A SINGULARLY PERTURBED MODEL FOR ROBUST CONTROL OF LINEAR SINGLE-LINK FLEXIBLE MANIPULATOR

Karimi H.R. Yazdanpanah M.J.

Control & Intelligent Processing Center of Excellence, Dept. of Electrical and Computer Engineering, Faculty of Engineering, University of Tehran, P. O. Box: 14395-515, Tehran, Iran.
E-mail: hrkarimi@ut.ac.ir

Abstract: This paper deals with the modelling of a single-link flexible manipulator utilizing the singular perturbation method. Authors' attention is focused on the robust regulation of the tip-position based on a new modelling approach under the assumption of norm-boundedness of the fast dynamics (deflection modes). In this approach, the deflection modes may be treated as norm-bounded disturbance. Hence, the controller synthesis is performed only for the certain dynamics of the system.

Keywords: Flexible link Manipulator; disturbance attenuation; singularly perturbed model.

1 INTRODUCTION

Flexible link manipulators are attractive because they avoid the large inertia forces associated with traditional, large-section, rigid-link manipulators. However, the introduction of flexibility and the consequent tendency of the links to oscillate during motion create a control problem for which a very accurate model of the flexible link system is required; see (Cannon and Schmitz, 1984; Hussain, et al., 1998). A new controller design for controlling a single-link flexible manipulator based on variable structure theory has been presented by (Qian and Ma, 1992). Also, a singular perturbation approach in (Sicilano and Book, 1988) has been developed for the control of lightweight flexible-link manipulators.
Singularly perturbed systems often occur naturally because of the presence of small parasitic parameters multiplying the time derivatives of some of the system states. Singularly perturbed control systems have been intensively studied for the past three decades; see (Kokotovic, et al., 1986). A popular approach adopted to handle these systems is based on the so-called reduced technique. The composite design based on separate designs for slow and fast subsystems has been systematically reviewed in (Saksena, et al., 1984). Also, the robust stabilization of singularly perturbed systems based on a new modeling approach has been investigated in (Karimi and Yazdanpanah, 2000).

The system under consideration, with slow and fast dynamics is described in the standard singularly perturbed form by

$$\dot{x}_s = a_{11}\, x_s + a_{12}\, x_f + b_1\, u, \qquad x_s(0) = \overline{\eta} \tag{1}$$

$$\varepsilon\, \dot{x}_f = a_{21}\, x_s + a_{22}\, x_f + b_2\, u, \qquad x_f(0) = \overline{\xi} \tag{2}$$

$$y = C\, x_s + F\, x_f \tag{3}$$

where $a_{11} \in R^{n\times n}$, $a_{12} \in R^{n\times m}$, $a_{21} \in R^{m\times n}$, $a_{22} \in R^{m\times m}$, $b_1 \in R^{n\times k}$, $b_2 \in R^{m\times k}$, $C \in R^{r\times n}$, $F \in R^{r\times m}$ are the certain matrixes and $x_s = [x_{s_1}, x_{s_2}, \ldots, x_{s_n}]^T \in R^n$, $x_f = [x_{f1}, x_{f2}, \ldots, x_{fm}]^T \in R^m$, $y(t) \in R^r$ and $u(t) \in R^k$ represent the state vectors of the slow and fast dynamics and measured output and control input, respectively. Also, $\overline{\eta}$ and $\overline{\xi}$ are, respectively, the initial states of $x_s(t)$ and $x_f(t)$. The singularly perturbed parameter ε is nonnegative and always represents the response time of the fast dynamics.

According to (Karimi and Yazdanpanah, 2000), our objective is to view a portion of the fast dynamics as norm-bounded uncertainty. Therefore, we call it *unmodeled dynamics*. Although the term unmodeled refers to a subsystem whose dynamics are not known, it is used to emphasize that the complete characteristics of this subsystem will not be utilized in the controller synthesis. If this is feasible, then the synthesis has to satisfy the design specifications for only the *known dynamics*, hereafter referred to as the *plant nominal dynamics*. The unmodeled dynamics, on the other hand, may be considered as a subsystem that is connected to the plant nominal dynamics.

