UNCERTAINTY IN CONTROL PROBLEMS: A SURVEY

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Abstract: Plant complexity inevitably leads to poor models that exhibit a high degree of parametric or functional uncertainty. The situation becomes even more complex if the plant to be controlled is characterized by multi-valued function or even if it exhibits a number of modes of behavior during its operation. Recently, control engineers and theorist have developed new control techniques under the framework of intelligent control to enhance the performance of the controller for more complex and uncertain plants. Basically, those techniques are based on incorporating models uncertainty which are proven to give more accurate control results under uncertain conditions. In this paper we survey some approaches that appear to be promising for enhancing the performance of intelligent control systems in the face of higher level of complexity and uncertainty.

Keywords: Uncertainty, Neural networks, Bayesian methods, Mixture density network, Conditional distributions, Adaptive control, Multiple model approaches

1. INTRODUCTION

In control engineering the main objective is to ensure that a system of interest performs according to predefined desired specifications, often under conditions of uncertainty. In general, uncertainty in control problems arises because of insufficient knowledge about the system itself or the environment in which it operates. This could be due to: the nondeterministic relationship between the input and the output variables of the system, the system itself is too complex to represent and the unexpected changes in the system characteristics because of failure or time varying properties.

A large number of control techniques, including classical control, optimal control, and robust control have been developed to achieve the above goal. Unfortunately, most of these developed control system design methods based on the availability of a mathematical model, which is generally characterized by parametric or functional uncertainty. Models uncertainty is usually inevitable when identifying highly complex systems. This is not simply because of the uncertainty arising from the estimated parameters or functions. It concerns also the structure and the complexity of the model and the appropriate choice of the cost function which provides basic ground to optimize model parameters. For such complex systems, neural networks have been the key solution for getting more accurate models and better generalization properties.

Although neural networks have helped estimating more accurate models, it has been shown that for most real world control problems with unpredictable disturbance and which exhibit a number of distinct modes of behavior during their operation, the predicted output of the neural network is inherently uncertain. So how could we proceed in such situations? The solution to this problem has been addressed from the point of view of estimating and incorporating uncertainty. It has been argued that for such complex systems, it may be the case that incorporating uncertainty in control design can help providing better control result.

Recently many publications have considered the use of knowledge of uncertainty to build a more robust controller.

Consequently new control algorithms have been developed for incorporating uncertainty in control design. The purpose of this paper is to provide an overview of some of the recently developed control algorithms that have been reported in the literature concerning this subject.

2. INTELLIGENT CONTROL SYSTEMS

In neuro-control field a large number of publications describing the use of neural network models for control of linear and nonlinear systems have been published this decade. As a result a number of design approaches have been developed in the literature. Almost all theses approaches assume the availability of an accurate mathematical model. This assumption however, is not reliable when speaking about complex nonlinear control problems which are often also exhibit a number of distinct modes of behavior during their operation. The purpose of this section is to give an overview for some of the current available control techniques, which are derived based on the above assumption.

One of the neural network control schemes that is based on supervised learning methods and the assumption of an accurate model is the inverse control methodology (Hunt *et al.*, 1992; Albus, 1975). In the direct inverse control scheme (Psaltis *et al.*, 1988), the inverse of the plant to be controlled is modeled offline. Here the unity operator between the reference input and the output is approximated by connecting the plant inverse model network in series with the plant in an open loop configuration. This scheme however, is not very robust due to the absence of feedback. A more robust scheme that belongs to the inverse control methodology, utilizes a second neural network that has been trained previously to model the dynamics of the plant (Hunt and Sbarbaro, 1991). It is called internal model control (IMC).

Adaptive control technique (Narendra and Annaswamy, 1989; Åström and Wittenmark, 1989) has been used in situations where the plant parameters are uncertain, because it is able to maintain adequate performance in the presence of unknown or time varying parameters. Mainly two methods have been reported in conventional adaptive control for handling adaptation: the direct and indirect methods (Narendra and Annaswamy, 1989; Åström and Wittenmark, 1989). Concepts from conventional adaptive control theory have been extended naturally to the neural control case (Narendra and Parthasarathy, 1990). This is because the neural network models approximate nonlinear functions by a parameterized model. The paper by Narendra and Parthasarathy (Narendra and Parthasarathy, 1990) is often considered as the pioneering paper in this field. It concentrated on discrete time systems and introduced four different classes of models. The paper has dealt with adaptive control for nonlinear plants. It focused on indirect adaptive control because of the lack of methods for directly adjusting the parameters of the controller using only output error between the plant and the reference input.

