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ADAPTIVE BACKSTEPPING MOTION CONTROL OF INDUCTION MOTOR DRIVES [1]

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Abstract: In this paper, an adaptive backstepping controller is proposed for the position tracking of a mechanical system driven by an induction motor (IM). The mechanical system is a single link fixed on the shaft of the induction motor. The backstepping methodology provides a simpler design procedure for a adaptive control scheme than the model reference adaptive control method, which is demonstrated in this paper. Another feature is that it provides a way to define the sliding surface if one wants to use the robust sliding-mode control. Thus, the backstepping control can be easily extended to be an adaptive sliding-mode controller. The final position control system is shown to be stable and robust to the parameter variations and external disturbances. The effectiveness of the proposed controllers are demonstrated by some experiments.

Keywords: Induction motors, sliding-mode, adaptive backstepping control.

1. INTRODUCTION

Since simple construction, ruggedness, reliability, low cost, and minimum maintenance of induction motor has widely used in many industry applications and, recently, even in the field of robotic applications (Hu, *et al.*, 1996). In such applications the mechanical load driven by induction motor must track a time-varying trajectory that specifies its desired position (Fusco, 2001). For counteract these variations, analyzing and designing the tracking performance of a position controller for torque-regulated induction motor is proposed in this paper.

Recently, the nonlinear state feedback theory is implemented to decouple the dynamics of the thrust force and the flux amplitude of the induction motor (Novotny and Lipo, 1996; Marino, *et al.*, 1993). The present trend is to develop torque control laws for a induction motor (Takahashi and Nogushi, 1986; Depenbrok, 1988). The advantages of DTC are quick torque response and lesser parameter dependence. However, torque ripples and high sample time request are drawbacks. An adaptive sliding-mode direct torque control scheme is proposed by Lin and Fang (2001) which improve the drawbacks of DTC.

A high performance motor drive must have good position command tracking and load regulating response. In real practice, the induction motor drive is influenced by the uncertainties, which usually compose of unpredictable plant parameter variations, external load disturbances, unmodelled and nonlinear dynamics of

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plant. Nonlinear control approaches have been developed to deal with such problems. The model reference adaptive control (MRAC) technique is one method to overcome parameter variations problem (Lin and Fang, 2001; Ko and Jeon, 1996). The other method is the adaptive backstepping control (Kanellakopoulos, *et al.*, 1991; Lin and Lee, 2000). The latter is simpler in the control design procedure. To compensate for uncertainties, many works developed the sliding-mode control schemes (Xia and Yu, 2000; Wang and Chen, 1999).

Usually, the backstepping control is incorporated with the nonlinear damping to increase the robustness (Krtic, *et al.*,1995; Hu, *et al.*, 1996). However, such an approach can only achieves semiglobal uniform ultimately bounded convergence (SGUUB) (Krtic, *et al.*, 1995). This paper tries to develop an adaptive sliding-mode backstepping position control scheme for an induction motor. This control scheme combine the adaptive backstepping method and the sliding-mode technology, so that it can adaptively tune the control gains with respect to the change of the system parameters and can also compensate for uncertainties. We first propose a new adaptive backstepping position control scheme. The resulting control law provides a way to assign the sliding surfaces for designing sliding-mode control. This special feature of the backstepping control methology is first demonstrated in this paper. The robustness of the proposed control scheme will be verified by an experiment with a sinusoidal disturbance.

This paper is organized as follows. Section 2 briefly describes the model, and reviews the sliding-mode direct torque control presented in (Lin and Fang, 2001). The proposed adaptive backstepping control is presented in Section 3. Section 4 considers the uncertainty case and extends the result to a robust version. The experiments are reported in Section 5. Finally, Section 6 draws the conclusions.