We showed in (Karimi and Yazdanpanah, 2000) that a portion of the fast dynamics may be treated as norm-bounded uncertainty and the remaining part can be augmented to the slow dynamics. In this view, (1-3) will read:

Nominal system:

$$E\dot{X} = A_X X + A_{Xv} v + B_X u \tag{4}$$

$$y = C_2 X + D_{21} v \tag{5}$$

Uncertain system:

$$\varepsilon \dot{v} = A_{vX} X + A_v v + B_v u \triangleq A_v v + \begin{bmatrix} A_{vX} & B_v \end{bmatrix} Z \tag{6}$$

$$y_v = v \tag{7}$$

$$Z = C_1 X + D_{12} u \triangleq \begin{bmatrix} I_{n+i-1} \\ 0 \end{bmatrix} X + \begin{bmatrix} 0 \\ I_k \end{bmatrix} u \tag{8}$$

such that

$$E \triangleq \begin{cases} diag\,(I_n, \varepsilon I_{i-1}) & i>1 \\ I_n & i=1 \end{cases}, \quad A_X = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix},$$

$$A_{Xv} = \begin{bmatrix} B_{11} \\ B_{21} \end{bmatrix}, \qquad B_X = \begin{bmatrix} B_{12} \\ B_{22} \end{bmatrix}, \qquad C_1 = \begin{bmatrix} C_{11} & C_{12} \end{bmatrix},$$

$$A_{vX} = \begin{bmatrix} A_{v1} & A_{v2} \end{bmatrix}, \; C_2 = \begin{bmatrix} C_{21} & C_{22} \end{bmatrix}.$$

In which $v = (v_i, v_{i+1}, ..., v_m)^T$ is the vector of fast dynamics, which is to be treated as a norm-bounded uncertainty and $X = (x_s, x_v)^T$ (in which $x_v = (v_1, v_2, ..., v_{i-1})^T$ for $i>1$) is the vector of dominant dynamics, where ***i*** is the index of the first state of the uncertainty dynamics. The value of this index will be determined using the algorithm mentioned in (Karimi and Yazdanpanah, 2000). The vectors η and ξ are the initial states.

The stability problem (ε-bound problem) in singularly perturbed systems differs from conventional linear systems, which can be designed as: characterizing an upper bound ε_0 of the positive perturbing scalar ε such that the stability of a reduced-order system would guarantee the stability of the original full-order system for all $\varepsilon \in (0, \varepsilon_0)$. Researchers have tried various ways to find either the stability bound ε_0 or a less conservative lower bound for ε_0, see (Chen and Lin, 1990; Karimi and Yazdanpanah, 2002b; Kokotovic, et al., 1986). Recently, the problem of robust stabilization and disturbance attenuation for a class of uncertain singularly perturbed systems with norm-bounded nonlinear uncertainties has been considered in (Karimi and Yazdanpanah, 2001). Also, the robust stability analysis and stability bound improvement of perturbed parameter (ε) in the singularly perturbed systems by using linear fractional transformations and structured singular values approach (μ) has been investigated in (Karimi and Yazdanpanah, 2002b).

The references (Karimi and yazdanpanah, 2002a; Karimi and Yazdanpanah, 2001) present the new results on control synthesis for robust stabilization and robust disturbance attenuation for linear state-delayed singularly perturbed systems with norm-bounded nonlinear uncertainties. The class of plants considered in this paper consists of systems in state-space form with linear nominal parts and norm-bounded nonlinear uncertainties only in the slow state variable.

The proposed methodology in (Karimi and Yazdanpanah, 2000) may be applied to many physical systems. The principle behind the proposed methodology is that the system under control should possess a two-time scale separation, namely, low and high frequency subsystems. The restriction imposed is that the high frequency subsystem should be stable to result in the norm-bounded property. One of the practical applications that fit into this framework is the single-link flexible manipulator, which is studied in this section. In fact the rigid dynamics that characterize the dominant motion of the joints correspond to the low frequency subsystem and the deflection dynamics due to flexibility of the links correspond to the high frequency subsystem. Assuming that all damping including the flexural damping, are positive and nonzero, the high frequency subsystem is stable and hence norm-bounded (Gawronski, 1993). The modelling approach is applicable to linear as well as nonlinear models of single-link flexible manipulators.