The most complex approach than the previously mentioned, but most powerful under uncertain conditions is the adaptive critic family. This approach approximates dynamic programming which is one of the optimal methods for deriving the optimal control law for stochastic nonlinear control problems. Its application to real world nonlinear stochastic systems is proven to be powerful and computationally feasible.

However, all neural control techniques can hardly give good control result when the plant is operating in suddenly-changing environment or when it exhibits different features in different zones of its input. This type of plant complexity is called multimodality (Fabri and Kadirkamanathan, 2001) and usually are not accommodated for in the above mentioned neural control techniques. In conventional control methods, this type of plant complexity has been handled by the use of multiple models. The multiple model control approach has been developed from the partitioning theory of adaptive control (Lainiotis, 1976). The successful use of multiple models in real time applications for control has been widely reported in the literature (Rauch, 1995; Gustafson and Maybeck, 1994; Maybeck and Stevens, 1991; Narendra *et al.*, 1995; Narendra and Balakrishnan, 1997; Narendra and Balakrishnan, 1994; Fabri and Kadirkamanathan, 2001; Jacobs and Nowlan, 1991; Johansen and Foss, 1995). Multiple model control techniques have been recently applied to the neural network models. Consequently, a number of multiple neural network methods such as the mixture of expert approach (Jacobs and Nowlan, 1991; Jordan and Xu, 1993; Jordan and Jacobs, 1994; Bishop, 1995), multiple paired forward and inverse models (Wolpert and Kawato, 1998) and the mixture density network approach (Herzallah and Lowe, 2004*b*) have been developed.

3. INCORPORATING UNCERTAINTY

In this section a discussion about how uncertainty knowledge is incorporated in the three neural control methods discussed in the previous section is provided. The next section will survey some of the multiple models approaches for incorporating uncertainty.

3.1 Direct Inverse Adaptive Control

In the direct inverse adaptive control, the controller is learning to recreate the input that created the desired output of the plant (Hunt and Sbarbaro, 1991; Psaltis *et al.*, 1988; Albus, 1975; Baughman and Liu, 1995).

Here the error for adapting the controller is the command error. Using the command error for adapting the controller rather than trajectory error has several drawbacks (Hunt and Sbarbaro, 1991; Psaltis *et al.*, 1988). Mainly, the learning procedure is not goal directed, since in control minimizing the trajectory error (the difference between the system and the desired outputs) rather than command error is required. In addition, obtaining the inverse of the system may not be possible in problems where the mapping is not one-to-one.

To overcome these problems researchers considered the use of model uncertainty (Herzallah and Lowe, 2004*b*; Herzallah and Lowe, 2004*a*). In (Herzallah and Lowe, 2004*b*) a novel inversion-based neurocontroller for solving control problems involving uncertain nonlinear systems which could also compensate for multi-valued system (where the mapping is not one-to-one) is introduced. The approach is based on modeling the conditional distributions of both forward and inverse models.

In their work (Herzallah and Lowe, 2004*b*) the conditional distributions of the models are assumed to be Gaussian. The Gaussian assumptions for the forward and inverse models are based on Theorem 4.2.1 in (Gersho and Gray, 1992). The theorem states that minimum mean squared error (MMSE) estimate of a random vector y given another random vector x is simply the conditional expectation of y given x, $\hat{\mathrm{y}} = E(\mathrm{y} \mid x)$. Based on this theorem the variance for each input pattern x is shown to be given by $\| \mathrm{y} - \hat{\mathrm{y}} \|^2$ (Herzallah and Lowe, 2004*b*). Another neural network which takes this variance as a target value has been used in (Herzallah and Lowe, 2004*b*) to model the conditional expectation of that variance, $\sigma^2(x) = E(\| \mathrm{y} - \hat{\mathrm{y}} \|^2 \mid x)$.

Rather than taking the conditional expectation from the inverse controller to represent the control signal to be forwarded to the plant, they suggested searching for the optimal control signal by generating samples from the conditional distribution of the inverse controller. Based on importance sampling from that distribution, the optimal control law is taken to be the one that minimizes the following performance index

$$J(k) = \underset{u \in U}{Min} \underset{v}{E}[(\hat{\mathrm{y}}(k+d) - \mathrm{y}_{ref}(k+d))^2 + \sigma_\xi^2], \quad (1)$$

where U is a vector containing the sampled values from the control signal distribution, E is the expected value of the cost function over the random noise variable v, and σ_ξ^2 is the variance of the uncertainty of the forward model.