2. SLIDING-MODE DIRECT TORQUE CONTROL

The mathematical model of a three-phase, Y-connected induction motor in a stator-fixed frame (a_s, b_s) can be described by five nonlinear differential equations with four electrical variables [stator currents (i_{as}, i_{bs}) and rotor fluxes $(\varphi_{ar}, \varphi_{br})$], a mechanical variable [rotor speed (ω_m)], and two control variables [stator voltages (u_{ds}, u_{qs})] (Novotny and Lipo, 1996; Leonhard, 1996) as follows:

$$\dot{i}_{as} = -\gamma i_{as} + \frac{K}{T_r}\varphi_{ar} + pK\omega\varphi_{br} + \alpha u_{as} \tag{1}$$

$$\dot{i}_{bs} = -\gamma i_{bs} + \frac{K}{T_r}\varphi_{br} - pK\omega\varphi_{ar} + \alpha u_{bs} \tag{2}$$

$$\dot{\varphi}_{ar} = \frac{M}{T_r} i_{as} - \frac{1}{T_r}\varphi_{ar} - p\omega\varphi_{br} \tag{3}$$

$$\dot{\varphi}_{br} = \frac{M}{T_r} i_{bs} - \frac{1}{T_r}\varphi_{br} + p\omega\varphi_{ar} \tag{4}$$

$$\dot{\omega} = -\frac{B}{J}\omega + \frac{T_e}{J} - \frac{T_L}{J} \tag{5}$$

where R_s and R_r are the stator and rotor resistance, L_s, L_r, and M are the stator, rotor, and mutual inductance, B and J are the friction coefficient and the moment of inertial of the motor, p is the number of pole-pairs, T_e and T_L are the electromagnetic torque and external load torque, $\tau_r = L_r/R_r$ is the rotor time constant, the parameters are $\sigma \equiv 1 - M^2/(L_s L_r)$, $K \equiv M/(\sigma L_s L_r)$, $\alpha \equiv 1/(\sigma L_s)$, and $\gamma \equiv R_s/(\sigma L_s) + R_r M^2/(\sigma L_s L_r^2)$. Note that

$$T_e = k_T(i_{bs}\varphi_{ar} - i_{as}\varphi_{br}) \tag{6}$$

where $k_T \equiv (3p/4)(M/L_r)$.

The direct torque control (DTC) scheme is to control the electromagnetic torque T_e to be the desired one by the voltage inputs. A sliding-mode direct torque control scheme with a sliding-mode flux observer presented in (Lin and Fang, 2001) is briefly introduced in the appendix. This direct torque control scheme will construct a voltage controller $\mathbf{u} = [u_{as}\ u_{bs}]^T$ to ensure that the electromagnetic torque T_e follows the desired torque trajectory $T_{e,ref}$.

The sliding-mode direct torque control scheme is

$$\mathbf{u} = -\mathbf{D}^{-1}\left(\mathbf{b} + k_c\mathbf{s} + \begin{bmatrix} \mu_{c1}\ \mathrm{Sat}(s_1) \\ \mu_{c2}\ \mathrm{Sat}(s_2) \end{bmatrix}\right) \tag{7}$$

where $\mathbf{s} = [s_1,\ s_2]^T$ are the sliding surfaces of torque and flux, $\mathbf{D}$, $\mathbf{b}$, k_c, and $(\mu_{c1},\ \mu_{c2})$ are the nonlinear control factor which are defined in the appendix.

Note that the saturation function $\mathrm{Sat}(s_i)$ is defined as

$$\mathrm{Sat}(s_i) = \frac{s_i}{|s_i| + \lambda} \tag{8}$$

where $\lambda > 0$ is a smooth factor.

For more details about the stability proof and experiments, the reader is referred to (Lin and Fang, 2001).

3. ADAPTIVE BACKSTEPPING MOTION CONTROL

This paper tried to develop a new backstepping control law for motion tracking of an induction motor, provided that an inner loop of torque control is implemented. Especially, we use the sliding-mode direct torque control

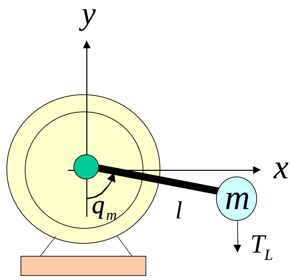

Fig. 1. Mechanical system of a motor with a rod fixed on the shaft.

scheme (Lin and Fang, 2001) described above as the inner loop. The following context is then concentrated on the motion tracking of a mechanical system driven by an induction motor.

