \eta_i(t) + \eta_i(t - h_i)$ and $\tilde{\eta}_{ij} = \tilde{\theta}_{ij}(t) + \eta_{ij}(t) + \eta_{ij}(t - h_{ij})$, $\bar{D}_i = \text{diag}\{|d_i^1| \ldots |d_i^k| \ldots |d_i^{m_i}|\}$ and d_i^k are the entries of D_i. The "artificial" vector

$$\eta_i^{*k} = r_0(2d_i)^{-1}[1,\ 0, \ldots,\ 0]^T. \quad (25)$$

has the same dimension as Θ_{fi}^k, and r_0 is an as yet unspecified positive constant.

Using (21) and (22) the time derivatives of the components of (24) along (20) can be written

$$\dot{V}_{ei} = -\hat{e}_i^T(t)Q_i\hat{e}_i(t) + \sum_{j=1,j\neq i}^{M} 2\hat{e}_i^T(t)P_i\bar{B}_iD_iD_{\phi ij}L^T\hat{e}_j(t) + \sum_{j=1,j\neq i}^{M} 2\hat{e}_i^T(t)P_i\bar{B}_iD_iC_{\phi ij}Z_{eij}(t) \quad (26)$$

$$\dot{V}_{zij} = -\sum_{j=1,j\neq i}^{M} Z_{eij}^T(t)Q_{zij}Z_{eij}(t) + \sum_{j=1,j\neq i}^{M} 2Z_{eij}^T(t)P_{zij}\hat{B}_{\phi ij}L_j^T\hat{e}_j(t) \quad (27)$$

$$\dot{V}_{\eta i} = -\sum_{k=1}^{l_i}\left(\gamma_i^k\right)^{-1}|d_i|\Big[2\widetilde{\Theta}_{li}^{kT}\eta_i^k(t) + 2\eta_i^{*kT}\eta_i^k(t) + \left\|\eta_i^k(t)+\eta_i^k(t-h_i)\right\|\Big] \quad (28)$$

$$\dot{V}_{\eta ij} = -\,\mathrm{tr}\Big(\eta_{ij}(t)\Gamma_i\bar{D}_i\tilde{\Theta}_{ij}^T(t) + \tilde{\Theta}_{ij}(t)\Gamma_i\bar{D}_i\eta_{ij}^T(t)\Big) - \|\eta_{ij}(t)+\eta_{ij}(t-h_{ij})\|_{\Gamma_i\bar{D}_i} \quad (29)$$

Further using (25), (26)–(29) and dropping negative terms we obtain

$$\dot{V}|_{(20)} \leq \sum_{i=1}^{M}\Big[-\hat{e}_i^T(t)Q_i\hat{e}_i(t) - r_0\hat{e}_i^TP_i^T\hat{B}_i\hat{B}_i^TP_i\hat{e}_i - \sum_{j=1,j\neq i}^{M} Z_{eij}^T(t)Q_{zij}Z_{eij}(t) + \sum_{j=1,j\neq i}^{M} 2\hat{e}_i^T(t)P_i\bar{B}_iD_iD_{\phi ij}L^T\hat{e}_j(t) + \sum_{j=1,j\neq i}^{M} 2\hat{e}_i^T(t)P_i\bar{B}_iD_iC_{\phi ij}Z_{eij}(t) + \sum_{j=1,j\neq i}^{M} 2Z_{eij}^T(t)P_{zij}\hat{B}_{\phi ij}L_j^T\hat{e}_j(t) \quad (30)$$

We can estimate the mixed terms of (30) as follows

$$\begin{aligned}
&2\hat{e}_j^T(t)P_i\hat{B}_iD_iD_{\phi ij}L^T\hat{e}_j(t)\\
\leq&\hat{e}_j^T(t)P_i\hat{B}_i\Upsilon_{1ij}\hat{B}_i^TP_i\hat{e}_j(t)+\hat{e}_j^T(t)G_i\hat{e}_j(t)\\
&2\hat{e}_i^T(t)P_i\hat{B}_iD_iC_{\phi ij}Z_{eij}(t)\\
\leq&\hat{e}_i^T(t)P_i\hat{B}_i\Upsilon_{2ij}\hat{B}_i^TP_i\hat{e}_i(t)+Z_{eij}^T(t)G_iZ_{eij}(t)\\
&2Z_{eij}^T(t)P_{zij}B_{\phi ij}L^T\hat{e}_j(t)\\
\leq&Z_{eij}^T(t)G_iZ_{eij}(t)+\hat{e}_j^T(t)\Upsilon_{3ij}\hat{e}_j(t)
\end{aligned} \quad (31)$$

where $G_i = G_i^T > 0$ is a some constant matrix and

$$\begin{aligned}
\Upsilon_{1ij} =& D_iD_{\phi ij}L^TG_i^{-1}LD_{\phi ij}^TD_i^T\\
\Upsilon_{2ij} =& D_iC_{\phi ij}G_i^{-1}C_{\phi ij}^TD_i^T\\
\Upsilon_{3ij} =& LB_{\phi ij}^TP_{zij}G_i^{-1}P_{zij}B_{\phi ij}L^T
\end{aligned} \quad (32)$$

