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A VEHICLE MODEL FOR THE DESIGN OF PERFORMANCE-ORIENTED YAW-CONTROL SYSTEMS

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Abstract: In this paper a complete dynamic model of a road vehicle is derived and discussed, which can be suitably used to develop the new concept of "performance-oriented" yaw-control system.

Keywords: Dynamic modelling, nonlinear models, tyres, vehicle dynamics, vehicle suspension.

1. INTRODUCTION

In this paper, a dynamic model of a road vehicle is developed and discussed with the objective to develop a new concept of "performance-oriented" yaw-dynamics controller. Among the multitude of electronic automatic control devices a modern GT-car is equipped with, the yaw-control system is probably the most critical and challenging. Despite its inherent complexity, in the last two years the interest of car manufacturers in yaw-controller devices has been booming, due to the huge potential benefits a yaw-controller can deliver to the overall dynamic behavior of a car. The standard way a yaw control system is designed is "safety-oriented"; namely, a car is designed and tuned in order to have a "passive" (i.e. without electronic assistance) behavior smooth, highly stable, and easily controllable by non-professional drivers (Bittanti *et al.*, 2001; Valtolina *et al.*, 2001). Hence, the yaw-control system is expected to be active only in extreme conditions, when the car takes unusual and dangerous attitudes. A completely different and new approach consists in designing a car having a "rough" passive behavior: non-smooth and almost unstable (the typical tuning of a racing car). Accordingly, the car must be equipped with a yaw-controller device which is intended and aimed to assist a non-professional driver to control the lateral and yaw dynamics of the car. In this framework, the yaw-controller is expected to be always active. The notion of "performance-oriented" yaw-controller is very appealing for extreme GT-cars manufacturers like Ferrari, since it can help to overcome the typical trade-off between stability/driveability and performance. The design of a performance-oriented yaw control system is a very new, unexplored and challenging task, which first requires the development of a suitable model of a car. Specifically, the standard single-track model typically used for yaw-controller design is inadequate, since many peculiar features and behaviors of a car must be explicitly taken into account, namely:

- the load transfer among the four tires;
- the non-linearity of the tires with respect to load changes;
- the anti-roll torsion springs.

The goal of the present paper is to develop and propose a complete model of the car specifically developed and suited for performance-oriented yaw-dynamics control systems. Needles to say, this is just the first part of a research work that has been developing by the Innovation Division of Ferrari S.p.A and the Politecnico di Milano, having the objective of designing and prototyping this new yaw-control concept. The outline of the paper is the following: in Section 2 the coordinate and reference systems which will be used in the rest of the paper are preliminary presented; Sections 3 to 5 are devoted to the development of the model, constituted by models of the tires, of the body, of the suspensions, and of the anti-roll springs. Section 6 ends the paper with some final remarks and future work outline.

2. CO-ORDINATE SYSTEMS

The co-ordinate systems used in the following are:

- ***CoG*** - the chassis (*Center of Gravity*) co-ordinate system, which has its origin at the vehicle center of gravity. The following conventions are applied:
  - *x axis*: longitudinal axis of the vehicle, positive forward in the direction of motion;
  - *z axis*: vertical axis of the vehicle, positive upward with respect to the vehicle;
  - *y axis*: lateral axis of the vehicle, positive with reference to a right-hand orthogonal co-ordinate system.

  Rotations around the X_{CoG} , Y_{CoG} and Z_{CoG} axes are named *roll* (φ), *pitch* (χ) and *yaw* (ψ) respectively.
- ***Un*** - the undercarriage co-ordinate system, which differs from the CoG only in the pitch (χ) e and roll (φ) angles. The origin of the undercarriage co-ordinate system lies at road-level in the middle of the perpendicular projection of the rear axle.
- ***W*** - the wheel co-ordinate system. To be precise there is a co-ordinate system for each individual wheel.
- ***In*** - the fixed inertial coordinate system.