The mechanical system considered is an induction motor with a rod fixed on the shaft axis of the motor and is shown in Fig. 1. The dynamics of the mechanical system are

$$\begin{aligned} J\ddot{\theta}_m &= -B\dot{\theta}_m - mgl\sin(\theta_m+\theta_0) + k_T u_T \\ &= -B\dot{\theta}_m - mgl\cos\theta_0\sin\theta_m - mgl\sin\theta_0\cos\theta_m \\ &\quad + k_T u_T \end{aligned} \tag{9}$$

where θ_m is the angular displacement of the shaft, m is the mass of the rod, l is the distance from the shaft center to the center of mass of the rod, g is the gravitational acceleration, , θ_0 is the null angle from the line of gravity. Furthermore, (9) is simplified as

$$\ddot{\theta}_m = -B_J\dot{\theta}_m - L_s\sin\theta_m - L_c\cos\theta_m + K_J u_T \tag{10}$$

where $B_J \equiv B/J$, $L_s \equiv mgl\cos\theta_0/J$, $L_c \equiv mgl\sin\theta_0/J$, $K_J \equiv k_T/J$. Note that $J > 0$.

The control objective is to design a controller u_T that forces the position variable θ_m to track a desired trajectory denoted by θ_m^* which is secon-order continuously differentiable.

Define the tracking error as $e_p = \theta_m^* - \theta_m$. The system (10) can be rewritten as

$$\begin{cases} \dot{e}_p = e_s = \dot{\theta}_m^* - \dot{\theta}_m \\ \dot{e}_s = \ddot{e}_p = \ddot{\theta}_m^* + B_J\dot{\theta}_m + L_s\sin\theta_m + L_c\cos\theta_m - K_J u_T \end{cases} \tag{11}$$

The concept of the backstepping is first to consider only one of the states. We consider e and let Lyapunov-like function be $V_0 = e_p^2/2$. The derivative of V_0 along the trajectory of e_p is

$$\dot{V}_0 = e_p\dot{e}_p = -c_1e_p^2 + e_p(e_s + c_1e_p) \tag{12}$$

The purpose of the special form of (12) is to achieve $\dot{V}_0 = -c_1e_p^2 < 0$ for $e_p \neq 0$ if e_s were kept to be $-c_1e_p$. However, e_s cannot be arbitrarily assigned. The backstepping design is then to consider the error $z \equiv e_s - (-c_1e_p)$. According to (11), the dynamics of z are

$$\dot{z} = K_J\left(\mathbf{h}^T\bar{\mathbf{x}} - u_T\right) \tag{13}$$

where

$$\mathbf{h} = \begin{bmatrix} 1/K_J \\ B_J/K_J \\ L_s/K_J \\ L_c/K_J \end{bmatrix}, \qquad \bar{\mathbf{x}} = \begin{bmatrix} \ddot{\theta}_m^* + c_1(\dot{\theta}_m^* - \dot{\theta}_m) \\ \dot{\theta}_m \\ \sin\theta_m \\ \cos\theta_m \end{bmatrix} \tag{14}$$

Note that the parameters of $\mathbf{h}$ are assumed unknown. We need to design an adaptive backstepping controller to estimate these parameters on line. The estimates of the unknown parameters are denoted by $\hat{\mathbf{h}}$ and the estimation error is $\tilde{\mathbf{h}} = \mathbf{h} - \hat{\mathbf{h}}$. Now, consider a new Lyapunov-like function:

$$V_1 = \frac{1}{2}\left(e_p^2 + z^2 + K_J\tilde{\mathbf{h}}^T\mathbf{\Gamma}\tilde{\mathbf{h}}\right) \tag{15}$$

where Γ is a positive definite matrix. The derivative of V_1 along the trajectory of the system (11) is

