STOCHASTIC STABILITY ANALYSIS AND FUZZY LINEAR CONTROL OF STOCHASTIC NONLINEAR TIME-DELAY SYSTEMS

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Abstract: This paper presents the fuzzy linear control design method for a class of stochastic non-linear time-delay systems with state feedback. First, the Takagi and Sugeno fuzzy linear model is employed to approximate a non-linear system. Next, based on the fuzzy linear model, a fuzzy linear controller is developed to stabilize the non-linear system. The control law is obtained to ensure stochastically exponential stability in the mean square, independent of the time-delay. The sufficient conditions for the existence of such a control are proposed in terms of certain linear matrix inequality. Finally, a simulation example is given to illustrate the applicability of the proposed design method.

Keywords: Fuzzy linear control; Linear matrix inequality; Time-delay systems; Stochastic systems; Exponential stability.

1. INTRODUCTION

Most of the systems, which are encountered in control engineering, contain various nonlinearities and are affected by random disturbance signals. Non-linear systems with time-delay constitute basic mathematical models of real phenomena, for instant in biology, mechanics and economics, see e.g. (Hale, 1997; Niculescu, et al., 1997). Control of time-delay systems has been a subject of great practical importance, which has attracted a great deal of interest for several decades. On the other hand, it turns out that the delayed state is very often the cause for instability and poor performance of systems. Moreover, considerable attention has been given to both the problems of robust stabilization and robust control for linear systems with unavoidable time-varying parameter uncertainties in modelling dynamical systems and certain types of time-delays (Malek-Zavarei and Jamshidi, 1987).

Since the introduction of fuzzy set theory by Zadeh (1973), many people have devoted a great deal of time and effort to both theoretical research and implementation technique for fuzzy logic controllers (Mamdani and Assilian, 1974; Tanaka and Wang, 2001). With the development of fuzzy systems, it is known that the qualitative knowledge of a system can also be represented in non-linear functional form. On the basis of this idea, some fuzzy models based control system design methods have appeared in the fuzzy control field (Chen, et al., 1993; Tanaka and Wang, 2001; Wang, et al., 1996). These methods are conceptually simple and straightforward. Fuzzy controllers are usually characterized using Mamdani and T-S type. In general, Mamdani type fuzzy controllers are designed empirically. However, T-S controllers can be designed using the information of several local linearized models of a given system via the so-called parallel-distributed compensation scheme. Various stability conditions of fuzzy systems have been obtained by employing Lyapunov stability theory (Chen, et al., 1999; Hwang and Lin, 1992; Lam, et al., 2001), passivity theory (Sio and Lee, 1998), and other methods (Feng, et al., 1997; Lee, et al., 2001; Tanaka and Wang, 2001). Problem of control design based on the state feedback for T-S fuzzy systems using LMI approach has been studied by Xiadong and Qingling (2002) and the delay-independent stability of T-S fuzzy model for a class of non-linear time-delay systems investigated by Gu, et al., (2001). Extension of the T-S fuzzy model approach to the stability analysis and control design for both continuous and discrete-time non-linear systems with time-varying delay has been considered by Cao and Frank (2000) and also Lee, et al. (2000) presented design of an output feedback robust H_∞ controller based on T-S fuzzy model for uncertain fuzzy dynamic systems with time-varying delayed state.

Recently, several criteria of input-to-bounded state (IBS) stabilization and bounded-input-bounded-output (BIBO) stabilization in mean square for non-linear and quasi-linear stochastic control systems with time-varying uncertainties has been investigated by Feng and Liao (2003), also, another stability concepts in the mean-square sense such as mean-square stability (MSS) and the internal mean-square stability (IMSS) have been studied by Lu and Skelton (2002). The stabilization of stochastic systems with multiplicative noise has been studied since the late sixties, particularly in the context of linear quadratic optimal control, see e.g., (Mclane, 1971; Willems, 1983). Also, a stochastic fuzzy control has been proposed by applying the stochastic control theory, instead of using a traditional fuzzy reasoning (Watanabe, 1995) and a class of fuzzy stochastic control systems with random delays investigated by Sinha, et al. (2002).