Generating samples from the conditional distribution of the inverse controller and then finding the control signal that minimizes a performance index of the form given in (1) makes the direct inverse control approach goal directed in terms of minimizing trajectory error rather than command error. Moreover, searching for the control signal that minimizes the performance index in (1) rather than using gradient information of (1) guarantees obtaining the absolute minimum of the performance index rather than a relative minimum (Bellman, 1962). Finally, for systems driven by a random forcing component the searching method allows approximating the integral of the utility function over the random variable, which is not easy to be done analytically, by the finite sum as given in (1).

A Bayesian framework for deriving the conditional distribution of the inverse controller has been proposed in (Herzallah and Lowe, 2004*a*). Here, the negative log posterior with respect to the control signal is minimized instead of minimizing the mean squared error function in the conventional direct inverse control method. The proposed Bayesian scheme was for a general nonlinear plant having the following form

$$\mathrm{y}(k+d) = f(\mathrm{y}(k), \ldots, \mathrm{y}(k-n), u(k), \ldots, u(k-n-1)), \quad (2)$$

where $\mathrm{y}(k)$ is the measured plant output vector, $u(k)$ is the measured plant input vector, n is the plant order, d is a known plant delay, and $f(.)$ is an unknown nonlinear function.

Based on Theorem 4.2.1 (Gersho and Gray, 1992) and the result reported in (Bishop, 1995) , the stochastic model of the system given in (2) is firstly identified. It is assumed to be described by a Gaussian probability density function (pdf) with a global covariance matrix, R^{-1}.

$$p(\mathrm{y}(k+d) \mid \mathbf{y}(k), u(k)) \propto \exp\Big(-\frac{1}{2}[\mathrm{y}(k+d) - \bar{f}(\mathbf{y}(k), u(k))] R^{-1}[\mathrm{y}(k+d) - \bar{f}(\mathbf{y}(k), u(k))]\Big), \quad (3)$$

where $\mathbf{y}(k) = [\mathrm{y}(k), \ldots, \mathrm{y}(k-n), u(k-1), \ldots, u(k-n-1)]$ is the vector of previous outputs and inputs values and $\bar{f}(\mathrm{y}(k), u(k))$ is the conditional expectation of the forward model.

Given the conditional distribution of the forward model (3), and a Gaussian prior distribution of the control signal denoted by $p(u(k) \mid \mathrm{y}_d(k+d), \mathbf{y}(k))$, where $\mathrm{y}_d(k+d)$ is the desired system output, the posterior probability distribution of the control signal, $p(u(k) \mid \mathrm{y}(k+d), \mathrm{y}_d(k+d), \mathbf{y}(k))$ is shown, using Bayes rule, to be given by

$$p(u(k) \mid \mathrm{y}(k+d), \mathrm{y}_d(k+d), \mathbf{y}(k)) = \frac{p(\mathrm{y}(k+d) \mid u(k), \mathbf{y}(k)) p(u(k) \mid \mathrm{y}_d(k+d), \mathbf{y}(k))}{p(\mathrm{y}(k+d) \mid \mathbf{y}(k))}. \quad (4)$$

The optimal control law $u^{opt}(k)$, is then derived by minimizing the negative log posterior of (4) with respect to $u(k)$

$$-\log p(u(k) \mid \mathrm{y}(k+d), \mathrm{y}_d(k+d), \mathbf{y}(k)) \propto \frac{1}{2}[\mathrm{y}(k+d)-\bar{f}(\mathbf{y}(k), u(k))]R^{-1}[\mathrm{y}(k+d)-\bar{f}(\mathbf{y}(k), u(k))] + \frac{1}{2}[u(k) - \hat{u}(k)]\hat{P}^{-1}[u(k) - \hat{u}(k)] - \frac{1}{2}[\mathrm{y}(k+d) - \hat{\mathrm{y}}(k+d)]Q^{-1}[\mathrm{y}(k+d) - \hat{\mathrm{y}}(k+d)], \tag{5}$$

where $\hat{P}^{-1}$ is the covariance matrix of the prior distribution of control signals, $\hat{u}(k)$ is the mean of the prior which is equal to the estimated control signal value, $\hat{\mathrm{y}}(k+d)$ is the best prediction of the system output given an estimate of the control signal, and Q^{-1} is the covariance matrix of the evidence, $p(\mathrm{y}(k+d) \mid \mathbf{y}(k))$.