2 SINGLE-LINK FLEXIBLE MANIPULATOR DYNAMICS

Only the linearized model of a single-link flexible manipulator in the modal coordinates will be considered. The model is given by (Cannon and Schmitz, 1984)

$$\dot{X} = AX + Bu \tag{9}$$

$$y = CX \tag{10}$$

with $X = (x_0^T, x_1^T, \ldots, x_m^T)^T$, $x_0 = (\theta, \dot{\theta})^T$, $x_i = (q_i, \dot{q}_i)^T$ for $i = 1, 2, \ldots, m$ and $A = diag\,(A_0, A_1, \ldots, A_m)$,

$$A_0 = \begin{pmatrix} 0 & 1 \\ 0 & -\alpha \end{pmatrix}, \quad A_i = \begin{pmatrix} 0 & 1 \\ -w_i^2 & -2\xi_i\, w_i \end{pmatrix}, \quad B_0 = \tfrac{1}{I_T}\begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

$$B = (B_0^T, B_1^T, \ldots, B_m^T)^T, \; B_i = \tfrac{1}{I_T}\begin{pmatrix} 0 \\ \dot{\phi}_i(0) \end{pmatrix}, \qquad C_0 = (L \quad 0),$$

$C = (C_0, C_1, \ldots, C_m)$, $C_i = (\phi_i(L) \quad 0)$, where y is the tip-position, m is the arbitrarily large number of flexible modes, α denotes the corresponding pole of the rigid dynamics, (ω_i, ξ_i) are the frequency and damping ratio of the i th deflection mode, L is the length of the link, I_T is the total inertia about the rotation axis and ϕ_i represents i th mode shape.

3 A SINGULARLY PERTURBED MODEL

Following (Sicilano and Book, 1988), a singularly perturbed model of (9-10) can be obtained as follows, where the singularly perturbed parameter ε is defined as

$$\varepsilon \underset{=}{\Delta} \frac{1}{\min\{w_i\}}, \qquad i = 1, \ldots, \infty \tag{11}$$

Now with defining $q_i = \varepsilon^2\, \delta_i$, we can reformulate (9-10) to a singularly perturbed system in this form

$$E\,\dot{x} = \hat{A}\,x + B\,u \tag{12}$$

$$y = \hat{C}\,x \tag{13}$$

with $x = (x_0^T, \hat{x}_1^T, \ldots, \hat{x}_m^T)^T$, $\hat{x}_i = (\hat{\delta}_{1i} \quad \hat{\delta}_{2i})^T$,

$$E = \begin{bmatrix} I_2 & 0 \\ 0 & \varepsilon I_{2m} \end{bmatrix}, \hat{C} = (C_0, \varepsilon^2 C_1, \ldots, \varepsilon^2 C_m),$$

$$\hat{A} = diag\,(A_0, \hat{A}_1, \ldots, \hat{A}_m), \hat{A}_i = \begin{bmatrix} 0 & 1 \\ -\tilde{w}_i^2 & -2\xi_i\,\tilde{w}_i \end{bmatrix}, \quad \tilde{w}_i = \varepsilon\, w_i$$

and $\hat{\delta}_{1i} = \delta_i$.

To apply the modelling approach of (Karimi and Yazdanpanah, 2000) to (12-13), we arrange the state of fast dynamics (12), on the basis of decreasing order of their performance levels. Let

$$T_f = diag\,(T_{f1}, \ldots, T_{fm}) \tag{14}$$

denote the balancing transformation matrix for the fast dynamics, where T_{f_i} is the balancing transformation of i th fast subsystem (Shahruz and Behtash, 1988) and it is represented by

$$T_{f_i} = \frac{1}{\tilde{w}_i^2\,\sqrt{1-\xi_i^2}} \begin{bmatrix} \xi_i & \sqrt{1-\xi_i^2} \\ -\tilde{w}_i & 0 \end{bmatrix}. \tag{15}$$