The main contribution of this paper, is the fuzzy linear control problem for a class of stochastic non-linear time-delay systems has been investigated and their attention were focused on the design of state feedback controller which ensures stochastically exponentially stable in the mean square, independent of the time- delay. Finally, the simulation results show that fuzzy linear state feedback controller can achieve the robust stability in the mean square and independent of the time-delay.

2. PRELIMINARIES AND PROBLEM FORMULATION

Consider a class of non-linear continuous-time state delayed stochastic system described by

$$dx(t) = [A(x(t))x(t) + A_d(x(t))x(t-h) + B(x(t))u(t)]dt + E_1\, dw(t) \tag{1}$$

$$x(t) = \varphi(t), \qquad t \in [-h, 0] \tag{2}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in \Re^n$ is the state vector, $u(t) = [u_1(t), u_2(t), \ldots, u_m(t)]^T \in \Re^m$ is the control input, h is the unknown state delay, $\varphi(t)$ is the continuous vector valued initial function and $w(t) = [w_1(t), w_2(t), \ldots, w_n(t)]^T \in \Re^n$ is a scalar Brownian motion defined on the probability space $(\Omega, F, \{F_t\}_{t\geq 0}, P)$.

A fuzzy dynamic model has been proposed by Takagi and Sugeno (1985) to represent local linear input-output relations of non-linear systems. This fuzzy linear model is described by fuzzy If-Then rules and will be employed here to deal with the control design problem of the non-linear system (1-2). The *i*th rule of this fuzzy model for the non-linear system (1-2) is of the following form (Hwang and Lin, 1992; Takagi and Sugeno, 1985; Wang, et al., 1996):

Plant Rule i :

$$\begin{aligned} &\textit{If } z_1(t) \textit{ is } F_{i1} \textit{ and} \ldots \textit{and } z_g(t) \textit{ is } F_{ig}, \\ &\textit{Then } dx(t) = [A_i\, x(t) + A_{id}\, x(t-h) + B_i\, u(t)]dt + E_1\, dw(t) \end{aligned} \tag{3}$$

for $i = 1, 2, \ldots, L$ where F_{ij} is the fuzzy set, $A_i \in \Re^{n\times n}$, $A_{id} \in \Re^{n\times n}$, $B_i \in \Re^{n\times m}$, L is the number of If-Then rules, and $z_1(t), z_2(t), \ldots, z_g(t)$ are the premise variables.

The overall fuzzy system is inferred as follows:

$$\begin{aligned} dx(t) &= \frac{[\sum_{i=1}^{L} \mu_i(z(t))\,(A_i x(t) + A_{id} x(t-h) + B_i u(t))]}{\sum_{i=1}^{L} \mu_i(z(t))} dt + E_1 dw(t) \\ &= \sum_{i=1}^{L} h_i(z(t))\,(A_i x(t) + A_{id} x(t-h) + B_i u(t))\,dt + E_1 dw(t) \end{aligned} \tag{4}$$

where

$$z(t) = [z_1(t), z_2(t), \ldots, z_g(t)] \tag{5}$$

$$\mu_i(z(t)) = \prod_{j=1}^{g} F_{ij}(z_j(t)) \tag{6}$$

$$h_i(z(t)) = \frac{\mu_i(z(t))}{\sum_{i=1}^{L} \mu_i(z(t))} \tag{7}$$

and $F_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in F_{ij}.

Assumption 1: we assume $\mu_i(z(t)) \geq 0$ for $i = 1, 2, \ldots, L$ and $\sum_{i=1}^{L} \mu_i(z(t)) > 0$ for all *t*.