Indices are also introduced for the individual wheels: $_{FL}$ for the front-left wheel, $_{FR}$ for the front-right wheel, $_{RL}$ for the rear-left wheel and $_{RR}$ for the rear-right wheel.
The transformation from the undercarriage co-ordinate system to the fixed (inertial) co-ordinate system corresponds to a yaw angle (ψ) rotation about the Z_{CoG} axis. The transformation matrix $\underline{T}_{UnIn}$ has the following expression:

$$\underline{T}_{UnIn} = \underline{T}_{RotZ} = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (1)$$

3. WHEEL MODEL

3.1 *Lateral friction forces*

A common assumption in most of tire friction models is that the lateral tire friction force F_s is a nonlinear function of the tire side slip angle α, of the vertical wheel-ground contact force F_z and of the camber angle γ (the inclination of a wheel outward from the body). One of the most well-known models of this type is Pacejka's model (Pacejka and Bakker, 1991), also known as the "*magic formula*". This model has been shown to suitably match experimental data, obtained under particular conditions of constant linear and angular velocity. The Pacejka model has the form:

$$\begin{aligned} &F_s(\alpha,F_z,\gamma)=S_v \\ &+D\,\sin\left\{C\,\tan^{-1}\left[B\,\alpha^*-E\left(B\,\alpha^*-\tan^{-1}(B\,\alpha^*)\right)\right]\right\} \end{aligned} \quad (2)$$

where the analytic expression of the different terms which appear in the formula (2) depends on α, F_z, γ and on 18 parameters $(a_0, \ldots, a_{17})$, whose set completely define the static behaviour of the wheel:

$$\begin{cases} \alpha^* &= \alpha+S_h \\ S_h &= a_8 F_z+a_9+a_{10}\gamma \\ S_v &= a_{11}F_z+a_{12}+(a_{13}F_z^2+a_{14}\cdot F_z)\gamma \\ C &= a_0 \\ D &= \left[(a_1 F_z+a_2)(1-a_{15}\gamma^2)\right]F_z \\ B &= \frac{1}{CD}\left[a_3 \sin\left(2\,\tan^{-1}\left(\frac{F_z}{a_4}\right)\right)(1-a_5|\gamma|)\right] \\ E &= (a_6\cdot F_z+a_7)((1-(a_{16}t\gamma+a_{17})\mathrm{sign}(\alpha^*)) \end{cases}$$

Pacejka's model aims at describing the experimental shapes force vs. tire side slip angle via static map $F_s(\alpha, F_z, \gamma) : \{\alpha, F_z, \gamma\} \mapsto F_s$. Nevertheless, it is well known that lateral friction force may also depend on the vehicle velocity v_{CoG} and varies when the road characteristics change. The effect of vehicle velocity v_{CoG} on the lateral friction force can be approximated by a decreasing exponential law,

$$F_s(\alpha,F_z,\gamma,v_{CoG})=F_s(\alpha,F_z,\gamma)\cdot e^{-\tau_s(v_{nom}-v_{CoG})} \quad (3)$$

where v_{nom} is the constant nominal velocity which the identified a_i parameters refer to.

Relaxation dynamics
When a rolling pneumatic tire is given a step change in side slip angle α, the lateral friction force F_s tends to a steady-state condition $F_{s_{ss}}$ which value can be calculated by the "magic formula" (2). The mechanism is not an instantaneous phenomenon, mainly due to the time required for the deflection of the tire sidewalls in the lateral direction. The lag is closely related to the rotation of the tire, typically taking between one-half and one full revolution of the tire to effectively reach the steady-state force condition. This distance is often referred to as the "*relaxation length*" S_{0s}. The time lag can be suitably approximated by $\tau = \frac{S_{0s}}{v_w}$, where v_w is the wheel velocity, and the

relaxation dynamics can be approximated by a first order filter with time constant τ:

$$\dot{F}_s = \frac{v_w}{S_{0s}} \cdot \left(F_{s_{ss}} - F_s\right) \tag{4}$$

where the steady-state force condition $F_{s_{ss}} = f(\alpha, F_z, \gamma, v_{CoG})$ is given by equation (3).

