CONTROLLER GAINS ADJUSTMENT FOR CLOSED-LOOP MODAL REQUIREMENTS

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Abstract: This paper deals with a post-synthesis problem : how can a nominal controller be adjusted, in order to correct the assignment of some closed-loop poles ? A parametric sensitivity tool, called the PRABI, is here proposed to mathematically solve this controller adjustment problem. It finds the most relevant combination of controller gains, which masters a defined modal variation. This solution is more simple if the nominal controller has a readable structure, like an observer-based structure. All these theoretical results are applied successfully to the adjustment of flight control laws of a flexible aircraft.

Keywords: controller adjustment, eigenvalue assignment, observers.

1. INTRODUCTION

The modern control synthesis techniques are able to take several kinds of specifications into account (modal, frequency characteristics, parametric robustness). But they often lead to high order controllers, so that implementation problems can arise in industrial applications.

That is the reason why the engineers prefer reducing the order of the final controller. Some methods consists in building a controller from a reduced **synthesis model** (also called "**on-board model**", when this model is implemented in an observer-based controller). Other methods propose to reduce the controller order after the synthesis phase (e.g. a balanced reduction, see (Moore, 1981)). One can notice that, in this last case, an on-board model can be built *a posteriori*, cf. (Alazard and Apkarian, 1999). Thus, one can easily obtain a controller with order much lower than the full order model ("**validation model**"), but such a controller does not ever respect exactly the synthesis objectives, when applied to the validation model.

Today, at this stage, engineers can only perform last-minute adjustments based on physical considerations and would be interested in automatic tools to adjust the controller to any evolution of the model and/or specifications. This post-synthesis phase depends on the adjustment specification and on the controller structure.

The purpose followed in this article is not to replace the reduction phase, but to adjust some controller parameters in order to meet a specified assignment of the closed-loop poles (it could be no more correct because of the reduction). The adjustment method is required to directly consider the validation model associated with its reduced controller. As this paper focuses on post-synthesis modal corrections, an observer-based structure of the controller is here suggested (the way to compute a given controller into an observer-based one is explained in (Alazard and Apkarian, 1999).). Unfortunately this structure can not observe all modes of the validation model, because of difference of order. The maximum number of observed poles is equal to the controller order augmented with the number of outputs (LUENBERGER structure). Then the separation principle is no more verified between controller dynamics and system dynamics. Therefore the pole assignment by state feedback is no more exact ((Cumer *et al.*, 2004)). And the classical results of modal theory can not be here applied. That is the reason why a theoretical tool measuring the parametric sensitivity is proposed to be used for these closed-loop modal corrections. This parametric sensitivity analysis tool, called the PRABI [1], was already exploited for robust analysis ((Gauvrit and Manceaux, 1997) (Cumer, 2000)) and is here extended to a kind of controller adjustment.

In the first part of this paper, the major principles of the PRABI tool are presented and the mathematical expressions are interpreted. It also explains how the PRABI tool is adapted to perform controller adjustment. But the PRABI needs a special layout of the problem, which is obtained in the second part. It specifies also the interest on observer-based structure for controller adjustment. And finally a particular application - namely, the adjustment of flight control laws of a flexible aircraft- proves the efficiency of this new method. Indeed, (Cumer *et al.*, 2004) proposed to tune some controller gains, extracted from an observer-based architecture and physical considerations. In this paper, the method based on PRABI tool allows these adjustments to be improved analytically.

2. THE PRABI TOOL PRINCIPLE AND ITS EXTENSION TO THE CONTROLLER ADJUSTMENT

The PRABI exploits the final results that one would obtain at the end of a Bayesian identification of uncertain parameters. The subjacent idea is : the more identifiable a parameter is, the more sensitive the system is to this parameter ((Lavigne, 1994)).

2.1 Bayesian identification

Consider the system (S) :

$$(S)\begin{cases} \dot{x} = \mathcal{A}(\theta)x + w \\ y = \mathcal{C}x + v \\ y_k = y(k\Delta t) \end{cases} \quad (1)$$

[1] *"Parameter Robust Analysis by Bayesian Identification"*

where v and w are independent gaussian centered white noises with the following stochastic properties [2] :

$$\begin{aligned} \mathrm{E}[w(t)w(\tau)^T] &= Q\delta(t-\tau) \\ \mathrm{E}[v(t)v(\tau)^T] &= R\delta(t-\tau). \end{aligned}$$

Q and R are the covariances of state noises and measure noises respectively. y_k represents the sampled output (and, as explained below, used for the fictitious parametric identification). Δt denotes the sampling period.