Therefore, we get

$$h_i(z(t)) \geq 0 \tag{8}$$

for $i = 1, 2, \ldots, L$ and

$$\sum_{i=1}^{L} h_i(z(t)) = 1 . \tag{9}$$

Therefore, from (1) we get (Chen, et al. 1999)

$$\begin{aligned} dx(t) &= [A(x(t))x(t) + A_d(x(t))x(t-h) + B(x(t))u(t)]dt + E_1 dw(t) \\ &= [\textstyle\sum_{i=1}^{L} h_i(z(t))\,(A_i x(t) + A_{id} x(t-h) + B_i u(t)) + \{(A(x) - \\ &\textstyle\sum_{i=1}^{L} h_i(z(t))A_i)x(t) + (A_d(x) - \sum_{i=1}^{L} h_i(z(t))A_{id})x(t-h) + (B(x) \\ &\textstyle- \sum_{i=1}^{L} h_i(z(t))B_i u(t))\}]dt + E_1 dw(t) \end{aligned} \tag{10}$$

where

$$\begin{aligned} &\{(A(x) - \textstyle\sum_{i=1}^{L} h_i(z(t))\,A_i)x(t) + (A_d(x) \\ &\textstyle- \sum_{i=1}^{L} h_i(z(t))\,A_{id})x(t-h) + (B(x) - \sum_{i=1}^{L} h_i(z(t))\,B_i u(t))\} \end{aligned} \tag{11}$$

denots the approximation error between the non-linear system (1) and the fuzzy model (4).

Suppose the following fuzzy controller is employed to deal with the above control system design:

Control Rule j :

$$\begin{aligned} &\textit{If } z_1(t) \textit{ is } F_{j1} \textit{ and} \ldots \textit{and } z_g(t) \textit{ is } F_{jg}, \\ &\textit{Then } u(t) = K_j x(t) \end{aligned} \tag{12}$$

for $j = 1, 2, \ldots, L$. Hence, the overall fuzzy controller is given by

$$u(t) = \frac{\sum_{j=1}^{L} \mu_j(z(t))(K_j x(t))}{\sum_{j=1}^{L} \mu_j(z(t))} = \sum_{j=1}^{L} h_j(z(t))\,K_j x(t) \tag{13}$$

where $h_j(z(t))$ is defined in (8) and (9) and K_j are the control parameters.

Substituting (13) into (10) yields the closed-loop non-linear control system as follows:

$$dx(t) = [\{\sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z(t))h_j(z(t))(A_i + B_iK_j)x(t) + A_{id}x(t-h)\}$$
$$+\Delta A + \Delta A_d + \Delta B]dt + E_1\,dw(t) \quad (14)$$

where

$$\Delta A = (A(x(t)) - \sum_{i=1}^{L} h_i(z(t))A_i)\, x(t) \ , \quad (15)$$

$$\Delta A_d = (A_d(x(t)) - \sum_{i=1}^{L} h_i(z(t))A_{id})\; x(t-h)\,, \quad (16)$$

$$\Delta B = \sum_{i=1}^{L} h_i(z(t))\sum_{j=1}^{L} h_j(z(t))(B(x(t)) - B_i)K_j x(t). \quad (17)$$

Assumption 2: There exist bounding matrices ΔA_i, ΔA_{id} and ΔB_i such that for all trajectory $x(t)$

$$\|\Delta A\| \le \left\|\sum_{i=1}^{L} h_i(z(t))\,\Delta A_i\; x(t)\right\|, \quad (18)$$

$$\|\Delta A_d\| \le \left\|\sum_{i=1}^{L} h_i(z(t))\,\Delta A_{id}\; x(t-h)\right\|, \quad (19)$$

$$\|\Delta B\| \le \left\|\sum_{i=1}^{L} h_i(z(t))\sum_{j=1}^{L} h_j(z(t))\,\Delta B_i\; K_j x(t)\right\|, \quad (20)$$

and the bounding matrices ΔA_i, ΔA_{id} and ΔB_i can be described by

$$\begin{bmatrix} \Delta A_i \\ \Delta A_{id} \\ \Delta B_i \end{bmatrix} = \begin{bmatrix} \delta_i A_p \\ \delta_{id} A_{pd} \\ \eta_i B_p \end{bmatrix} \quad (21)$$

where $\|\delta_i\| \le 1$, $\|\delta_{id}\| \le 1$ and $\|\eta_i\| \le 1$, for $i = 1,2,\ldots,L$ (Boyd, et al. 1994).
According to assumption 2, we get