Using T_f in (12-13), we obtain the balanced system in this form

$$E\,\dot{x} = \overline{A}\,x + \overline{B}\,u \tag{16}$$

$$y = \overline{C}\,x \tag{17}$$

where $x = (x_0^T, \overline{x}_1^T, \ldots, \overline{x}_m^T)^T$, $\overline{x}_i = (\delta_{1i} \quad \delta_{2i})^T$, $\overline{A} = diag(A_0, \overline{A}_1, \ldots, \overline{A}_m)$, $\overline{B} = [B_0^T, \overline{B}_1^T, \ldots, \overline{B}_m^T]^T$, and $\overline{C} = [C_0, \overline{C}_1, \ldots, \overline{C}_m]$. The matrices $\overline{A}_i$, $\overline{\mathrm{B}}_i$ and $\overline{C}_i$ are defined as follows:

$$\overline{A}_i = \begin{bmatrix} -\xi_i\,\tilde{w}_i & \tilde{w}_i\,\sqrt{1-\xi_i^2} \\ -\tilde{w}_i\,\sqrt{1-\xi_i^2} & -\xi_i\,\tilde{w}_i \end{bmatrix},$$

$$\overline{B}_i = \frac{\tilde{w}_i\,\dot{\phi}_i(0)}{I_T} \begin{bmatrix} -\sqrt{1-\xi_i^2} \\ \xi_i \end{bmatrix},$$

and

$$\overline{C}_i = \frac{\varepsilon^2 \phi_i(L)}{\tilde{w}_i^2\,\sqrt{1-\xi_i^2}} \begin{bmatrix} \xi_i & \sqrt{1-\xi_i^2} \end{bmatrix}.$$

Consequently, for simplicity of design, the first state is taken as the output, i.e., the tip-position. Taking the output as the first state implies that the first row of $\overline{A}$ should be changed (Yazdanpanah, et al., 1997). The state space model in this case is

$$E\,\dot{x} = A\,x + B\,u \tag{18}$$

$$y = C\,x \tag{19}$$

with $x = (y, \dot{\theta}, \overline{x}_1^T, \ldots, \overline{x}_m^T)^T$ and

$$A = \begin{bmatrix} \overline{A}_0 & \vec{A} \\ 0 & \overleftarrow{A} \end{bmatrix}, \quad \overline{A}_0 = \begin{bmatrix} 0 & L \\ 0 & -\alpha \end{bmatrix}, \; \overleftarrow{A} = diag\,(\overline{A}_1, \ldots, \overline{A}_m),$$

$$\vec{A} = \begin{bmatrix} r_1 & \ldots & r_m \\ 0 & \ldots & 0 \end{bmatrix}, \quad B = \overline{B}, \quad C = \begin{bmatrix} 1 & 0_{2\times m+1} \end{bmatrix},$$

$$r_i = \begin{bmatrix} -\dfrac{\varepsilon\,\phi_i(t)\,\xi_i}{\tilde{w}_i\,\sqrt{1-\xi_i^2}} & 0 \end{bmatrix} \quad (i = 1, \ldots, m).$$

Now, let $v = [\overline{x}_i^T, \ldots, \overline{x}_m^T]^T$, $(i \geq 1)$ denote the part of the state that is to be treated as uncertainty corresponding to unmodeled dynamics. The certain dynamics in this setting corresponds to the state vector $X = [\,y, \dot{\theta}, x_1^T, \ldots, x_{i-1}^T]^T$. Therefore, the state-space representation of the system is in this form

$$E\,\dot{X} = A_X\, X + A_{Xv}\, v + B_X\, u, \qquad y_X = C_X\, X \tag{20}$$

$$\varepsilon\,\dot{v} = A_v\, v + B_v\, u, \qquad y_v = v \tag{21}$$

with

$$A_X = \begin{bmatrix} \overline{A}_0 & \vec{A}_{i-1} \\ 0 & \overleftarrow{A}_{i-1} \end{bmatrix}, \quad A_{Xv} = \begin{bmatrix} \overleftrightarrow{A}_i \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad \overleftarrow{A}_{i-1} = diag\,(\overline{A}_1, \ldots, \overline{A}_{i-1}),$$

$$\vec{A}_{i-1} = \begin{bmatrix} r_1 & \ldots & r_{i-1} \\ 0 & \ldots & 0 \end{bmatrix}, \; \overleftrightarrow{A}_i = \begin{bmatrix} r_i & \ldots & r_m \\ 0 & \ldots & 0 \end{bmatrix}, \; A_v = diag\,(\overline{A}_i, \ldots, \overline{A}_m),$$

$$B_X = \begin{bmatrix} B_0^T & B_1^T & \ldots & B_{i-1}^T \end{bmatrix}^T, \quad B_v = \begin{bmatrix} B_i^T & \ldots & B_m^T \end{bmatrix}^T.$$