This system is characterized by some uncertain parameters, which are considered as random variables. These variables are gathered together into a vector θ. With the assumption that only the matrix $\mathcal{A}$ varies ($\mathcal{A} = \mathcal{A}(\theta)$) [3], the following result has already been demonstrated in (Gauvrit, 1982) (Lavigne, 1994) :

$$p(\theta_0 + \Delta\theta/y^k) = \Lambda \exp\left[-\mathrm{Trace}(M_0^{-1}\Delta M)\right] \quad (2)$$

where

- y^k represents the set of sampled outputs from y_0 to y_k,
- $p(\theta_0 + \Delta\theta/y^k)$ is the probability that θ is equal to $\theta_0 + \Delta\theta$ (y^k known),
- Λ is a constant,
- $M_0 = \mathrm{E}[(y_k - \hat{y}_{k/k-1})(y_k - \hat{y}_{k/k-1})^T]$ where $\hat{y}_{k/k-1}$ is obtained by the KALMAN filter used for the estimation of (S) and tuned at θ_0,
- ΔM is such that $M_0 + \Delta M = \mathrm{E}[(y_k - \hat{y}_{k/k-1})(y_k - \hat{y}_{k/k-1})^T]$, where $\hat{y}_{k/k-1}$ is now obtained by an another KALMAN filter, used for the estimation of (S) too, but tuned at $\theta_0 + \Delta\theta$ (untuned filter).

In (Gauvrit, 1982; Lavigne, 1994), it is also proved that

$$\mathrm{Trace}(M_0^{-1}\Delta M) = \Delta\theta^T G_{\theta_0}^{-1}\Delta\theta.$$

Consequently the probability $p(\theta_0 + \Delta\theta/y^k)$ is a gaussian law according to the direction $\Delta\theta$. G_{θ_0} is the covariance matrix of the θ identification error around θ_0.

2.2 Interpretation and use of $G_{\theta_0}^{-1}$ to correct a pole assignment

The equation (2) expresses the identification sharpness of the vector θ_0 into the direction $\Delta\theta$.

Since $G_{\theta_0}^{-1}$ is the inverse of covariance matrix for the gaussian law $p(\theta_0 + \Delta\theta/y^k)$, its normalized singular vector $\Delta\theta_m$ associated with its minor singular value λ_m represents the direction $\Delta\theta$ of minimal sensitivity -i.e. this direction is the set of

[2] $\mathrm{E}[x]$ denotes the expected value of random variable x. $\delta(t)$ is the DIRAC function.
[3] It is also possible to consider variations in $\mathcal{C}$.

parametric variations to which the system is the least sensitive. Indeed, the following mathematical expressions :

- $G_{\theta_0}^{-1}\Delta\theta_m = \lambda_m \Delta\theta_m$,
- and $p(\theta_0 + \alpha\Delta\theta_m / y^k) = \Lambda \exp(-\lambda_m \alpha^2)$,

lead to a gaussian law with a maximal standard deviation. In other words, this parametric direction is the least identifiable (see Figure 1 (b)). Reciprocally the normalized singular vector $\Delta\theta_M$ associated with the major singular value λ_M of $G_{\theta_0}^{-1}$ is the direction $\Delta\theta$ of maximal sensitivity. The identification of θ variations is good in this direction (see Figure 1 (a)).

It is also necessary to precise that this result is only local around θ_0.

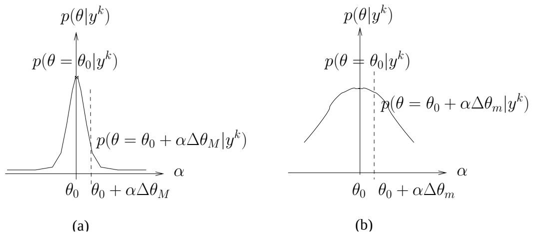

Fig. 1. Different qualities of bayesian identification - a) along the maximal sensitivity direction, b) along the minimal sensitivity direction.