$$\begin{aligned} \dot{V}_1 &= -c_1e_p^2 + e_pz + zK_J\left(\mathbf{h}^T\bar{\mathbf{x}} - u_T\right) + K_J\tilde{\mathbf{h}}^T\mathbf{\Gamma}\dot{\tilde{\mathbf{h}}} \\ &= -\boldsymbol{\varepsilon}^T\mathbf{F}\boldsymbol{\varepsilon} \end{aligned} \tag{16}$$

where

$$\boldsymbol{\varepsilon} = \begin{bmatrix} e_p \\ z \end{bmatrix}, \qquad \mathbf{F} = \begin{bmatrix} c_1 & -1/2 \\ -1/2 & c_2 \end{bmatrix} \tag{17}$$

if the controller and the adaptive law are, respectively,

$$u_T = \hat{\mathbf{h}}^T\mathbf{x} \tag{18}$$

$$\dot{\hat{\mathbf{h}}} = z\mathbf{\Gamma}^{-1}\mathbf{x} \tag{19}$$

where $\mathbf{x}^T = \bar{\mathbf{x}}^T + [c_2z, \quad 0, \quad 0, \quad 0]$. It is easy to show that the symmetrical matrix $\mathbf{F}$ is positive definite and then $\dot{V}_1 \leq 0$ if $c_1c_2 > 1/4$.

Proposition 1. Consider the system (10). The angular displacement θ_m of the system will asymptotically converge to the desired trajectory θ_m^* if the controller and the adaptive law are, respectively, (18) and (19) with $c_1c_2 > 1/4$.

Proof. V_1 in (15) is a Lyapunov-like function, so we cannot directly apply the Lyapunov stability theory.

However, V_1 is bounded below and non-increasing, which implies that $\lim_{t\to\infty} V(t)_1 = V_{1\infty}$ exists (Ioannou and Sun, 1996). Thus, e_p, z, $\tilde{\mathbf{h}} \in L_\infty$, so that $\hat{\mathbf{h}} \in L_\infty$ since $\mathbf{h}$ is constants. It then follows from (11) and (13) that $\dot{e_p}$,$\dot{z} \in L_\infty$. Integrating (16), we obtain $V_1(t)|_{t=0} - V_{1\infty} \geq \int_0^\infty \boldsymbol{\varepsilon}^T\mathbf{\Gamma}\boldsymbol{\varepsilon}$, and then $\boldsymbol{\varepsilon} \in L_2$. A corollary of Barbalat's lemma (Ioannou and Sun, 1996) states that $\boldsymbol{\varepsilon} \in L_\infty$ and $\boldsymbol{\varepsilon} \in L_2$ imply $\boldsymbol{\varepsilon} \to \mathbf{0}$ as $t \to \infty$. This completes the proof. Q.E.D.

It should be remarked that u_T in (18) is used as the reference active torque u_{Tref} for the inner loop torque control.

4. EXTENSION TO ROBUSTNESS

The above mechanical model is an ideal case. We now consider a more practical case by introducing an uncertainty in (10) to obtain

$$\ddot{\theta}_m = -B_J\dot{\theta}_m - L_s \sin\theta_m - L_c\cos\theta_m + K_J u_T + \Delta \tag{20}$$

where $\Delta \equiv K_J\Delta_1$ is a bounded uncertainty satisfying $|\Delta_1| \le \rho$, in which $\rho > 0$ is an unknown bound. After introducing the uncertainty, (13) should also be modified as

$$\dot{z} = K_J\left(\mathbf{h}^T\bar{\mathbf{x}} - \Delta_1 - u_T\right) \tag{21}$$

Let the sliding surface be $\mathbf{s} = \boldsymbol{\varepsilon}$ and define the Lyapunov function as $V = (1/2)\mathbf{s}^T\mathbf{s}$. It can be shown that a sliding-mode controller $u_T = \mathbf{h}^T\mathbf{x} + \rho\ \mathrm{sign}(z)$ can draw the overall system to the sliding surface $\mathbf{s} = \mathbf{0}$ and then θ_m asymptotically approaches the target θ_m^*, if all system parameters are known. However, we assume that the parameters are unknown. Thus, we require the following adaptive sliding-mode backstepping controller.