In the above representation form, the subsystem (21) may play the role of an uncertainty coupled to the certain subsytem (20) provided that the H_∞ norm of the uncertainty is bounded. The above representation is shown schematically in Figure 1. The plant has two inputs (v, u) and two outputs (Z, y). The first input $y_v = v$ represents the disturbances to be rejected. The second input is the control input *u* that is used for feedback design. The controlled output *Z* represents a penalty variable, which may include a tracking error, as well as a cost of the control input, needed to achieve the prescribed goal. The second output is the measurement output that is made on the system. This is used to generate the control input, which in turn is the tool we have to minimize the effect of the exogenous input on the controlled output. A constraint that is imposed is that the mapping from the measurement to the control input should be such that the closed-loop system is internally stable. The effect of the exogenous input on the controlled output after closing the loop is measured in terms of their energies and the worst-case disturbance of the closed-loop system. Our measure is the closed-loop H_∞ norm, which is simply the L_2 induced norm. Suppose the objective is to only stabilize the system, i.e., the system has no exogenous input. By virtue of the small gain theorem, if the plant is stable, the overall system would remain stable if the product of the L_2 gains of the plant and unmodelled dynamics is less than one.

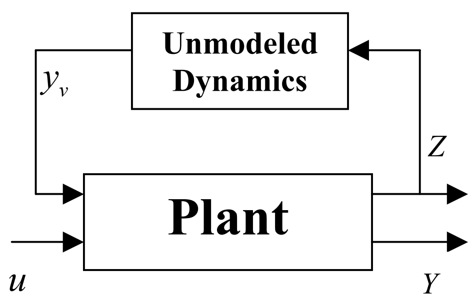

Fig. 1. Partitioning the flexible-link dynamics into two subsystems: plant and unmodelled dynamics.

4 SIMULATION RESULTS

In this section the methodology proposed in this paper is applied to a single–link flexible manipulator of 14th –order with six modes of deflection as the fast dynamics. The objective is to design a regulator so that the tip-position $x_{s1}(t)$ is robustly regulated to zero. The control design proceeds by utilizing the five steps introduced in (Karimi and Yazdanpanah, 2000). The link parameters as well as the natural modes and the corresponding damping ratios used for design and simulation are given in (Yazdanpanah, et al., 1997). By utilizing the procedure of (Karimi and Yazdanpanah, 2000), the minimum value for *i* was found to be four. In other words, three deflection modes are eligible to be considered as uncertainty. By using the relation (11), we obtain $\varepsilon = 0.011$, also, $x_1(0) = 1$ and the initial value of other dynamics are zero. In the mean time, we obtain $\gamma_1 = 0.00372$ and $\gamma_2 = 67.5$ and $\gamma_3 = 1$.

4.1 The state feedback control

Consider $\gamma = 68$ and according to Theorem A1, we apply the state feedback controller to *nominal system* (20). Figure 2, depicts the regulation of tip-position ($x_{s1}(t)$) and other states of the *nominal system,* also Figure 3, depicts the regulation of uncertainty dynamics (Δ). The controller has been depicted in Figure 4 and Figure 5 depicts correctness of the attenuation level of uncertainty dynamics on the controlled output.