Tire side slip angle α

If the single wheel is considered, the tire side slip angle α_{ij} is the angle between the wheel plane and the velocity $v_{w_{ij}}$ of the wheel ground contact point. Since the directions of the wheel ground contact point velocities are known from Eqs. (12), the four side slip angles can easily be derived geometrically:

$$\begin{aligned}
\alpha_{FL} &= \delta_w - \arctan\left(\frac{v_{CoG}\,\sin(\beta)+\dot{\psi}\,r_{FL}\,\cos(\vartheta_{FL})}{v_{CoG}\,\cos(\beta)-\dot{\psi}\,r_{FL}\,\sin(\vartheta_{FL})}\right)\\
\alpha_{FR} &= \delta_w - \arctan\left(\frac{v_{CoG}\,\sin(\beta)+\dot{\psi}\,r_{FR}\,\sin(\vartheta_{FR})}{v_{CoG}\,\cos(\beta)+\dot{\psi}\,r_{FR}\,\cos(\vartheta_{FR})}\right)\\
\alpha_{RL} &= \arctan\left(\frac{v_{CoG}\,\sin(\beta)-\dot{\psi}\,r_{RL}\,\sin(\vartheta_{RL})}{v_{CoG}\,\cos(\beta)-\dot{\psi}\,r_{RL}\,\cos(\vartheta_{RL})}\right)\\
\alpha_{RR} &= \arctan\left(\frac{v_{CoG}\,\sin(\beta)-\dot{\psi}\,r_{RR}\,\cos(\vartheta_{RR})}{v_{CoG}\,\cos(\beta)+\dot{\psi}\,r_{RR}\,\sin(\vartheta_{RR})}\right)
\end{aligned} \tag{5}$$

where the vehicle body side slip angle β has the following expression:

$$\beta = \arctan\left(\frac{\dot{y}_{CoG}}{\dot{x}_{CoG}}\right)_{In} - \psi \tag{6}$$

where ψ is the yaw angle and $(\dot{x}_{CoG}, \dot{y}_{CoG})$ are the longitudinal and lateral components of the CoG-velocity, expressed in the inertial co-ordinate system.

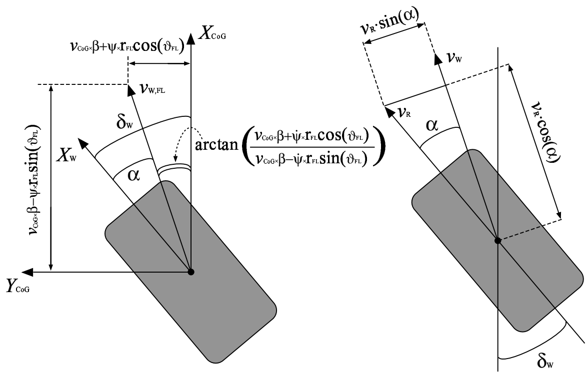

Fig. 1. Calculation of the front left tire side slip angle and the longitudinal slip λ.

3.2 *Longitudinal friction forces*

The friction force in the tire/road interface is the main mechanism for converting wheel angular acceleration (or deceleration), due to the motor torque T_{Drive} (or to the brake torque T_{Br}), to forward vehicle acceleration (or deceleration).

Once again, a common assumption in most tire friction models is that the longitudinal tire friction force F_l is a nonlinear function of the normalized relative velocity between the road and the tire (longitudinal slip coefficient λ), of the vertical load F_z and of the camber angle γ. As an alternative to the static maps $F_l(\lambda, F_z, \gamma) : \{\lambda, F_z, \gamma\} \mapsto F_l$, dynamic models based on the dynamic friction models of (Dahl, 1976) and Lugre (Canudas De Wit *et al.*, 1995) can be found in literature (Canudas De Wit and Tsiotras, 1999).

Once again, Pacejka's model is used to describe the longitudinal tire force. The following equation is formally identical to (2):

$$\begin{aligned}
&F_l(\lambda, F_z, \gamma) = \\
&+M\,\sin\left\{L\,\tan^{-1}\left[N\,\lambda^* - P\left(B\,\lambda^* - \tan^{-1}(N\,\lambda^*)\right)\right]\right\}
\end{aligned} \tag{7}$$

where the analytic expression of the different terms which appear in the formula (7) now depends on λ, F_z and on 11 parameters $(b_0, \ldots, b_{10})$:

$$\begin{cases}
\lambda^* &= \lambda + K_h\\
K_h &= b_9 F_z + b_{10}\\
L &= b_0\\
M &= (b_1 F_z + b_2) F_z\\
N &= \frac{1}{L\,M}\left(b_3 F_z^2 + b_4 F_z\right) e^{-b_5 F_z}\\
P &= b_6 F_z^2 + b_7 F_z + b_8
\end{cases}$$

which can be identified by matching experimental data, as shown in (Bakker *et al.*, 1987).