Proposition 2. Consider the system (20). The angular displacement θ_m of the system will asymptotically converge to the desired trajectory θ_m^* if the controller and the adaptive law are, respectively,

$$u_T = \hat{\mathbf{h}}^T\mathbf{x} + \hat{\rho}\ \mathrm{sign}(z) \tag{22}$$

$$\dot{\hat{\mathbf{h}}} = z\mathbf{\Gamma}^{-1}\mathbf{x} \tag{23}$$

$$\dot{\hat{\rho}} = \gamma_\rho^{-1}|z| \tag{24}$$

with $c_1c_2 > 1/4$ for $\mathbf{x}$ and $\gamma_\rho > 0$.

Proof. Let the Lyapunov-like function V_2 be

$$V_2 = \frac{1}{2}\left(\boldsymbol{\varepsilon}^T\boldsymbol{\varepsilon} + K_J\tilde{\mathbf{h}}^T\mathbf{\Gamma}\tilde{\mathbf{h}} + K_J\gamma_\rho\tilde{\rho}^2\right) \tag{25}$$

where $\tilde{\rho} = \rho - \hat{\rho}$. Applying (22), we obtain the derivative of V_2 along the trajectory of the system (20) as

$$\begin{aligned}\dot{V}_2 &= -\boldsymbol{\varepsilon}^T\mathbf{F}\boldsymbol{\varepsilon} - zK_J(\Delta_1 + \hat{\rho}\ \mathrm{sign}(z)) + K_J\gamma_\rho\tilde{\rho}\dot{\tilde{\rho}} \\ &\le -\boldsymbol{\varepsilon}^T\mathbf{F}\boldsymbol{\varepsilon} + K_J(\rho|z| - \hat{\rho}|z|) + K_J\gamma_\rho\tilde{\rho}\dot{\tilde{\rho}} \\ &= -\boldsymbol{\varepsilon}^T\mathbf{F}\boldsymbol{\varepsilon} \le 0 \end{aligned} \tag{26}$$

where $\mathbf{F}$ is defined in (17). Note that $-\Delta_1 z \le |\Delta_1 z| \le \rho|z|$. Then V_2 is bounded below and non-increasing. The rest of the proof is similar to the last part of the proof of Proposition 1 and is omitted. Q.E.D.

It should be remarked that the sign function in (22) is replaced with the Sat function in (8) in the implementation.

5. EXPERIMENTS

The overall position control scheme will be verified by experiments. The experiment system is a PC-based control system. A servo control card on the ISA bus of the PC provides eight A/D converters, four D/A converters, and an encoder counter. The sampling time for the overall control is 0.3ms. The ramp comparison modulation circuit is to generate the PWM for driving the IGBT module inverter. The induction motor in the experiment system is a 4-pole, 5HP, and 220V motor with the rated current, speed, and torque of 13.4.A, 1730rpm, and 18Nm, respectively. The encoder has 4096 counters per revolution. The modeling parameters of the motor are $R_s = 0.3\Omega$, $R_r = 0.36\Omega$, $L_s = 48mH$, $L_r = 48mH$, and $L_m = 45mH$. Those of the mechanical system (cf. Fig. 1) are $J \approx 0.0042\mathrm{kgm}^2$, $l \approx 0.5$m, and $m \approx 1.7$kg.

Two experiments are reported in the following: 1) reference trajectory generated by set-point position, 2) sinusoidal trajectory.