4.2 Two-Time Scale Sliding-Mode Control

To illustrate the methodology proposed in this paper and its performance we compare it with at least one of the robust control design approaches according to (Alvarez-Gallegos and Silva-Navarro, 1997). Therefore, we consider the singularly perturbed system

$$\dot{x} = A_{11}x + A_{12}z + B_1 u, \quad x \in \Re^n, u \in \Re^r \tag{22}$$

$$\varepsilon \dot{z} = A_{21}x + A_{22}z + B_2 u, \; z \in \Re^m \tag{23}$$

where A_{ij} and $B_i \, (i, j = 1,2)$ are non-singular matrices with appropriate dimensions. The slow subsystem and the fast subsystem are given by

$$\dot{x}_s = A_0 x_s + B_0 u_s \tag{24}$$

$$\frac{d\hat{\eta}}{d\tau} = A_{22}\hat{\eta} + B_2 u_f \tag{25}$$

where

$$A_0 = A_{11} - A_{12}A_{22}^{-1}A_{21}, \; B_0 = B_1 - A_{12}A_{22}^{-1}B_2 \cdot$$

Suppose that both slow and fast subsystems are stabilizable and slow switching surface $\sigma_s(x_s) = S_s\, x_s$ and fast switching surface $\sigma_f(\hat{\eta}) = S_f\, \hat{\eta}$ with constant matrices S_s and S_f are such that

$$\Re e \lambda_i \{[I_n - B_0(S_s B_0)^{-1} S_s] A_0\} \le -c_s < 0$$

$$\Re e \lambda_i \{[I_m - B_2(S_f B_2)^{-1} S_f] A_{22}\} \le -c_f < 0 \,.$$

Then the two-time scale sliding-mode control

$$u = -G_s\, x - G_f\,(z - Hx) \tag{26}$$

with

$$G_s = (S_s B_0)^{-1}(S_s A_0 + L_s S_s),$$
$$G_f = (S_f B_2)^{-1}(S_f A_{22} + L_f S_f),$$
$$H = -A_{22}^{-1}(A_{21} - B_2 G_s)$$

stabilizes the singularly perturbed syste asymptotically. Select L_s and L_f as positive defin matrices such that $\Re e\,\lambda(-L_s) \le -c_s < 0$ a $\Re e\,\lambda(-L_f) \le -c_f < 0$.

The design parameters for the sliding-mode contro are selected as

$$S_s = [10\verb|^|7 \quad -10\verb|^|4],$$
$$S_f = [2 \quad -2 \quad 2 \quad -2 \quad 2 \quad -2 \quad 2 \quad -2 \quad 2 \quad -2 \quad 2 \quad -2],$$
$$L_s = L_f = 1.$$

We apply the two-time scale sliding-mode control to single-link flexible manipulator. Figure 6, depicts the regulation of tip-position ($x_{s1}(t)$), also Figure 7, depicts the regulation of other states of the *system*. The controller has bin depicted in Figure 8.

5 CONCLUSION

In this paper the modelling of a single–link flexible manipulator utilizing the singular perturbation method was presented. The robust regulation of the tip-position was considered based on a new modelling approach under the assumption of norm-bounded ness of the fast dynamics. In this approach, norm-bounded disturbances and their effect on the tip-position are minimized. Hence, the controller synthesis was performed only for the certain dynamics of the singularly perturbed system. In the comparison between the results obtained with state feedback control and two-time scale sliding-mode control, it was observed that the methodology proposed in this paper achieved a desirable performance.