Applying (31) to (30) and selecting values of r_o, Q_{zij} and Q_i from the inequalities $(i,j=1,\ldots,M)$

$$\begin{aligned}
\tfrac{r_o}{M-1} >& \lambda_{max}\left(\Upsilon_{1ij}+\Upsilon_{2ij}\right)\\
\lambda_{min}(Q_{zij}) >& \lambda_{max}\left(2G_i\right)\\
\lambda_{min}(Q_i) >& 2\lambda_{max}\left(G_j+\Upsilon_{3ji}\right)
\end{aligned} \quad (33)$$

where $\lambda_{min}(\star)$ and $\lambda_{max}(\star)$ are the minimum and maximum eigenvalues of $(\star)$, respectively we obtain

$$\dot{V}|_{(20)} \leq -\sum_{i=1}^{M}\Big(\hat{e}_i^T(t)Q_i\hat{e}_i(t) + \sum_{j=1,j\neq i}^{M} Z_{eij}^T(t)Q_{zij}Z_{eij}(t)\Big) \leq 0 \quad (34)$$

This implies (Hale and Lunel, 1993) that V and, therefore, $\hat{e}_i(t)$, $e_i(t)$, $Z_{eij}(t)$, $\widetilde{\theta}_{li}$, θ_{li}, $\widetilde{\theta}_{ij}$, $\theta_{ij} \in \mathsf{L}_\infty$. The remainder of the stability analysis follows directly using the steps in (Ioannou and Sun, 1996).

Remark 5. We note that r_o and the matrices Q_{zij} and Q_i are used only for analysis and do not influence the control law. Decentralized controller gains adjust automatically to counter the non-desirable effects of interconnections and parameter uncertainties.

5. ROBUSTNESS OF THE DECENTRALIZED CONTROLLER

As in adaptive centralized control theory (Ioannou and Sun, 1996), the coordinated adaptive controller proposed here can be shown to be robust with respect to disturbances and unmodeled dynamics by introducing modifications to the adaptive law. A brief example of this property is mentioned below where the integral adaptation law is discussed for ease of exposition. If disturbance $f_i(t), (\|f_i\| \leq f_{oi})$ is present in the subsystems, and e.g. a σ-modification as in (Ioannou and Sun, 1996) is used, the error equation (19) and the adaptation algorithms (23) can be rewritten as

$$e_i = W_{mi}(s)S_iD_i\Big(\big[\widetilde{\Theta}_{li}^{1T}\Omega_{li}^1\ldots\widetilde{\Theta}_{li}^{m_iT}\Omega_{li}^{m_i}\big] + \hat{f}_i(t) + \sum_{j=1,j\neq i}^{M}\big[\widetilde{\Theta}_{ij}^T\omega_{ij} + \hat{\Phi}_{ij}(s)[e_{xj}](t)\big]\Big) \quad (35)$$

$$\begin{aligned}
\dot{\Theta}_{li}^k(t) =& -\eta_i^k(t) - \sigma_i^k\Theta_{li}^k\\
\dot{\Theta}_{ij}^T(t) =& -\eta_{ij}(t) - \sigma_{ij}\Theta_{ij}^T
\end{aligned} \quad (36)$$

where $\hat{f}_i(t) = \bar{\Phi}_i(s)f_i(t)$ is guaranted to be bounded with the bound $\hat{f}_{oi}$ due to the boundedness of $f_i(t)$ and the stability of $\bar{\Phi}_i(s) = I - \Phi_i(s)$.

Instead of $V_{\eta i}$ and $V_{\eta ij}$ in (24) we use the following functions

$$\begin{aligned}
V_{\eta i} =& \sum_{k=1}^{l_i}\left(\gamma_{li}^k\right)^{-1}|d_i|\tilde{\eta}_i^{kT}(t)\tilde{\eta}_i^k(t)\\
V_{\eta ij} =& \sum_{j=1,j\neq i}^{M}\mathrm{tr}\Big(\tilde{\eta}_{ij}(t)\Gamma_i^{-1}\bar{D}_i\tilde{\eta}_{ij}^T(t)\Big)
\end{aligned} \quad (37)$$

with $\tilde{\eta}_i^k(t) = \tilde{\theta}_{li}^k(t) + \eta_i^{*k}$ and $\tilde{\eta}_{ij}(t) = \tilde{\theta}_{ij}(t)$

Using (35) and (36) and in view of (37) after some simplifications, we obtain the time derivative of (24)

$$\dot{V} \leq \sum_{i=1}^{M} \Big[-\hat{e}_i^T(t)Q_i\hat{e}_i(t) - \sum_{k=1}^{l_i} \sigma_i^k|d_i|\tilde{\eta}_i^{kT}(t)\tilde{\eta}_i^k(t) + \beta_i^* - \sum_{j=1,j\neq i}^{M} \Big(Z_{eij}^T(t)Q_{zij}Z_{eij}(t) - \sigma_{ij}\mathrm{tr}\big(\tilde{\eta}_{ij}(t)\bar{D}_i\tilde{\eta}_{ij}^T(t)\big)\Big)\Big] \quad (38)$$

where

$$\beta_i^* = \hat{f}_{oi}^2\,\|G_i\| + \sum_{k=1}^{l_i} \sigma_i^k|d_i|\,\big\|\theta_{li}^{*k} - \eta_i^{*k}\big\|^2 + \sum_{j=1,j\neq i}^{M} \sigma_{ij}\mathrm{tr}\big(\eta_{ij}^*\bar{D}_i\eta_{ij}^{*T}\big) \quad (39)$$