In the first experiment, the motor is asked to go to $\theta_m = \pi/2$ at $t = 0.5$s, then to $\theta_m = \pi$ at $t = 5$s, and finally to return to $\theta_m = \pi/2$ again at $t = 8$s. However, the desired trajectory is generated by the reference model of

$$\ddot{\theta}_m^* = -k_t\dot{\theta}_m^* - k_s\theta_m^* + k_s\theta_r \tag{27}$$

where θ_r is the angular displacement command, and k_t and k_s are positive constants, which can be selected that $s^2 + k_ts + k_s = (s+p_1)(s+p_2)$ with $p_1, p_2 > 0$. The gains of the reference model are $k_t = 10$ and $k_s = 24$. It should be remarked that the reference active torque u_{Tref} in the inner loop is equal to u_T generated by the adaptive sliding-mode backstepping controller stated in Proposition 2, while the reference flux ϕ_{ref} is given as a constant of 0.185 which is the square of rotor flux norm value (0.43 Web). The experiment results are shown in Fig. 3. It can be seen that the steady-state error is negligible, and the transient response also meets the reference model. The history of the estimated torque shows that the values are around zero for $\theta_m = \pi$ and around about 9Nm for $\theta_m = \pi/2$, which is consistent with the physical property.

The desired trajectory in the second experiment, a sinusoidal position trajectory:

$$\theta_r = (1 - e^{-10t})^2\pi\sin(2t) \tag{28}$$

which makes the starting smooth, since $\ddot{\theta}_r(0) = \dot{\theta}_r(0) = \theta_r = 0$. The experimental results are shown in Fig. 4. The tracking error $\theta_m - \theta_m^*$ in Fig. 4 is also negligible. Consequently, these two experiments verify the control theory and support that the proposed adaptive sliding-mode backstepping motion control has a good performance and can be applied to the position control of an induction motor.

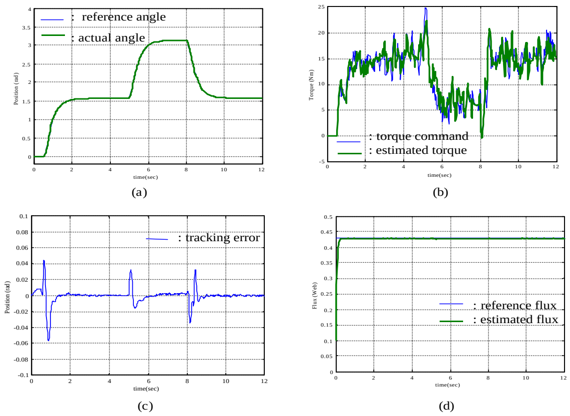

Fig. 2. Responses of a set point position command: (a) position; (b) torque command and estimated torque; (c) tracking error $(\theta_m^* - \theta_m)$; (d) rotor flux.

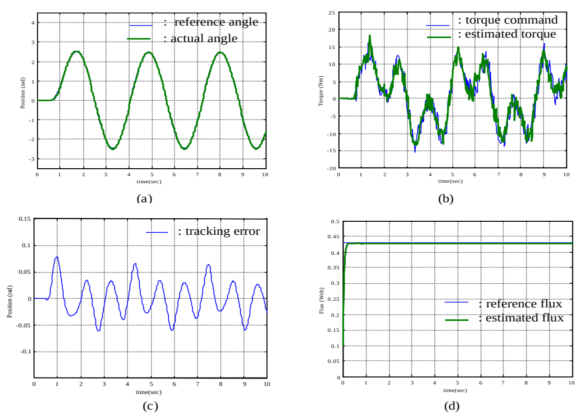

Fig. 3. Responses of a sinusoidal position command: (a) position; (b) torque command and estimated torque; (c) tracking error $(\theta_m^* - \theta_m)$; (d) rotor flux.

6. CONCLUSIONS

This paper presents a new adaptive backstepping motion control for a mechanical system driven by an induction motor. We adopt the sliding-mode direct torque control proposed in (Lin and Fanf, 2001) as the inner loop controller, which ensures that the electromagnetic torque of the motor will closely follow the torque command. The main topic of this paper is then only to design a position controller, which generates the torque command to the inner loop controller so that the asymptotical stability can be ensured. This position controller is derived based on the backstepping methodology. On the other hand, the backstepping method provides a way to define the sliding surface for the sliding-mode control. We use this concept to extend the result to the system with an uncertainty. The proposed control scheme is the so-called adaptive sliding-mode backstepping controller stated in Proposition 2. The control system is implemented on a PC-based system to control an induction motor with a rod fixed on the shaft. Both set-point and tracking position control experiments verify the control theory and show that the proposed control scheme is useful for industrial applications.