Consider now the controller adjustment problem for modal requirements. Suppose that a parametric vector $\Delta\theta = \mathrm{vec}(\delta p, \delta K)$ can be highlighted, such that δp is the desired variation of a modal characteristic (i.e. of dimension 1) and δK groups the controller gains to be adjusted. In this case, the direction $\Delta\theta$ of minimal sensitivity, found by the PRABI, gives the ΔK value which counters a δp quantity. But this controller variation can imply a displacement of all closed-loop modes.

Then, δp must take all possible variations (pulsation and ratio damping) of all system modes into account, and the search for the direction of minimal sensitivity must be able to fix some modal variations at 0 (adjustment constraints).

Mathematically, the aim is to find a direction $\Delta\theta$ that minimizes $\Delta\theta^T G_{\theta_0}^{-1} \Delta\theta$ under some modal constraints. The controller adjustment will be perfect, if $\Delta\theta^T G_{\theta_0}^{-1} \Delta\theta$ is equal to 0 and if the constraints are verified.

3. CONTROLLER ADJUSTMENT ALGORITHM BASED ON THE PRABI

3.1 Extraction of the fundamental controller parameters in a observer-based architecture

As explained in the introduction, once an onboard model is computed, the reduced controller is easily structured into an observer-based architecture, basically composed of a state estimator, a state feedback and a dynamic YOULA parameter (as shown on figure 2) [4] .

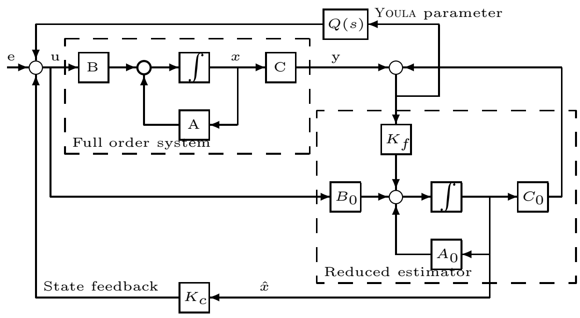

Fig. 2. Observer-based structure of a reduced controller.

When the plant order allows a full estimation of the states (dimensions of x and $\hat{x}$ are equal), a standard eigenvalue assignment can be performed for a modification *a posteriori* of the closed-loop dynamics. However, if the plant order is not low enough, this assignment is no more possible. Indeed, the state-space dynamics $(\mathrm{spec}(A_0 - B_0 K_c))$, the state estimator dynamics $(\mathrm{spec}(A_0 - K_f C_0))$, and the YOULA parameter dynamics $(\mathrm{spec}(Q(s))$ are not separated and do not correspond to the closed-loop dynamics.

The adjustment procedure aims to find the best combination of some gains extracted from this control law structure, which meets the pole assignment requirement. It is clear that the elements of the matrices K_c and K_f are the most important gains of this structure for pole assignment. Given the dimension of these matrices, only the K_c gains will be here considered, because they control the modes resulting from the plant [5] .

The figure 3 illustrates how controller gains, to be tuned, can be extracted from K_c. Matrices M_k and N_k are introduced, so that the state feedback matrix is: $K_c + M_k \mathrm{diag}(\delta K) N_k$. **Thus presented, the adjustment procedure will finally give a particular value of the vector δK.**

3.2 Description of the modal parameters to be modified

For a modification of a modal characteristic δp (for example, a variation of a flexible mode damp-

[4] One can point out that there are several solutions according to the distribution of the closed-loop eigenvalues between the dynamics $(\mathrm{spec}(A_0 - K_f C_0))$, $(\mathrm{spec}(A_0 - B_0 K_c))$ and $(\mathrm{spec}(Q(s)))$. Some remarks in (Alazard and Apkarian, 1999) can help to make this choice.

[5] With the aim of generality, the notation K is used in the next paragraphs and can refer to the gains of K_c or of both K_c and K_f.