$$(\Delta A)^T(\Delta A) \le (A_p x(t))^T(A_p x(t))\,, \quad (22)$$

$$(\Delta A_d)^T(\Delta A_d) \le (A_{pd}x(t-h))^T(A_{pd}x(t-h))\,, \quad (23)$$

$$(\Delta B)^T(\Delta B) \le (\sum_{j=1}^{L} h_j(z(t))B_pK_jx(t))^T(\sum_{j=1}^{L} h_j(z(t))B_pK_jx(t)). \quad (24)$$

Next, observe the closed-loop system (14) and let $x(t;\zeta)$ denote the state trajectory from the initial data $x(\theta) = \zeta(\theta)$ on $-h \le \theta \le 0$ in $L^2_{F_0}([-h,0];\Re^{2n})$. Clearly, the system (14) admits a trivial solution $x(t;0) \equiv 0$ corresponding to the initial data $\zeta = 0$. We introduce the following stability and stabilizability concepts.

Definition 1 (Wang, et al., 2001): For the system (14) and every $\zeta \in L^2_{F_0}([-h,0];\Re^{2n})$, the trivial solution is asymptotically stable in the mean square if

$$\lim_{t\to\infty} E|x(t;\zeta)|^2 = 0\,, \quad (25)$$

and is exponentially stable in the mean square if there exist constants $\alpha > 0$ and $\beta > 0$ such that

$$E|x(t;\zeta)|^2 \le \alpha\, e^{-\beta t} \sup_{-h\le\theta\le 0} E|\zeta(\theta)|^2. \quad (26)$$

Definition 2 (Wang, et al., 2001): we say that the system (1-2) is exponentially stabilizable in mean square if, for every $\zeta \in L^2_{F_0}([-h,0];\Re^{2n})$, there exists a fuzzy linear control law (13) such that the resulting closed-loop system is exponentially stable in mean square.

The objective of this paper is to design a fuzzy linear control for the stochastic non-linear time-delay system (1-2). More specifically, we are interested in seeking the control parameters K_j, for $j = 1,2,\ldots,L$, such that the closed-loop system (14) is exponentially stable in mean square, independent of the unknown time-delay.

3. MAIN RESULTS AND PROOFS

We first give the following lemma, which will be used in the proof of our main results.
Lemma 1 (Zhou and Khargonekar, 1988): For any matrices X and Y with appropriate dimensions and for any constant $\eta > 0$, we have:

$$X^T Y + Y^T X \le \eta\, X^T X + \frac{1}{\eta} Y^T Y\,. \quad (27)$$

3.1. Stochastic Stability Analysis

In this section, assuming that the fuzzy linear control is known and we will study the conditions under which the closed-loop system is stochastically exponentially stable in the mean square. The following theorem will play a key role in the stability analysis of closed-loop system and design of the expected fuzzy linear control.

Theorem 1: Let the control parameters K_j, for $j = 1,2,\ldots,L$, be given. If the fuzzy controller (13) is employed in the non-linear system (1-2) and there exists positive scalars $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4$ and a positive definite matrix $P = P^T$ such that the following matrix inequalities

$$(A_i + B_iK_j)^T P + P(A_i + B_iK_j) + (\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4)P^2$$
$$+\varepsilon_1^{-1} A_{id}^T A_{id} + \varepsilon_2^{-1} A_p^T A_p + \varepsilon_3^{-1} A_{pd}^T A_{pd} + \varepsilon_4^{-1}(B_pK_j)^T(B_pK_j) < 0 \quad (28)$$

are satisfied for all $i, j = 1,2,\ldots,L$, then the closed-loop non-linear system (14) is exponentially stable in the mean square and independent of the unknown time-delay h.