Thus, if we select Q_{zij} and Q_i from (33) and the values of α and r_o from the inequalities

$$\alpha = \min\Big[\tfrac{\lambda_{min}(Q_i)}{\lambda_{max}(P_i)},\ \tfrac{\lambda_{min}(Q_{zij})}{\lambda_{max}(P_{zij})},\ \sigma_i^k\gamma_i^k,\ \tfrac{\sigma_{ij}\min_i|d_i|}{\lambda_{max}(\Gamma_i^{-1}\bar{D}_i)}\Big] \quad (40)$$

and

$$\tfrac{r_o}{M-1} > \lambda_{max}\left(\Upsilon_{1ij} + \Upsilon_{2ij} + \Upsilon_{4i}\right) \quad (41)$$

where $\Upsilon_{4i} = D_iG_i^{-1}D_i$ we obtain after some manipulations from (38)

$$\dot{V} = -\alpha V + \sum_{i=1}^{M} \beta_i^* \quad (42)$$

Furthermore, using standard arguments from the robust adaptive control theory, see, e.g. (Ioannou and Sun, 1996), we conclude that all closed-loop signals are bounded, and $e_i(t)$ converges to the residual set bounded by the some constant. We can show that by decreasing the values σ_i and σ_{ij} sufficiently, the upper bound on the steady-state error e_i may be made as small as desired. The system designer can e.g. tune the size of the residual set by adjusting these parameters which were introduced in the adaptation laws described in (36).

6. SUMMARY

In this paper an adaptive coordinated decentralized controller is proposed for large-scale linear systems with MIMO subsystems which admits output decentralized model reference adaptive designs with zero asymptotical errors. We develop decentralized MRAC on the base of *a priory* information about only the local subsystems gain frequency matrices without additional *a priory* knowledge about the full system gain frequency matrix. A novel Lyapunov-Krasovskii type functional derived to design the adaptive laws and in order to prove stability. The decentralized controller is also shown to be robust to disturbances. This paper represents the first results for decentralized MRAC of large-scale system with MIMO subsystems.

REFERENCES

Costa, R. R., L. Hsu, A. K. Imai and P. Kokotović (2003). Lapunov-based adaptive control of MIMO systems. *Automatica* **39**(7), 1251–1257.

Gavel, D. T. and D. D. Šiljak (1989). Decentralized adaptive control: structural conditions for stability. *IEEE Transactions on Automatic Control* **34**(4), 413–426.

Hale, J. K. and S. M. V. Lunel (1993). *Introduction to Functional Differential Equations*. Springer-Verlag. New York.

Ioannou, P. A. (1986). Decentralized adaptive control of interconnected systems. *IEEE Transactions on Automatic Control* **31**, 310–314.

Ioannou, P. A. and J. Sun (1996). *Robust Adaptive Control*. Prentice-Hall. New Jersey.

Jiang, Zh. P. (2000). Decentralized and adaptive nonlinear tracking of large-scale systems via output feedback. *IEEE Transactions on Automatic Control* **45**(11), 2122–2128.

Krishmanurthy, P. and F. Khorrami (2003). Decentralized control and disturbance attenuation for large-scale nonlinear systems in generalized output-feedback canonical form. *Automatica* **39**(10), 1923–1933.

Mirkin, B. M. (1999). Adaptive decentralized control with model coordination. *Automation and Remote Control* **60**(1), 73–81. (Translated from Avtomatika i telemekhanika, no. 1, 1999, pp. 90–100).

Mirkin, B. M. (2003). Comments on "Exact output tracking in decentralized adaptive control". *IEEE Transactions on Automatic Control* **48**(2), 348–350.

Mirkin, B. M. and P. O. Gutman (2003). Decentralized output-feedback MRAC of linear state delay systems. *IEEE Transactions on Automatic Control* **48**(9), 1613–1619.

Morse, A. S. (1993). A gain matrix decomposition and some ot its applications. *Systems and Contr. Letters* **21**, 1–10.

Narendra, K. S. and A. M. Annaswamy (1989). *Stable Adaptive Systems*. Prentice-Hall. New York.

Narendra, K. S. and N. O. Oleng' (2002). Exact output tracking in decentralized adaptive control. *IEEE Transactions on Automatic Control* **47**(2), 390–395.

Ortega, R. (1996). An energy amplification condition for decentralized adaptive stabilization. *IEEE Transactions on Automatic Control* **41**, 285–289.

Sastry, S. and M. Bodson (1989). *Adaptive control. Stability, Convergence, and Robustness*. Prentice-Hall. Englewood Cliffs, New Jersey.

Tao, G. (2003). *Adaptive Control Design and Analysis*. John Wiley. New York.