For linear systems the update equation for the control signal, $u^{opt}(k)$, and its variance, P^{opt}, is shown to be given by (Herzallah and Lowe, 2004*a*)

$$\begin{aligned} u^{opt}(k) &= \hat{u}(k) + \Gamma e(k), \\ P^{opt} &= (I - \Gamma \hat{B})\hat{P}, \end{aligned} \tag{6}$$

where Γ is known as the Bayesian gain, $e(k)$ is the error, and $\hat{B}$ is the derivative of the forward linear model with respect to $u(k)$

For nonlinear systems the optimal control law is shown to be given by deriving equation (5) with respect to $u(k)$ and setting the derivative equal to zero (Herzallah and Lowe, 2004*a*).

$$[\mathrm{y}(k+d) - \bar{f}(\mathrm{y}(k), u(k))]\frac{\partial \bar{f}(\mathbf{y}(k), u(k))}{\partial u(k)}R^{-1} = (u(k) - \hat{u}(k))\hat{P}^{-1} = 0. \tag{7}$$

A nonlinear optimization method is then used to calculate the optimal control law. The variance of the optimal control signal is shown to be given by,

$$P^{opt} =< (u(k) - u^{opt}(k))^2 >, \tag{8}$$

see (Herzallah and Lowe, 2004*a*) for more details.

Although the direct inverse control approach does not require the availability of forward model of the plant to be controlled, The proposed Bayesian method in (Herzallah and Lowe, 2004*a*) does. However, the direct inverse control approach does not consider knowledge of uncertainty in deriving the optimal control law, while the Bayesian method uses knowledge of uncertainty from both the forward and the inverse models of the plant to obtain the optimal estimate of the control signals. This is an advantage of the Bayesian method over the direct inverse control method. Moreover, the Bayesian method for deriving the optimal control law provides a systematic procedure for estimating the conditional distribution of the inverse controller.

3.2 Neural Network Adaptive Control

In neural adaptive control scheme, a combined off-line followed by on-line adaptation is often adopted to determine the parameter vector of the controller. This reduces prior uncertainty of unknown parameters and assure stability of the overall control system. Only later on-line control is started using the initial structures of both identifier and controller that are already substantially close to the optimal.

In their book (Fabri and Kadirkamanathan, 2001) Fabri and Kadirkamanathan made the argument that the above procedure for adaptive control defeats its main objective because most of the uncertainty existing prior to application of the control can be reduced during the off-line training phase. They proposed dual adaptive control scheme which avoided the pre-control neural network training phase by taking into consideration the parameters uncertainty and its effect on tracking, in the on-line control phase. Their proposed scheme was for stochastic affine class of nonlinear discrete time system having the general form

$$\mathrm{y}(k) = f[x(k-1)] + g[x(k-1)]u(k-1) + e(k), \tag{9}$$

where $\mathrm{y}(k)$ is the system output, u(k) is the control signal, $x(k-1) = [\mathrm{y}(k-n), \ldots, \mathrm{y}(k-1), u(k-1-p), \ldots, u(k-2)]^T$ is the vector of previous output and input values, $f[x(k-1)], g[x(k-1)]$ are unknown nonlinear functions of the delay vector and $e(k)$ is an additive noise signal which is assumed to be independent and has zero mean Gaussian distribution of variance σ^2.

Using equation (9) and neural network approximation models for the nonlinear functions of the delay vector, the affine nonlinear discrete system given in (9) is represented in the following state form

$$\begin{aligned} w^*(k+1) &= w^*(k) \\ \mathrm{y}(k) &= h(w^*(k), x(k-1), u(k-1)) + e(k), \end{aligned} \tag{10}$$

where

$$h(w^*, x(k-1), u(k-1)) = \hat{f}[x(k-1), \hat{w}_f] + \hat{g}[x(k-1), \hat{w}_g]u(k-1), \tag{11}$$

could be a nonlinear or linear function of the unknown optimal parameters $w^* = [w^*_{f_1}, \ldots, w^*_{f_i}, w^*_{g_1}, \ldots, w^*_{g_i}]$ if a multi layer perceptron or a Gaussian radial basis function networks are used to approximate the nonlinear functions of delay vector respectively. Since the parameters appear linearly in the Gaussian radial basis controller a Kalman filter is used to estimate the parameters and their uncertainty. The multilayer perceptron controller however, is more complicated since the unknown parameters do not appear linearly in the model equations. Consequently, an extended Kalman filter is used for parameter estimation.