Proof: Fix $\zeta \in L^2_{F_0}([-h,0];\Re^{2n})$ arbitrarily, and write $x(t;\zeta) = x(t)$. We define the Lyapunov function candidate

$$\Upsilon(x(t),t) = x^T(t)\,P\,x(t) + \int_{t-h}^{t} x^T(s)\,Q\,x(s)\,ds \quad (29)$$

where $P = P^T$ is the positive definite solution to the matrix inequality (28) and $Q = Q^T > 0$ is defined by

$$Q := \varepsilon_1^{-1}(\sum_{i=1}^{L} h_i(z(t))\,A_{id})^T(\sum_{i=1}^{L} h_i(z(t))\,A_{id}) + \varepsilon_3^{-1} A_{pd}^T\, A_{pd}\,. \quad (30)$$

The stochastic differential of Υ along a given trajectory is obtained as

$$d\Upsilon(x(t),t) = \{x^T(t)(\{\sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z(t))h_j(z(t))(A_i + B_iK_j)\}^T P$$
$$+Q)x(t) + x^T(t-h)(\sum_{i=1}^{L} h_i(z(t))\,A_{id})^T Px(t) + x^T(t)P(\sum_{i=1}^{L} h_i(z(t))\,A_{id})$$
$$\times x(t-h) + x^T(t)P(\sum_{i=1}^{L}\sum_{j=1}^{L} h_i(z(t))h_j(z(t))(A_i + B_iK_j))x(t) + (\Delta A$$
$$+\Delta A_d + \Delta B)^T Px(t) + x^T(t)P(\Delta A + \Delta A_d + \Delta B) - x^T(t-h)Qx(t-h)\}dt$$
$$+2x^T(t)PE_1\,dw(t)\,. \quad (31)$$

Now, by *Lemma 1*, it is trivial to show that for any positive scalars of $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4$ the following matrix inequalities hold:

$$((\sum_{i=1}^{L} h_i(z(t))A_{id})x(t-h))^T P x(t)$$
$$+x^T(t)P((\sum_{i=1}^{L} h_i(z(t))A_{id})x(t-h)) \le \varepsilon_1 x^T(t)P^2 x(t) + \varepsilon_1^{-1} x^T(t-h)$$
$$\times(\sum_{i=1}^{L} h_i(z(t))A_{id})^T(\sum_{i=1}^{L} h_i(z(t))A_{id})x(t-h) \quad (32)$$

and

$$(\Delta A)^T P x(t) + x^T(t)P(\Delta A) \le x^T(t)(\varepsilon_2 P^2 + \varepsilon_2^{-1} A_p^T A_p)x(t) \quad (33)$$

$$(\Delta A_d)^T P x(t) + x^T(t)P(\Delta A_d) \le \varepsilon_3 x^T(t)P^2 x(t) + \varepsilon_3^{-1} x(t-h)^T A_{pd}^T A_{pd} x(t-h) \quad (34)$$

$$(\Delta B)^T P x(t) + x^T(t)P(\Delta B) \le x^T(t)(\varepsilon_4 P^2 + \varepsilon_4^{-1}(\sum_{j=1}^{L} h_j(z(t))B_p K_j)^T(\sum_{j=1}^{L} h_j(z(t))B_p K_j))x(t) \quad (35)$$

Then, noticing the definition (30), substituting (32-35) into (31) results in

$$d\Upsilon(x(t),t) \le -\sum_{i=1}^{L}\sum_{j=1}^{L} \lambda_{\min}(-\Pi_{ij})x^T(t)\,x(t)\,dt + 2x^T(t)PE_1\,dw(t) \quad (36)$$

where

$$\Pi_{ij} := (A_i + B_i K_j)^T P + P(A_i + B_i K_j) + (\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4)P^2 + \varepsilon_1^{-1} A_{id}^T A_{id} + \varepsilon_2^{-1} A_p^T A_p + \varepsilon_3^{-1} A_{pd}^T A_{pd} + \varepsilon_4^{-1}(B_p K_j)^T(B_p K_j) \quad (37)$$

Then, according to the inequality (28), we find

$$\Pi_{ij} < 0, \quad for\ i,j = 1,2,\ldots,L\,. \quad (38)$$

Consequently, the inequalities (36) and (38) mean that the non-linear stochastic time-delay closed-loop system (14) is asymptotically stable (in the mean square) by the fuzzy control law (13).