The effect of vehicle velocity v_{CoG} on the longitudinal friction force can be approximated by a decreasing exponential law,

$$F_l(\lambda, F_z, \gamma, v_{CoG}) = F_l(\lambda, F_z, \gamma) \cdot e^{-\tau_l (v_{nom} - v_{CoG})} \tag{8}$$

where v_{nom} is the constant nominal velocity which the identified b_i parameters refer to.

Finally, by introducing the relaxation dynamics the following expression is obtained:

$$\dot{F}_l = \frac{v_w}{S_{0l}} \cdot \left(F_{l_{regime}} - F_l\right) \tag{9}$$

where the steady-state force condition $F_{l_{ss}} = f(\lambda, F_z, \gamma, v_{CoG})$ is given by equation (8).

Tire longitudinal slip λ

In general, because of tire deformation which reduce the real circumference of the pneumatic, wheel ground contact point velocity v_w is not equal to the product $\omega\,r$. Denoting by v_r the velocity of a pure rolling wheel whose radius is r and angular velocity is w, the longitudinal slip coefficient λ can be defined as (see Figure 1):

$$\begin{aligned}
\lambda &= \frac{v_r - v_w}{\max(|v_r|, |v_w|)}\\
&= \frac{\omega\,r\,\cos(\alpha) - v_w}{\max\left(|\,\omega\,r\,\cos(\alpha)\,|,\ |\,v_w\,|\right)}
\end{aligned} \tag{10}$$

where the tire side slip angle α, the wheel rotational speed ω and the wheel ground contact point velocity v_w are given by Eqs. (5), (11) and (12) respectively.

3.3 *Wheel dynamics*

To calculate the angular wheel velocity ω, a torque balance is formed for each wheel:

$$J_w\,\dot{\omega} = T_{Drive} - T_{Br} - r\,F_l \tag{11}$$

where, J_w is the moment of inertia of the wheel, r is the static tire radius and F_l is the longitudinal friction force (given by eq. (7)). The accelerating torque is the drive torque T_{Drive} of the driveline, while the decelerating effects comes from the braking torque T_{Br} and tire friction torque $r\,F_l$. The wheel is considered not deformable (i.e. the effective dynamic rolling radius is equal to the static tire radius).
The derivation of the wheel ground contact point velocities $v_{w_{ij}}$ is carried out in the undercarriage co-ordinate system. Looking at the vehicle during a turn from a bird's eye view, the assumption that the vehicle velocity can be described as a superposition of a pure rotational motion, with magnitude v_{CoG} (vehicle center of gravity velocity) and direction β (vehicle body side slip angle) to the longitudinal axis, and a purely rotational motion with yaw rate $\dot{\psi}$ around the Z_{CoG} axis, is made. By denoting with r_{ij} and ϑ_{ij} the distances and the angles between the wheel ground contact points and the undercarriage axes X_{Un} and Y_{Un} respectively (see Figure 2), the wheel velocities $v_{w_{ij}}$ are given by the geometric superposition of the *CoG* velocity and a part of the magnitude $|r_{ij}\,\dot{\psi}|$ in the direction specified in Figure 2.
If the wheel casters are neglected, geometric calculations result in the following equations for the distances: $r_{FL} = r_{FR} = \sqrt{l_F^2 + (\frac{b_F}{2})^2}$, $r_{RL} = r_{RR} = \sqrt{l_R^2 + (\frac{b_R}{2})^2}$ and for the angles: $\vartheta_{FL} = \arctan\left(\frac{b_F}{2\,l_F}\right)$, $\vartheta_{FR} = \arctan\left(\frac{2\,l_F}{b_F}\right)$, $\vartheta_{RL} = \arctan\left(\frac{2\,l_R}{b_R}\right)$, $\vartheta_{RR} = \arctan\left(\frac{b_R}{2\,l_R}\right)$. The wheel ground contact point velocities can be divided into longitudinal and lateral components, with respect to the vehicle *CoG* co-ordinate system:

$$\begin{aligned}
v_{w_{FL}} &= \left(v_{CoG}\,C_\beta - \dot{\psi}\,r_{FL}\,S_{\vartheta_{FL}}\right)\overrightarrow{e_x} \\
&\quad + \left(v_{CoG}\,S_\beta + \dot{\psi}\,r_{FL}\,C_{\vartheta_{FL}}\right)\overrightarrow{e_y} \\
v_{w_{FR}} &= \left(v_{CoG}\,C_\beta + \dot{\psi}\,r_{FR}\,C_{\vartheta_{FR}}\right)\overrightarrow{e_x} \\
&\quad + \left(v_{CoG}\,S_\beta + \dot{\psi}\,r_{FR}\,S_{\vartheta_{FR}}\right)\overrightarrow{e_y} \\
v_{w_{RL}} &= \left(v_{CoG}\,C_\beta - \dot{\psi}\,r_{RL}\,C_{\vartheta_{RL}}\right)\overrightarrow{e_x} \\
&\quad + \left(v_{CoG}\,S_\beta - \dot{\psi}\,r_{RL}\,S_{\vartheta_{RL}}\right)\overrightarrow{e_y} \\
v_{w_{RR}} &= \left(v_{CoG}\,C_\beta + \dot{\psi}\,r_{RR}\,S_{\vartheta_{RR}}\right)\overrightarrow{e_x} \\
&\quad + \left(v_{CoG}\,S_\beta - \dot{\psi}\,r_{RR}\,C_{\vartheta_{RR}}\right)\overrightarrow{e_y}
\end{aligned} \tag{12}$$

where C_t and S_t stand for $\cos(t)$ and $\sin(t)$, while $\overrightarrow{e_x}$ and $\overrightarrow{e_y}$ denote the longitudinal and lateral *CoG* co-ordinate directions respectively.

4. VEHICLE CHASSIS DYNAMICS

As in (Kiencke and Nielsen, 2000), the chassis is considered as a single rigid body with 6 degrees of freedom: center of gravity position $(x_{CoG}, y_{CoG}, z_{CoG})$ and rotation angles around the principal axes of the *CoG* co-ordinate system (ψ, φ, χ). The chassis dynamics study is divided into two parts: the *chassis translatory motion*, for the calculation of displacement, velocity and acceleration in the direction of the three co-ordinate axes and the *chassis rotational motion*, for the calculation of yaw rate $\dot{\psi}$, roll rate $\dot{\varphi}$, pitch rate $\dot{\chi}$ and their integrals.

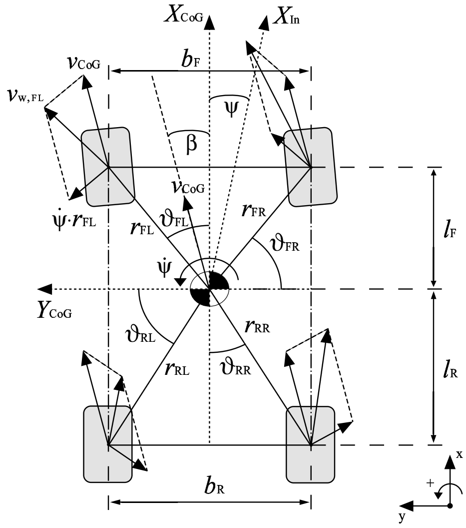

Fig. 2. Vehicle from a bird's eye view: velocities and their superposition

4.1 *Chassis translatory motion*

The translatory variables are calculated in the inertial co-ordinate system, because all of the forces acting on the chassis and relevant for the translatory motion are contact forces to the environment (carrying out the integration in the inertial system no apparent forces have to be considered). The force balance for the three co-ordinate directions leads to the following array equation:

$$m_{CoG}\begin{bmatrix}\ddot{x}_{CoG}\\ \ddot{y}_{CoG}\\ \ddot{z}_{CoG}\end{bmatrix}_{In} = \underline{T}_{UnIn}\underbrace{\begin{bmatrix}\sum_{ij} F_{X_{ij}} + F_{wind,X} + F_{Gx}\\ \sum_{ij} F_{Y_{ij}} + F_{wind,Y} + F_{Gy}\\ \sum_{ij} F_{Zc_{ij}} + F_{wind,Z} + F_{Gz}\end{bmatrix}}_{\text{Forces in the } Un \text{ co-ordinate system}}$$