Appendix A

A.1 *Sliding-mode direct torque control*

The active torque (u_T) and the square of the flux norm (ϕ) are defined as

$$u_T = i_{bs}\varphi_{ar} - i_{as}\varphi_{br} \tag{A.1}$$

$$\phi = \varphi_{ar}^2 + \varphi_{br}^2 \tag{A.2}$$

and the errors as $e_T \equiv u_T - u_{Tref}$ and $e_\phi \equiv \phi - \phi_{ref}$, where u_{Tref} and ϕ_{ref} are the reference values of the active torque and the square of the flux norm.

The sliding surface $\mathbf{s} = [s_1; s_2]^T$ are selected as

$$s_1 = e_T + k_1 \int_0^t e_T dt \tag{A.3}$$

$$\begin{aligned} s_2 &= \frac{d}{dt} e_\phi + k_2 e_\phi \\ &= k_2 e_\phi + \frac{2}{T_r}[M(i_{as}\varphi_{ar} + i_{bs}\varphi_{br}) - \phi] \\ &\quad -\dot{\phi}_{ref} \end{aligned} \tag{A.4}$$

where k_1 and k_2 are positive gains. If the system stays stationary on the surface, then $s_1 = s_2 = \dot{s}_1 = \dot{s}_2 = 0$.

The sliding-mode direct torque control scheme is

$$\mathbf{u} \equiv \begin{bmatrix} u_{as} \\ u_{bs} \end{bmatrix} = -\mathbf{D}^{-1}\left(\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} + k_c \mathbf{s} + \begin{bmatrix} \mu_{c1}\ \mathrm{Sat}(s_1) \\ \mu_{c2}\ \mathrm{Sat}(s_2) \end{bmatrix} \right) \tag{A.5}$$

where $k_c > 0$ is a control gain and

$$\mathbf{D} = \begin{bmatrix} -\alpha\varphi_{\mathrm{br}} & \alpha\varphi_{ar} \\ 2\alpha\dfrac{M}{T_r}\varphi_{ar} & 2\alpha\dfrac{M}{T_r}\varphi_{br} \end{bmatrix} \tag{A.6}$$

$$\begin{aligned} b_1 &= \left(k_1 - \frac{1}{T_r} - \gamma\right) u_T - p\omega\left(\phi_d + K\phi\right) \\ &\quad -k_1 u_{Tref} - \dot{u}_{Tref} \end{aligned} \tag{A.7}$$

$$\begin{aligned} b_2 &= \frac{2M}{T_r}\left[\frac{M}{T_r} m_i - \left(\frac{1}{T_r} + \gamma\right)\phi_d + \frac{K}{T_r}\phi + p\omega u_T\right] \\ &\quad + \left(k_2 - \frac{2}{T_r}\right)\dot{\phi} - k_2\dot{\phi}_{ref} - \ddot{\phi}_{ref} \end{aligned} \tag{A.8}$$

$$\phi_d = i_{as}\varphi_{ar} + i_{bs}\varphi_{br} \tag{A.9}$$

$$m_i = i_{as}^2 + i_{bs}^2 \tag{A.10}$$

It is shown in (Lin and Fang, 2001) that the overall system will asymptotically converge to $u_T = u_{Tref}$ and $\phi = \phi_{ref}$, if the controller (A.5) applies.