The expected exponential stability (in the mean square) of the closed-loop system (14) can be proved by making some standard manipulation on (36), see (Mao, 1996). Let β_{ij} be the unique root of the equation

$$\lambda_{\min}(-\Pi_{ij}) - \beta_{ij}\,\lambda_{\max}(P) - \beta_{ij}\,h\,\lambda_{\max}(Q)e^{\beta_{ij}h} = 0 \quad (39)$$

where Π_{ij} and Q are defined, respectively, in (37) and (30) and P is the positive definite solution to (28) and h is the unknown time-delay. Then, by Wang and Burnham, (2001), we have

$$E|x(t)|^2 \le \lambda_{\min}^{-1}(P)\,([\lambda_{\max}(P) + h\,\lambda_{\max}(Q)] + \beta_{ij}\,\lambda_{\max}(q)h^2 e^{\beta_{ij}h})\ \underset{-h\le\theta\le 0}{Sup}\ E|\zeta(\theta)|^2 e^{-\beta_{ij}t} \quad (40)$$

Notice that, according to (40), the definition of exponential stable in *Definition 1* is satisfied and this completes the proof of *Theorem 1*.
The result of Theorem 1 may be conservative due to the use of inequalities (32-35). However, such conservativeness can be significantly reduced by appropriate choices of the parameters ε_1, ε_2, ε_3, ε_4 in a matrix norm sense.

3.2. Fuzzy Control Design

This subsection is devoted to the design of control parameters K_j, for $j = 1,2,\ldots,L$, by using the result in Theorem 1. We will show that the design of control parameters problem can be solved via the resolution of matrix inequalities. Our approach follows the one developed by Gahinet for the deterministic case (Fu and Liao, 2003). The key tool, which makes this possible, is the stochastic version of the Bounded Real Lemma. From deterministic H_∞ control theory we will need the following lemma, so-called, *Projection Lemma*.

Lemma 2 (Xu and Chen, 2002): Given a symmetric matrix $\mathrm{H} \in \Re^{m\times m}$ and two matrices $\mathrm{N} \in \Re^{l\times m}$ and $\mathrm{M} \in \Re^{n\times m}$, consider the problem of finding some matrix X such that

$$\mathrm{H} + \mathrm{N}^T \mathrm{X}^T \mathrm{M} + \mathrm{M}^T \mathrm{X} \mathrm{N} < 0 \quad (41)$$

Then, (41) is solvable for X if and only if

$$\mathrm{N}^{T\perp} H\, \mathrm{N}^{T\perp T} < 0, \quad \mathrm{M}^{T\perp} H\, \mathrm{M}^{T\perp T} < 0 \quad (42)$$

here, if $\Sigma \in \Re^{n\times m}$ and rank $\Sigma = r$, the orthogonal complement $\Sigma^\perp$ is defined as a possibly nonunique $(n-r)\times n$ matrix with rank $n-r$, such that $\Sigma^\perp\Sigma = 0$.

By using the *Schur complement formula*, inequality (28) is equivalent to

$$\begin{bmatrix} (A_i + B_i K_j)^T P + P(A_i + B_i K_j) + \Psi_i^T \Psi_i & (B_p K_j)^T & P \\ B_p K_j & -\varepsilon_4 I & 0 \\ P & 0 & -(\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4)^{-1} I \end{bmatrix} < 0 \quad (43)$$

where

$$\Psi_i := \begin{bmatrix} \varepsilon_1^{\frac{-1}{2}} A_{id} \\ \varepsilon_2^{\frac{-1}{2}} A_p \\ \varepsilon_3^{\frac{-1}{2}} A_{pd} \end{bmatrix} \quad (44)$$