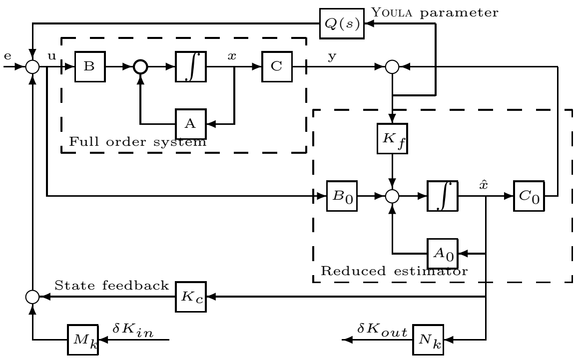

Fig. 3. Extraction of K_c gains.

ing ratio), a relevant combination of controller gains is a value along a direction of δK which could balance a value of the selected variation δp of a closed-loop pole. In other terms the adjusted controller puts the considered pole in the opposite direction induced by the fictitious variation δp. Note that δp is called " fictitious closed-loop variation", because it only appears in the closed-loop system for the computation of the best direction of δK.

To act on a closed-loop mode, the system presented on figure 3 needs to be written in a state-space realization. This realization (A_{cl}, B_{cl}, C_{cl}) has the same order than the closed-loop system in figure 2, but the number of inputs and outputs is increased by the size of the vector δK. For reasons of clarity, the matrix D_{cl} is omitted.

Figure 4 shows a way to modify A_{cl}. The vector $\Delta\theta$, composed of the perturbations δp and δK, acts on the closed-loop system *via* a static feedback. One can remark that this system could be presented as a $M - \Delta$ standard form with a structured perturbation $\Delta = \Delta\theta = \text{diag}(\delta p, \delta K)$.

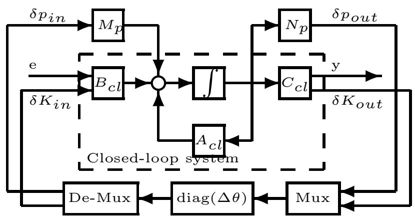

Fig. 4. Full $\Delta\theta$ perturbation feedback.

Subsequently the PRABI will help to find the best controller gains adjustment which counters the artificial closed-loop variation.

3.3 Definition of the system (S) *in (1)*

The matrix $G_{\theta_0}^{-1}$ is highly dependent on the noise covariances Q, R, and on the observation matrix $\mathcal{C}$. The choice of these parameters influences the adjustment results. Since this identification is a fictitious one, it is possible to choose a different matrix C_{fic} only for the identification procedure (figure 5).

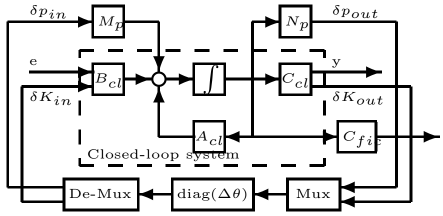

Fig. 5. Use of a fictitious output matrix C_{fic} for $G_{\theta_0}^{-1}$ computation.

For example, consider a closed-loop system composed only of a pair of complex conjugated modes :

$$\mathcal{A} = \begin{bmatrix} 0 & 1 \\ -\omega^2 & -2\xi\omega \end{bmatrix} \tag{3}$$

and the vector $\Delta\theta = [\delta\omega \quad \delta\xi]^T$ (here, the controller gains are not included in θ). A random choice for Q and R leads to a positive symmetric definite matrix $G_{\theta_0}^{-1}$:

$$G_{\theta_0}^{-1} = \begin{bmatrix} a & b \\ b & c \end{bmatrix}.$$

With such a $G_{\theta_0}^{-1}$, the minimum of $\Delta\theta^T G_{\theta_0}^{-1} \Delta\theta$ is achieved for $\delta\xi = -\frac{b}{c}$ when $\delta\omega$ is fixed to 1 (for example). If b is different from 0, it means that, from a PRABI point of view, a displacement of $\delta\omega$ can be compensated by a variation of $\delta\xi$.

Of course this possible compensation is not interesting for modifying the assignment of closed-loop poles. In this modal context, it is more judicious to have a decoupling between pulsation and damping ratio corrections. Thus, Q and R (and possibly C_{fic} too) will be chosen to ensure the nullity of b. A simple optimization procedure gives such Q, R and C_{fic}.

In a more general case, when the order of the closed-loop system is higher than 2, the matrix A_{cl} is first reordered into a block diagonal representation (blocks are similar to $\mathcal{A}$ in equation (3)) and the optimization step is performed for each block independently. The final Q, R and C_{fic} are the concatenation of the successive results ; i.e. they are block diagonal matrices.