where, $F_{X_{ij}}$ and $F_{Y_{ij}}$ are the horizontally acting wheel forces (13), $F_{Zc_{ij}}$ are the vertically acting forces due to the suspensions (23), F_{wind} is the aerodynamic force (15), F_G is the gravitational force (14) and $\underline{T}_{UnIn}$ is the transformation matrix from the undercarriage to the inertial co-ordinate system (1). The horizontally acting wheel forces (lateral and longitudinal friction forces) have to be transformed from the wheel to the undercarriage co-ordinate system. Rear wheels need no transformation, while for the front ones the steer angle δ_w must be taken into account:

$$\begin{cases}
F_{X_{FL}} = F_{l_{FL}}\cos(\delta_w) - F_{s_{FL}}\sin(\delta_w)\\
F_{Y_{FL}} = F_{s_{FL}}\cos(\delta_w) + F_{l_{FL}}\sin(\delta_w)\\
F_{X_{FR}} = F_{l_{FR}}\cos(\delta_w) - F_{s_{FR}}\sin(\delta_w)\\
F_{Y_{FR}} = F_{s_{FR}}\cos(\delta_w) + F_{l_{FR}}\sin(\delta_w)\\
F_{X_{RL}} = F_{l_{RL}}\ ,\ F_{Y_{RL}} = F_{s_{RL}}\\
F_{X_{RR}} = F_{l_{RR}}\ ,\ F_{Y_{RR}} = F_{s_{RR}}
\end{cases} \tag{13}$$

Gravitational force
Under the assumption of level road, both in the undercarriage and in the inertial co-ordinate system the gravitational force has the following form:

$$\begin{bmatrix} F_{Gx} \\ F_{Gy} \\ F_{Gz} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -m_{CoG} \cdot g \end{bmatrix} \tag{14}$$

where g is the gravitational acceleration.

Wind force
Under the assumption that both wind velocity and vertical motion are neglected, wind force has the following expression:

$$\begin{bmatrix} F_{wind,X} \\ F_{wind,Y} \\ F_{wind,Z} \end{bmatrix} = \begin{bmatrix} -c_{aer,X} \cdot v_{CoG,X}^2 \\ -c_{aer,Y} \cdot v_{CoG,Y}^2 \\ 0 \end{bmatrix} \tag{15}$$

where $c_{aer,X}$ and $c_{aer,Y}$ are the coefficients of *aerodynamic drag.*

4.2 *Chassis rotational motion*

The rotational variables can be calculated directly in the undercarriage co-ordinate system since the roll and pitch axes are assumed to lie at road level. The equations of angular dynamics can be expressed in vector form as $\dot{\Gamma}(t) = T^{(e)}(t)$, where Γ is the overall angular momentum of the vehicle and $T^{(e)}$ is the sum of external torques acting on the vehicle.

Roll rate
The torque balance around the vehicle longitudinal axis results:

$$\begin{aligned} J_X \ddot{\varphi} = & (F_{Zc_{FL}} - F_{bar_F}) \tfrac{b_F}{2} C_\varphi - (F_{Zc_{FR}} + F_{bar_F}) \tfrac{b_F}{2} C_\varphi \\ & + (F_{Zc_{RL}} - F_{bar_R}) \tfrac{b_R}{2} C_\varphi - (F_{Zc_{RR}} + F_{bar_R}) \tfrac{b_R}{2} C_\varphi \\ & - m_{CoG}\, h\, \ddot{z}_{CoG}\, S_\varphi + m_{CoG}\, h\, \ddot{y}_{CoG}\, C_\varphi \end{aligned}$$

where $F_{Zc_{ij}}$ and $F_{bar_{F,R}}$ are the vertically acting chassis forces due to suspensions (23) and anti-roll bars (24), b_F and b_R are the distances between wheels on front and rear axles, m_{CoG} is the vehicle mass, h is the height of the CoG, g is the gravitational constant, J_X is the moment of inertia about the vehicle longitudinal axis and $\ddot{y}_{CoG}$ and $\ddot{z}_{CoG}$ are the lateral and vertical CoG accelerations (in inertial co-ordinates) respectively.