Compared to the conventional neural adaptive control methods where the difference between the desired output $\mathrm{y}_d(k)$, and the system output $\mathrm{y}(k)$, $[\mathrm{y}(k) - \mathrm{y}_d(k)]^2$ is minimized to derive the optimal control law, the following form of the performance index has been suggested for incorporating model uncertainty

$$J = E\{[\mathrm{y}(k)-\mathrm{y}_d(k)]^2+qu^2(k-1)+re^2(k) \mid I^{k-1}\}, \tag{12}$$

where $E\{. \mid I^{k-1}\}$ denotes the mathematical expectation conditioned on the information state I^{k-1} which consists of all output measurements up to time $(k-1)$, denoted by $Y^{k-1} = \{\mathrm{y}(i)\}_{i=0}^{k-1}$, and all previous inputs U^{k-2}. The design parameters r, and q are scalar weighting factors.

The control law minimizing the above performance index J subject to the system equation (9), is then shown to be given by

$$u^*(k-1) = \frac{\{\mathrm{y}_d(k) - \hat{f}[.]\}\hat{g}[.] - (1+r)\mu_{gf}}{\hat{g}^2[.] + q + (1+r)\mu_{gg}}, \tag{13}$$

where the arguments $[.]$ of $\hat{f}$ and $\hat{g}$ are $[x(k-1), \hat{w}_f(k)]$ and $[x(k-1), \hat{w}_g(k)]$ respectively,

$$\mu_{gf} = \nabla_{h_g}(k)P_{gf}(k)\nabla^T_{h_f}(k)$$
$$\mu_{gg} = \nabla_{h_g}(k)P_{gg}(k)\nabla^T_{h_g}(k),$$

where P_{gf}, and P_{gg} are the partitioning matrices of the covariance matrix of the optimal neural network parameters w^* and ∇_{h_g}, ∇_{h_f} denote the gradients of the two components of the function h, $\hat{f}$ and $\hat{g}$, with respect to w^* evaluated at $w^* = \hat{w}(k)$ respectively.

In the above proposed scheme (Fabri and Kadirkamanathan, 2001) Fabri and Kadirkamanathan avoided the pre-control neural network phase by taking into consideration model parameters uncertainty. This is shown to be more convenient with the features expected from the adaptive control, and also more efficient and economical since off-line training is usually time consuming and expensive.

An alternative approach for incorporating uncertainty in functional adaptive control by neural networks was proposed in (Herzallah, 2003). The scheme is based on the idea of modeling and incorporating the uncertainty in the predicted output of the neural network model. Here the forward model of the plant is firstly identified using a neural network model. Similar to the discussion in Section 3.1 and based on theorem 4.2.1 (Gersho and Gray, 1992), the output of the system is shown to be given by (Herzallah, 2003)

$$\mathrm{y}(k) = \hat{\mathrm{y}}(k) + e(k), \tag{14}$$

where $\hat{\mathrm{y}}(k)$ is the conditional expectation of the system output modeled using a neural network or any function approximator, and $e(k)$ is the residual error of the output which is shown to be a random variable with zero mean Gaussian distribution of variance equal to the squared difference between the system output and its estimate, $\parallel \mathrm{y} - \hat{\mathrm{y}} \parallel^2$. This variance is input dependent as has been discussed in Section 3.1. The conditional expectation of this variance is modeled using another neural network (Herzallah, 2003).

Since the estimated variance, σ^2, around the predicted output of the system model is input dependent Herzallah (Herzallah, 2003), has shown that the derived control law from the conventional neural adaptive control method is not optimal. Instead of minimizing the difference between the system and the desired outputs, $\parallel \mathrm{y}(k)-\mathrm{y}_d(k) \parallel^2$ in the conventional adaptive control, it has been shown (Herzallah, 2003) that a performance index of the following form should be minimized

$$\begin{aligned} J &= E\{(\mathrm{y}(k) - \mathrm{y}_d(k))^2\} \\ &= (\mathrm{y}(k) - \mathrm{y}_d(k))^2 + \sigma^2. \end{aligned} \tag{15}$$