Then $G_{\theta_0}^{-1}$ can be decomposed as follows :

$$G_{\theta_0}^{-1} = \begin{bmatrix} G_{11} & G_{12} \\ G_{12}^T & G_{22} \end{bmatrix}$$

(where the sub-matrix G_{11} is diagonal)
so that, for $\Delta\theta = [\delta p \quad \delta K]^T$,

$$\Delta\theta^T G_{\theta_0}^{-1} \Delta\theta = \delta p^T G_{11} \delta p + 2\delta p^T G_{12} \delta K + \delta K^T G_{22} \delta K \tag{4}$$

3.4 Numerical solution of controller adjustment

Under these conditions the minimum of (4), when δK is fixed, is characterized by $\delta p = -G_{11}^{-1} G_{12} \delta K$ (similarly, when δp is fixed, $\delta K = -G_{22}^{-1} G_{12}^{T} \delta p$ gives the optimum).

In order to weight the shifting of all poles, the criterion used is the following :

$$Crit = (\delta p^* - \delta p^d)^T P (\delta p^* - \delta p^d) \qquad (5)$$

- δp^d is the desired vector of the closed-loop system parameters,
- δp^* is the optimal vector of the closed-loop system parameters, when δK is fixed in (4) : i.e. $\delta p^* = -G_{11}^{-1} G_{12} \delta K$,
- P is a diagonal weighting matrix, which gives different priorities between the closed-loop poles (or closed-loop system parameters).

The objective is to find δK which minimizes (5). The solution of this problem is given by :

$$G_{12}^T G_{11}^{-1} P G_{11}^{-1} G_{12} \delta K = G_{12}^T G_{11}^{-1} P \delta p^d \qquad (6)$$

In some cases (depending on $G_{12}^T G_{11}^{-1} P G_{11}^{-1} G_{12}$ rank), several solutions are possible. The choice of one of these solutions is then led by a secondary criterion (for example, the minimization of the δK norm).

3.5 Controller adjustment algorithm

Finally, for the controller adjustment problem, examined in this paper, three kinds of parameters need to be defined : the most relevant controller gains to be tuned (the adjustment result is a rectification of a combination of these gains), the closed-loop modal characteristics to be modified (characterizing the adjustment objective) and the PRABI parameters (guaranteeing the accuracy of the adjustment).

The controller adjustment algorithm is then composed of five steps :

(1) Group the gains of the (observer-based) controller you choose to tune into the vector K. The vector ΔK denotes their variations.
(2) Obtain a state-space realization of the varying closed-loop system and take into account the modal objectives of the post-synthesis adjustment. Obtain a standard $M - \Delta$ form, as illustrated on figure 4.
(3) Initialize the PRABI parameters, so that the submatrix G_{11} is diagonal.
(4) Then compute the matrix $G_{\theta_0}^{-1}$
(5) With an optimization procedure, find the best value of δK (i.e. the best value of a controller gains combination), which must be applied in order to modify the closed-loop modal characteristics (see equation (6)).

4. APPLICATION TO FLIGHT CONTROL OF A FLEXIBLE AIRCRAFT

The model used here is a linearized 60th order model of the lateral motion of a flexible aircraft around an equilibrium point. The state vector x contains :

- 4 rigid states (yaw angle β, roll rate p, yaw rate r, roll angle ϕ),
- 36 states that represent the 18 flexible modes modelled between 8 and 80 rd/s,
- 20 secondary states that represent dynamics of servo-control surfaces and aerodynamics lags.

The system is composed of 4 inputs and 6 outputs. The nominal controller for this system is described by a 20^{th} order state space representation (with 6 inputs and 4 outputs).

As effects of the control law, the spiral mode is moved far in the half left plane as the Dutch roll mode. The damping ratios of most of flexible modes increase. But the Dutch roll mode damping ratio is too weak and must be increased (closed-loop poles, obtained with the nominal controller, are represented by "o" on figures 6 and 7). Thus, different modal modifications can be defined. Observer-based controllers will be used (for reasons evoked above) and the efficiency of the controller adjustment method based on PRABI will be verified.