A.2 *Adaptive sliding-mode flux observer*

The above sliding-mode DTC requires the signals of ϕ, i_{as}, and i_{bs}. The currents i_{as} and i_{bs} can be measured, while ϕ is provided by the following flux observer (Lin and Fang, 2001):

$$\dot{\hat{i}}_{as} = -\gamma \hat{i}_{as} + \frac{K}{T_r}\hat{\varphi}_{ar} + pK\omega\hat{\varphi}_{br} + \alpha u_{as} + \Lambda_1 \quad \text{(A.11)}$$

$$\dot{\hat{i}}_{bs} = -\gamma \hat{i}_{bs} + \frac{K}{T_r}\hat{\varphi}_{br} - pK\omega\hat{\varphi}_{ar} + \alpha u_{bs} + \Lambda_2 \quad \text{(A.12)}$$

$$\dot{\hat{\varphi}}_{ar} = \frac{M}{T_r}\hat{i}_{as} - \frac{1}{T_r}\hat{\varphi}_{ar} - p\omega\hat{\varphi}_{br} + \Lambda_3 \quad \text{(A.13)}$$

$$\dot{\hat{\varphi}}_{br} = \frac{M}{T_r}\hat{i}_{bs} - \frac{1}{T_r}\hat{\varphi}_{br} + p\omega\hat{\varphi}_{ar} + \Lambda_4 \quad \text{(A.14)}$$

where $\hat{i}_{as}, \hat{i}_{bs}, \hat{\varphi}_{ar}, \hat{\varphi}_{br}$ are the estimates of $i_{as}, i_{bs}, \varphi_{ar}, \varphi_{br}$, respectively. Λ_i, $i = 1 \cdots 4$, are the observer inputs. Let the estimate errors be $\mathbf{e} = [e_1,\ e_2,\ e_3,\ e_4]^T = [\hat{i}_{as} - i_{as},\ \hat{i}_{bs} - i_{bs},\ \hat{\varphi}_{ar} - \varphi_{ar},\ \hat{\varphi}_{br} - \varphi_{br}]^T$. Using the adaptive sliding-mode theory, the inputs Λ_1 and Λ_2 are designed as

$$\begin{cases} \Lambda_1 = -\hat{\rho}_1 \operatorname{sign}(e_1) - \hat{\zeta}_1 \\ \Lambda_2 = -\hat{\rho}_2 \operatorname{sign}(e_2) - \hat{\zeta}_2 \end{cases} \quad \text{(A.15)}$$

with the adaptive laws of

$$\dot{\tilde{\boldsymbol{\rho}}} = \dot{\hat{\boldsymbol{\rho}}} = \begin{bmatrix} \dot{\hat{\rho}}_1 \\ \dot{\hat{\rho}}_2 \end{bmatrix} = \begin{bmatrix} |e_1| \\ |e_2| \end{bmatrix} \quad \text{(A.16)}$$

$$\dot{\tilde{\boldsymbol{\zeta}}} = \dot{\hat{\boldsymbol{\zeta}}} = \begin{bmatrix} \dot{\hat{\zeta}}_1 \\ \dot{\hat{\zeta}}_2 \end{bmatrix} = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \quad \text{(A.17)}$$

On the other hand, the other two inputs Λ_3 and Λ_4 are

$$\begin{bmatrix} \Lambda_3 \\ \Lambda_4 \end{bmatrix} = \begin{bmatrix} k_\phi & -p\omega \\ p\omega & k_\phi \end{bmatrix} \begin{bmatrix} \dfrac{K}{T_r} & pK\omega \\ -pK\omega & \dfrac{K}{T_r} \end{bmatrix}^{-1} \begin{bmatrix} \Lambda_1 \\ \Lambda_2 \end{bmatrix} - \begin{bmatrix} \rho_3 \ \mathrm{Sat}(e_3) \\ \rho_4 \ \mathrm{Sat}(e_4) \end{bmatrix} \quad \text{(A.18)}$$

where, $k_\phi > 0$ is a constant and $\left[\ \rho_3,\ \rho_4\ \right]^T$ are the upper bound of the uncertainty of estimate flux equations.

Some experiments in (Lin and Fang, 2001) have verified the asymptotic stability of the adaptive sliding-mode DTC. Then, we can assume that the active torque u_T follows the reference one u_{Tref}, so that the electromagnetic torque T_e generated by the motor can be approximately equal to $k_T u_{Tref}$.

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