Pitch rate
The torque balance around the vehicle lateral axis is given by:

$$\begin{aligned} J_Y \ddot{\chi} = & (F_{Zc_{RL}} + F_{Zc_{RR}})\, l_R\, C_\chi - (F_{Zc_{FL}} + F_{Zc_{FR}})\, l_F\, C_\chi \\ & - m_{CoG}\, h\, \ddot{z}_{CoG}\, S_\chi - m_{CoG}\, h\, \ddot{x}_{CoG}\, C_\chi \end{aligned}$$

where $F_{Zc_{ij}}$ are the vertically acting chassis forces due to suspensions (23), l_F and l_R are the distances from CoG to front and rear axles, m_{CoG} is the vehicle mass, h is the height of the CoG, g is the gravitational constant, J_Y is the moment of inertia about the vehicle lateral axis and $\ddot{x}_{CoG}$ and $\ddot{z}_{CoG}$ are the longitudinal and vertical CoG accelerations (in inertial co-ordinates) respectively.

Yaw rate
Neglecting caster effect, the torque balance around the vehicle vertical axis can be expressed as:

$$\begin{aligned} J_Z \ddot{\psi} = & (F_{Y_{FL}} + F_{Y_{FR}}) l_F - (F_{Y_{RL}} + F_{Y_{RR}}) l_R \\ & + (F_{X_{FR}} - F_{X_{FL}}) \tfrac{b_F}{2} + (F_{Y_{RR}} - F_{Y_{RL}}) \tfrac{b_R}{2} \end{aligned}$$

where $F_{X_{ij}}$ and $F_{Y_{ij}}$ are the horizontally acting chassis forces (13), l_F and l_R are the distances from CoG to front and rear axles, b_F and b_R are the distances between wheels on front and rear axles and J_Z is the moment of inertia about the vehicle vertical axis.

5. SUSPENSION AND ANTI-ROLL BARS

A quarter vehicle model (Gillespie, 1992) is used for each wheel. Each tire/suspension subsystem has an individual mass m_w, which is connected via wheel spring k_w to the road and via a spring-damper system (k_s,d_s) to the chassis (sprung mass). All four wheel suspensions are assumed to be vertical directed.
The vertical forces acting upon a single wheel are the wheel ground contact forces,

$$F_{z_{ij}} = k_w (z_{road} - z_{w_{ij}}) \tag{16}$$

the suspension spring forces,

$$F_{c_{ij}} = k_s (z_{w_{ij}} - z_{c_{ij}}) \tag{17}$$

and the suspension damping forces,

$$F_{d_{ij}} = d_s (\dot{z}_{w_{ij}} - \dot{z}_{c_{ij}}) \tag{18}$$

where z_{road}, $z_{w_{ij}}$ and $z_{c_{ij}}$ are the heights of the wheel/ground, wheel/suspension and suspension/chassis contact points respectively.

5.1 *Suspension geometry*

In this section the geometrical relations between the height $z_{c_{ij}}$ of the suspension/chassis contact points and the roll (φ) and pitch (χ) angles together with the height h of the CoG are calculated (the reference road level is $z_{road,0} = 0$). The following equations can be easily derived (see Figure 3):

$$\begin{aligned} & z_A = \tfrac{(z_{c_{FL}} + z_{c_{FR}})}{2}, \; z_B = \tfrac{(z_{c_{RL}} + z_{c_{RR}})}{2} \\ & z_C = \tfrac{(z_{c_{FL}} + z_{c_{RL}})}{2}, \; z_D = \tfrac{(z_{c_{FR}} + z_{c_{RR}})}{2} \end{aligned} \tag{19}$$