Hence, dropping off the variance of the system output which is also input dependent from the performance index to be minimized in deriving the control law can in no way give the optimal solution. Consequently the optimal control law is shown to be given by differentiation of (15) with respect to $u(k)$ and equating to zero.

$$\frac{\partial J}{\partial u(k)} = (\mathrm{y}(k) - \mathrm{y}_d(k))\frac{\partial \mathrm{y}(k)}{\partial u(k)} + \frac{\partial \sigma^2}{\partial u(k)} = 0. \tag{16}$$

Compared to the method proposed by (Fabri and Kadirkamanathan, 2001) this method can be seen to be more general for many reasons. Firstly, this method is shown to be suitable for the four different classes of models defined by Narendra and Parthasarathy in (Narendra and Parthasarathy, 1990), as long as the variance of the forward model could be estimated as input dependent variance. The method proposed in (Fabri and Kadirkamanathan, 2001) on the other hand was for a specific affine class of nonlinear discrete time system. Secondly, the variance of the residual error in (Herzallah, 2003) is the variance of the error of the predicted output from the neural network. This includes all possible sources of variation in the predicted output, whether it is due to noise affecting the output, noise affecting the input, or even due to parameters uncertainty. The variance in (Fabri and Kadirkamanathan, 2001) however, includes variations of model parameters only.

3.3 Adaptive Critic Control

This approach can be defined as a set of methods that approximate dynamic programming. It is based on the basic concept common to all forms of dynamic programming (Howard, 1960). The user needs to supply a utility function U and a stochastic model of the plant to be controlled. Dynamic programming is used to solve for another function called the cost function J, which is assumed to be a function of the state variable at time k of the plant to be controlled, $x(k)$.

Following the concept of dynamic programming, adaptive critic methods can be defined more precisely as designs that include two neural networks: the critic network which tries to approximate the cost function J or its derivatives, and the action network which should be adapted so as to maximize J in near term future. The input to both the action and the critic networks is the state vector $x(k)$. The cost function to be minimized is usually taken to be of the following form

$$J[x(k)] = U(x(k), u[x(k)]) + < J[x(k+1)] > . \tag{17}$$

Based on the output supposed to be approximated by the critic network and the method for adapting the action network, three different critic designs have been proposed in the literature: (1) Heuristic dynamic programming (HDP), which adapts a critic network whose output is an approximation of $J(x(k))$, (2) Dual heuristic programming (DHP), which adapts a critic network whose outputs represent the derivative of $J(x(k))$ (Balakrishnan and Biega, 1996), and (3) Globalized DHP $(GDHP)$, which adapts a critic network whose output is an approximation of $J(x(k))$, but adapts it so as to minimise errors in the implied derivatives of J, as well as J itself. The reader is referred to (Werbos, 1992; Prokhorov and Wunsch, 1997) for full discussion about critic designs.

The adaptive critic design methods are capable of deriving optimal control law over time in noisy nonlinear environments and under uncertain conditions. This comes from the fact that the adaptive critic methods are an approximation for the dynamic programming which is currently the only mathematical formalism under which an optimal controller can be designed under uncertain conditions. The fact that (17) takes the expected value of the cost function at time $k+1$, $< J[x(k+1)] >$, shows that models uncertainty can be accounted for in deriving the optimal control law, although non of the new researches considered this.

4. IMPROVING THE PERFORMANCE OF INTELLIGENT CONTROL USING MULTIPLE MODELS: DEALING WITH UNCERTAINTY

As discussed in Section 2, multiple model approaches have been proposed to handel problems with higher level of uncertainty and complexity, known as multi-modality.

In the control literature, three types of multi-modality are considered. The first one is temporal multi-modality: this situation occurs when the plant operates in suddenly changing environment or when a fault condition occurs. The second type of multi-modality is called spatial multi-modality: it occurs when the plant is characterized by a highly nonlinear complex function, which exhibits different characteristics over different operating zones or operating spaces. The third type of multi-modality occurs when one tries to acquire the inverse dynamics of the plant using supervised learning. This is an ill posed problem, where there is a well defined forward solution, but the solutions to the inverse problem are not unique. Most motor control problems are ill-posed in the sense that there is a well defined forward solution, but the inverse solution is not unique.

Although multiple approaches usually model the conditional probability of the i^{th} model, the controller is usually designed by ignoring knowledge of uncertainty. Two different methods have been suggested for designing the controller. In the first method the new control signal is taken to be the output of the controller with the highest conditional probability. In the second method, the new control signal is taken to be the probability weighted average of the outputs of all controllers.