The inequality (43) has the form

$$\Gamma_i + \mathrm{N}_i^T \Omega \mathrm{M} + \mathrm{M}^T \Omega^T \mathrm{N}_i < 0 \quad (45)$$

where

$$\Omega := K_j,\ \mathrm{M} := \begin{bmatrix} I & 0 & 0 \end{bmatrix},\ \mathrm{N}_i^T := \begin{bmatrix} PB_i \\ B_p \\ 0 \end{bmatrix} = \begin{bmatrix} P & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}\begin{bmatrix} B_i \\ B_p \\ 0 \end{bmatrix},$$

$$\Gamma_i := \begin{bmatrix} A_i^T P + PA_i + \Psi_i^T \Psi_i & 0 & P \\ 0 & -\varepsilon_4 I & 0 \\ P & 0 & -(\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4)^{-1} I \end{bmatrix}. \quad (46)$$

Then, we have the following result.

Theorem 2: The closed-loop fuzzy system (14) is exponentially stable in the mean square and independent of the unknown time-delay h, if the following conditions are satisfied, for $i = 1,2,\ldots,L$,

$$\begin{cases} \mathrm{N}_i^{T\perp}\,\Gamma_i\,\mathrm{N}_i^{T\perp T} < 0 \\ \mathrm{M}^{T\perp}\,\Gamma_i\,\mathrm{M}^{T\perp T} < 0 \\ P = P^T > 0 \end{cases} \quad (47)$$

where M, N_i and Γ_i are defined in (46).
Proof: The proof follows directly from Theorem 1 and Projection lemma.

Let $\begin{bmatrix} V_{1i} & V_2 \end{bmatrix} = \begin{bmatrix} B_i & B_p \end{bmatrix}^{T\perp}$ and, by some calculation, we have:

$$\mathrm{N}_i^{T\perp} = \begin{bmatrix} V_{1i} & V_2 & 0 \\ 0 & 0 & I \end{bmatrix}\begin{bmatrix} P^{-1} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}, \quad (48)$$

and

$$\mathrm{M}^{T\perp} = \begin{bmatrix} 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \quad (49)$$

Then, it follows from (47) that we have:

$$\mathrm{M}^{T\perp}\,\Gamma_i\,\mathrm{M}^{T\perp T} = \begin{bmatrix} -\varepsilon_4 I & 0 \\ 0 & -(\varepsilon_1+\varepsilon_2+\varepsilon_3+\varepsilon_4)^{-1} I \end{bmatrix} < 0 \qquad (50)$$

This further implies that $\mathrm{M}^{T\perp}\,\Gamma_i\,\mathrm{M}^{T\perp T} < 0$ is satisfied for $i = 1, 2, \ldots, L$ and

$$\mathrm{N}_i^{T\perp}\,\Gamma_i\,\mathrm{N}_i^{T\perp T} = \begin{bmatrix} \begin{bmatrix} V_{1i} & V_2 \end{bmatrix} \begin{bmatrix} P^{-1}(A_i^T P + PA_i + \Psi_i^T \Psi_i) P^{-1} & 0 \\ 0 & -\varepsilon_4 I \end{bmatrix} \begin{bmatrix} V_{1i}^T \\ V_2^T \end{bmatrix} & \begin{bmatrix} V_{1i} & V_2 \end{bmatrix} \begin{bmatrix} I \\ 0 \end{bmatrix} \\ \begin{bmatrix} I & 0 \end{bmatrix} \begin{bmatrix} V_{1i}^T \\ V_2^T \end{bmatrix} & -(\varepsilon_1+\varepsilon_2+\varepsilon_3+\varepsilon_4)^{-1} I \end{bmatrix} < 0 . \qquad (51)$$

Using the Schur complement formula, it is easy to see that (51) is equivalent to

$$A_i^T P + P A_i + (\varepsilon_1+\varepsilon_2+\varepsilon_3+\varepsilon_4) P^2 + \Psi_i^T \Psi_i < 0 . \qquad (52)$$

If the LMI in (52) have a positive-definite solution for P, then the closed-loop system (14) is exponentially stable in the mean square and independent of the unknown time-delay h. Moreover, in this case, a set of particular solutions of control parameters K_j, for $j = 1, 2, \ldots, L$, corresponding to a feasible solution P can be obtained by using the result of matrix inequality (52). Then, we obtain the following result:

Theorem 3: If there exist positive scalars $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4$ such that the linear matrix inequality (52) has positive definite solution P, then, the fuzzy control with parameters $\Omega := K_j$ for $j = 1, 2, \ldots, L$ can be easily obtained by solving (45) and will be such that the closed-loop system (14) is exponentially stable in the mean square and independent of the unknown time-delay h.