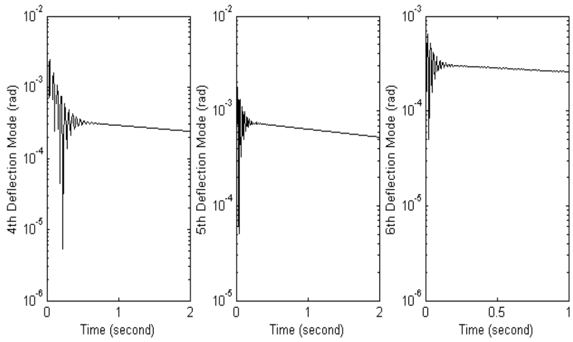

Fig. 3. Absolute values of 3rd-6th deflection modes

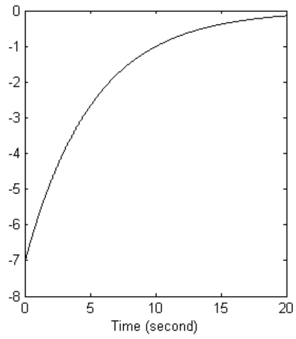

Fig. 4. H_∞ control using state feedback

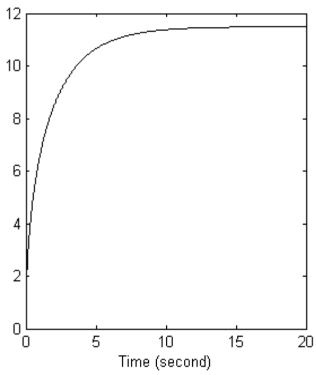

Fig. 5. Attenuation level

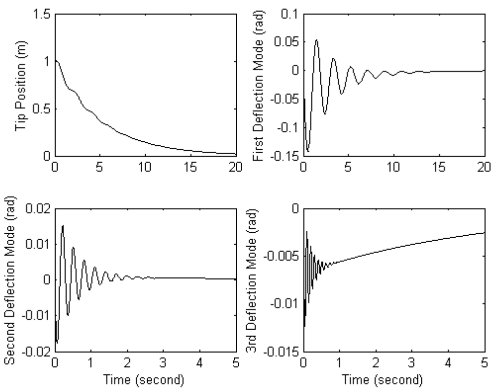

Fig. 2. Regulation of tip position along with other certain dynamics using state feedback

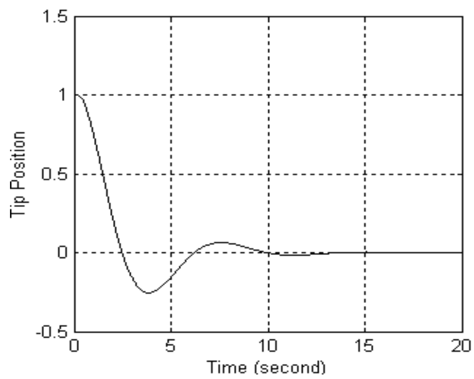

Fig. 6. Tip-position under sliding-mode control

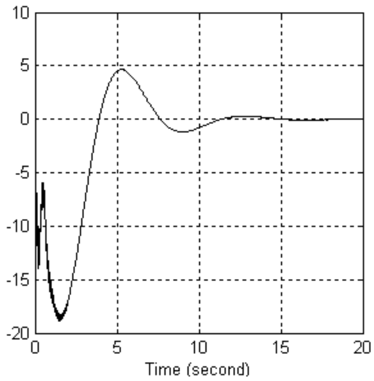

Fig. 7. Two-time sliding-mode control

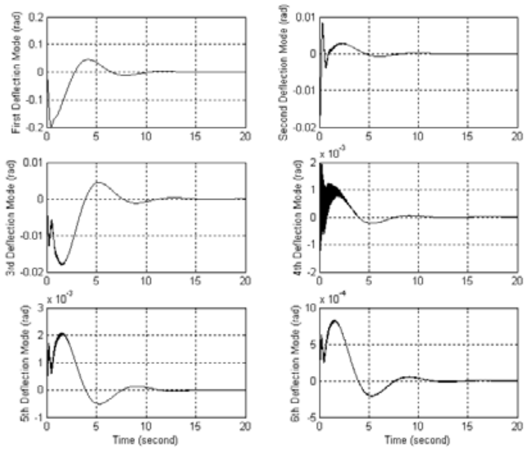

Fig. 8. Fast subsystem dynamics under sliding-mode control

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Appendix

Theorem A1 (Green and Limebeer, 1996)
Under the following assumptions on the *nominal system* (4-8):

i. $(A_X \quad B_X \quad C_2)$ *is stabilizable-detectable.*
ii. $(A_X \quad A_{Xv} \quad C_1)$ *is stabilizable-detectable.*
iii. Rank of matrix D_{12} *is* k *and rank of matrix* D_{21} *is* r.

Then, the algebraic Riccati equation

$$A_X^T X_\infty + X_\infty A_X + X_\infty(\gamma^{-2} A_{Xw} A_{Xw}^T - B_X B_X^T) X_\infty + C_1^T C_1 = 0$$

has a positive semi-definite solution $X_\infty \geq 0$ such that $A_X - (B_X B_X^T - \gamma^{-2} A_{Xw} A_{Xw}^T) X_\infty$ is asymptotically stable. Then the control law $u = -B_X^T X_\infty X \underline{\underline{\Delta}}\, kX$ is stabilizing and satisfies $\|T_{Zw}\|_\infty < \gamma$.