Different methods for incorporating uncertainty knowledge in the multiple model approaches have been recently appeared in control literature. In the following we discuss briefly two of the approaches.

The first method is a mixture of adaptive control to handle dynamic uncertainty, and multiple model techniques to handel the multi-modality. It is known as multiple model adaptive control scheme (Fabri and Kadirkamanathan, 2001). It is designed for a class of affine-nonlinear stochastic plant with temporal multi-modality of the following form

$$\mathrm{y}(k) = f_{m(k)}[x(k-1)] + g_{m(k)}[x(k-1)]u(k-1) + e(k), \tag{18}$$

where $\mathrm{y}(k)$ is the system output, $u(k)$ is the control signal, $x(k-1) = [\mathrm{y}(k-n), \ldots, \mathrm{y}(k-1), u(k-1-p), \ldots, u(k-2)]^T$ is the vector of previous output and input values, and $e(k)$ is an additive noise signal which is assumed to be independent and has zero mean Gaussian distribution of variance σ^2. The smooth nonlinear functions $f_{m(k)}[x(k-1)], g_{m(k)}[x(k-1)]$ could switch form at an arbitrary instant in time taking on any of the pairs $\{(f_1, g_1), (f_2, g_2), \ldots, (f_H, g_H)\}$ as indexed by $m(k) \in \{1, \ldots, H\}$.

A multiple model approach based on Gaussian radial basis function network is used to identify the nonlinear modes of the plant. H local neural network models, one per mode, are then used to identify the plant and to control it via an indirect adaptive techniques. Two Gaussian radial basis function networks are used for each local model to identify the two nonlinear functions (f_i) and (g_i) in (18)

$$\begin{aligned} \hat{f}_i[x, \hat{w}_{f_i}^T] &= \hat{w}_{f_i}^T \phi_{f_i}[x] \\ \hat{g}_i[x, \hat{w}_{g_i}^T] &= \hat{w}_{g_i}^T \phi_{g_i}[x]. \end{aligned} \tag{19}$$

As can be seen from the above equation, the unknown variables consist of the optimal output layer parameters of the networks in all local models, $w^*_{f_i}$, $w^*_{g_i}$; $i = 1, \ldots, H$.

From (18) and (19), the system dynamics during activity of the mode captured by local model i could be represented in the following state space form

$$\begin{aligned} w_i^*(k+1) &= w_i^*(k) \\ \mathrm{y}(k) &= w_i^{*^T}(k)\phi_i[x(k-1)] + e(k), \end{aligned} \quad (20)$$

where $w_i^{*^T} = [w_{f_i}^{*^T}(k), w_{g_i}^{*^T}(k)]$ and $\phi_i^T[x(k-1)] = [\phi_{f_i}^T[x(k-1)]\phi_{g_i}^T[x(k-1)]u(k-1)]$.

The number of local models to be estimated is determined by the number of plant modes if known a priori. Otherwise, a self organized scheme which allows adding new local models is used (Fabri and Kadirkamanathan, 2001). Since (20) is linear in the parameters, a Kalman filter is used to generate recursively the conditional minimum mean square predictive estimate $\hat{w}_i(k+1)$ of w_i^* and its covariance matrix $P_i(k+1)$ whenever the mode corresponding to local model i is active as could be detected by $m(k)$. As the mode index $m(k)$ is not actually known a mode estimation method is developed in (Fabri and Kadirkamanathan, 2001). Interested readers is referred to (Fabri and Kadirkamanathan, 2001) for the problem of mode and parameters estimation.

Following the discussion in Section 3.2 a performance index of the form given in (12) has been suggested for incorporating models uncertainty in the multiple model adaptive control scheme (Fabri and Kadirkamanathan, 2001). The difference here is that a number of local models are taken to represent the forward dynamics of the system as can be seen from (18).