Modification (a posteriori) of the Dutch roll damping ratio. In this case, an observer-based controller (26 states estimated with a Luenberger estimator) is conceived from an on-board model described by the 4 rigid modes and 22 states representing 11 flexible modes. Only the state feedback K_c is chosen to be modified (a better result could be found in considering gains of both K_c and K_f). The effects of the variation of K_c around its nominal value (applied on the validation model) are shown in figure 6.

The different weightings, grouped into the matrix P (see equation (5)), which lead to this combination of controller gains are : 1 for the Dutch roll pulsation, 1 for the Dutch roll damping ratio, 1 for the pulsation of the first flexible mode, 1 for the damping ratio of the first flexible mode, and 0.1 for all other closed-loop parameters (damping ratios and pulsations).

The controller adjustment direction, obtained by the PRABI method, makes it possible to move the Dutch roll mode along an iso-pulsation curve. The other plant poles are not sensitive to this direction of controller adjustment. As K_f is not taken into account, some controller modes are sensitive to this direction (the choice of weightings also influences the result). The final adjustment

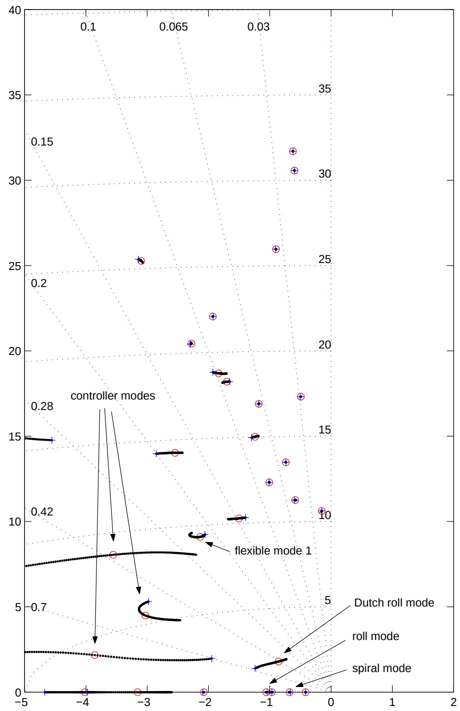

Fig. 6. Pole evolution around the nominal assignment (marked by the "o") due to a scalar variation of the controller gains combination, computed to correct the Dutch mode damping ratio.

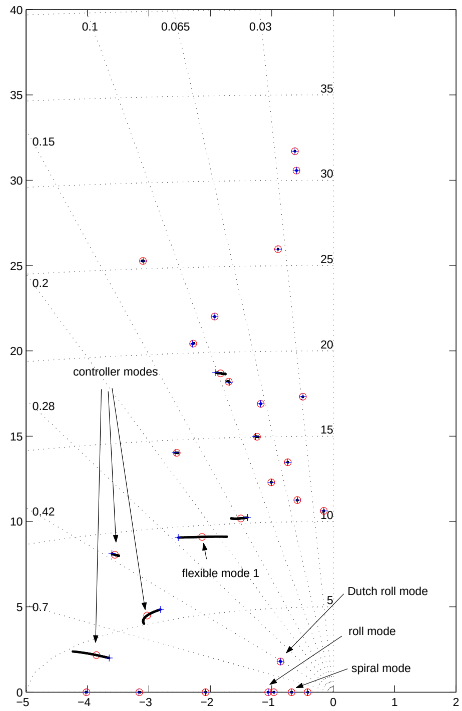

Fig. 7. Pole evolution around the nominal assignment (marked by the "o") due to a scalar variation of the controller gains combination, computed to modify the damping ratio of the first flexible mode.

corresponds to a chosen position of the poles, marked by '+'. The major modification concerns the damping ratio of the Dutch roll.

Correction of the damping ratio of the first flexible mode. The adjustment objective is the improvement of the damping ratio of the first flexible mode. The conception of the observer-based controller and the choice of the weightings are the same as in the precedent case. Figure 7 shows the effects of the extracted controller gains combination on the validation model. The pole placement marked by '+' is the final choice for this adjustment.

5. CONCLUSION

In this paper a controller adjustment method is presented to correct the initial closed-loop pole assignment and successfully applied to a high order flexible aircraft model. This adjustment tool has the significant advantage to include directly the validation model into the synthesis of controller gains combination to be tuned. At present, extensions of this promising method are studied in order to minimize a transfer norm, not considered in design of the first controller.

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