4. SIMULATION RESULTS

In this section, to illustatrte the effectiveness of the proposed method, we will design a fuzzy linear controller for the following stochastic non-linear time-delay system

$$dx(t) = [-0.06\,x(t)^3 + x(t-h) + u(t)]dt + dw(t) \qquad (53)$$

$$x(t) = 1, \qquad t \in [-h, 0] . \qquad (54)$$

Consider $h = 1$ second as the time-delay parameter. To use the fuzzy linear controller design, we consider a fuzzy model, which represents the dynamics of the non-linear plant. Therefore, we represent the system (53-54) by the following T-S fuzzy model

Plant Rule 1:

If $x(t)$ *is* F_{11},

Then $dx(t) = [-3\,x(t) + 0.5\,x(t-h) + 2\,u(t)]dt + dw(t)$

Plant Rule 2:

If x *is* F_{21},

Then $dx(t) = [-2\,x(t) + 0.1\,x(t-h) + u(t)]dt + dw(t)$

where the membership functions of F_{11} and F_{21} are given as follows:

$$F_{11} = 1 - \frac{1}{1+e^{-x^2}} , \quad F_{21} = 1 - F_{11} = \frac{1}{1+e^{-x^2}} ,$$

and the bounding matrices are chosen as $A_p = 0.5$, $A_{pd} = 0.5$ and $\mathrm{B}_p = 1$.

Substituting the above parameters into Theorem 3, using the LMI toolbox in MATLAB the solutions of (45), i.e., state feedback gains, can be obtained as $K_1 = 0.1$ and $K_2 = 0.1709$ and the positive scalars $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4$ found as $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_4 = 0.1$.

Robust stability of the state of system (53) in the presence of disturbance, i.e. Brownian motions has been depicted in Figure 1 and it is seen that due to Brownian motion as the external disturbance, state still is bounded. The overall fuzzy controller is shown in Figure 2.

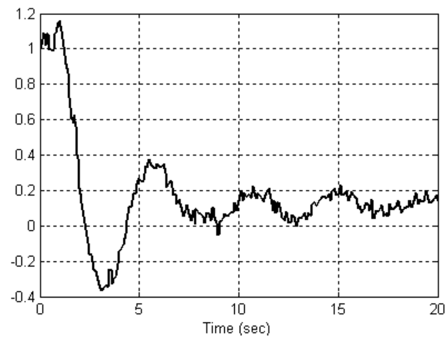

Fig. 1. Time behavior of the state of system

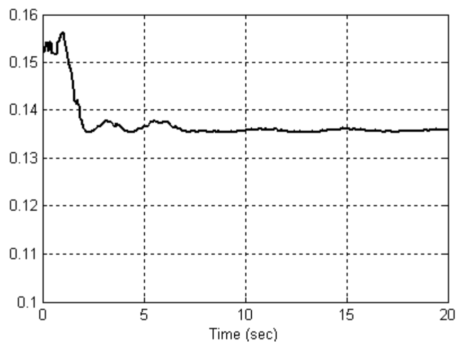

Fig. 2. Control input

5. CONCLUSIONS

In this paper, the fuzzy linear control design method for a class of stochastic non-linear time-delay systems with state feedback was developed. First, the Takagi and Sugeno fuzzy linear model was employed to approximate a non-linear system. Next, based on the fuzzy linear model, a fuzzy linear controller was developed to stabilize the non-linear system. The control law has been obtained such that ensures stochastically exponentially stable in the mean square, independent of the time-delay and the sufficient conditions for the existence of such a control was proposed in terms of certain linear matrix inequality. A simulation example was given to illustrate the applicability of the proposed design method.

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