Subject to (18) and knowledge of the mode sequence $S(k) = \{m(1), m(2), \ldots, m(k)\}$, the control law minimizing the performance index J stated in (12) is then shown to be given by

$$u^*(k-1) = \frac{\{\mathrm{y}_d(k) - \hat{f}_{m(k)}[.]\}\hat{g}_{m(k)}[.] - (1+r)\nu_{gf_{m(k)}}}{\hat{g}^2_{m(k)}[.] + q + (1+r)\nu_{gg_{m(k)}}}, \quad (21)$$

where

$$\begin{aligned} \hat{f}_m(k) &= \hat{w}^T_{f_{m(k)}}(k \mid S(k))\phi_f[x(k)] \\ \hat{g}_m(k) &= \hat{w}^T_{g_{m(k)}}(k \mid S(k))\phi_g[x(k)] \\ \nu_{gf_{m(k)}} &= \phi_g^T[x(k)]P_{gf_{m(k)}}(k \mid S(k))\phi_f[x(k)] \\ \nu_{gg_{m(k)}} &= \phi_g^T[x(k)]P_{gg_{m(k)}}(k \mid S(k))\phi_g[x(k)], \end{aligned} \quad (22)$$

and where $\hat{w}_{f_{m(k)}}(k \mid S(k))$ and $\hat{w}_{g_{m(k)}}(k \mid S(k))$ are sub-vectors of $\hat{w}_{m(k)}(k \mid S(k))$. Similarly $P_{gf_{m(k)}}(k \mid S(k))$ and $P_{gg_{m(k)}}(k \mid S(k))$ are sub-matrices of the covariance matrix $P_{m(k)}(k \mid S(k))$.

Compared to the conventional multiple model approaches, this approach has the advantage of incorporating uncertainty of model parameters. Again only parameters uncertainty is accounted for in this approach. All other sources of uncertainty have been ignored.

The second method, uses the mixture density network approach for representing general probability density functions of the inverse controllers. This method is applied to ill-posed control problems in which the solution to the inverse controller is not unique (Herzallah and Lowe, 2004*b*).

For multi-valued functions (ill-posed problems), it has been shown (Bishop, 1995; Herzallah and Lowe, 2004*b*) that mixture density networks (MDNs) provide a general framework for modeling the conditional probability density functions of inverse controllers, $p(u(k) \mid s(k))$. Here $s(k) = [\mathrm{y}_d(k+d), x(k)]$, where $x(k) = [\mathrm{y}(k), \ldots, \mathrm{y}(k-q+1), u(k-1), \ldots, u(k-p+1)]$. The distribution of the control signals, $u(k)$, is described by a parametric model whose parameters are determined by the output of a neural network, which takes $s(k)$ as inputs. The general conditional distribution function is given by

$$p(u(k) \mid s(k)) = \sum_{j=1}^{M} \alpha_j(s(k))\phi(u(k) \mid s(k)) \quad (23)$$

where $\alpha_j(s(k))$ represents the mixing coefficients, and can be regarded as prior probabilities (which depend on $s(k)$), $\phi_j(u(k)|s(k))$ are the kernel distributions of the mixture model (whose parameters are also conditioned on $s(k)$), and M is the number of kernels in the mixture model.

Different methods for calculating the output from the mixture density network has been suggested. For control applications where unique solutions cannot be found, and where the distribution of the target data consists of different numbers of distinct branches, one specific branch from the estimated conditional density of the MDN needs to be selected. Two examples of how to select a specific branch are the most likely, and the most probable output values. Interested readers are referred to (Bishop, 1995; Herzallah, 2003) for more details.

However it has been shown (Herzallah and Lowe, 2004*b*) that non of the two proposed methods lead to deriving the optimal control law. The argument was based on the fact that although mixture density network models the general distribution of the inverse controllers, people takes a specific quantity to represent the output of the mixture density network, either the most probable or the most likely value, and ignores all other information about uncertainty.

Consequently Herzallah (Herzallah, 2003) proposed an inversion based neuro-controller for incorporating uncertainty in the mixture density network. Similar to the discussion in Section 3.1 the approach is based on importance sampling this time from the non-Gaussian distribution of the inverse controller. the optimal control signal is searched for by generating samples from that distribution which are then forwarded to the plant model. The optimal control signal is then taken to be the one that minimizes a performance index of the form given in (1).

5. CONCLUSIONS

This paper has provided a survey of some of the recently developed methods in the neural control field for incorporating models uncertainty. The basic ideas, the strength and the weakness of each method, and relations with conventional methods are also summarized. The methods discussed in the paper are mainly based on utilizing statistical techniques for modeling the conditional distributions of the outputs or parameters of the neural networks. We explored advantages and disadvantages of each method and discussed the links between the different methods in a unified presentation and identified key areas for future research.

This survey is aimed at researchers currently working in control field. By putting together some of the publications related to incorporating uncertainty in control problems, we hope that interested researchers may find out about the current status of this field.

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