Trigonometric calculations result in the following equations:

$$z_B - z_A = (l_R + l_F) \sin(\chi) \tag{20}$$

$$z_C - z_D = \tfrac{(b_R + b_F)}{2} \sin(\varphi) \tag{20.1}$$

$$\tfrac{1}{2}(z_{A,C} + z_{B,D}) = z_{CoG} - h \cdot \cos(\chi) \cdot \cos(\varphi) \tag{20.2}$$

and under the assumption of *rigid* vehicle chassis:

$$z_{c_{RL}} - z_{c_{RR}} = z_{c_{FL}} - z_{c_{FR}} \tag{21}$$

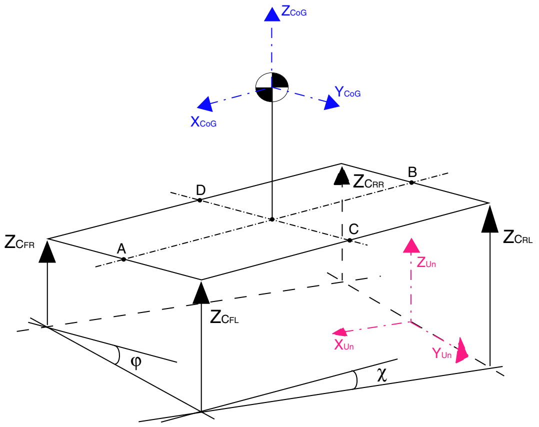

Fig. 3. Suspension geometry

From equations (19), (20) and (21) the following expressions for the heights $z_{c_{ij}}$ are obtained:

$$
\begin{aligned}
z_{c_{FL}} &= z_{CoG} - h\,\cos(\chi)\,\cos(\varphi) \\
&\quad + \tfrac{1}{4}\,(b_R + b_F)\,\sin(\varphi) - \tfrac{1}{2}\,(l_R + l_F)\,\sin(\chi) && (22)\\
z_{c_{FR}} &= z_{CoG} - h\,\cos(\chi)\,\cos(\varphi) \\
&\quad - \tfrac{1}{4}\,(b_R + b_F)\,\sin(\varphi) - \tfrac{1}{2}\,(l_R + l_F)\,\sin(\chi) && (22.1)\\
z_{c_{RL}} &= z_{CoG} - h\,\cos(\chi)\,\cos(\varphi) \\
&\quad + \tfrac{1}{4}\,(b_R + b_F)\,\sin(\varphi) + \tfrac{1}{2}\,(l_R + l_F)\,\sin(\chi) && (22.2)\\
z_{c_{RR}} &= z_{CoG} - h\,\cos(\chi)\,\cos(\varphi) \\
&\quad - \tfrac{1}{4}\,(b_R + b_F)\,\sin(\varphi) + \tfrac{1}{2}\,(l_R + l_F)\,\sin(\chi) && (22.3)
\end{aligned}
$$

For the explicit calculation of the suspension dumping forces equations (22) have to be differentiated.

5.2 *Suspension dynamics*

Assuming a level road (i.e. $z_{road} = 0$) and the constants k_w, k_s and d_s equal for the four wheels, the force balances at the quarter chassis and the wheels are:

$$m_{w_{ij}}\,\ddot{z}_{w_{ij}} = -d_s(\dot{z}_{w_{ij}} - \dot{z}_{c_{ij}}) - (k_w + k_s) z_{w_{ij}} + k_s\, z_{c_{ij}} \mp F_{bar_i}$$

By using the results obtained in section (5.1) and integrating the previous equations, the suspension vertical forces $F_{Zc_{ij}}$ can be calculated as the sum of the spring (17) and damping (18) components namely:

$$F_{Zc_{ij}} = k_s(z_{w_{ij}} - z_{c_{ij}}) + d_s(\dot{z}_{w_{ij}} - \dot{z}_{c_{ij}}) \qquad (23)$$

5.3 *Anti-Roll bars*

The anti-roll bar increases the effective spring rate of the suspension in roll, the extent to which is dependent on the stiffness K_{b_i} of the anti-roll bar itself and where it attaches to the suspension. As a result, the anti-roll bar vertical forces F_{bar_i} can be considered proportional to the difference between the heights of the suspension/chassis contact points which the bar is connected to:

$$
\begin{aligned}
F_{bar_F} &= K_{b_F}(z_{c_{FL}} - z_{c_{FR}}) = \tfrac{1}{2}\,K_{bF}\,(b_R + b_F)\,\sin(\varphi) \\
F_{bar_R} &= K_{b_R}(z_{c_{RL}} - z_{c_{RR}}) = \tfrac{1}{2}\,K_{bR}\,(b_R + b_F)\,\sin(\varphi)
\end{aligned} \qquad (24)
$$

where b_F and b_R are the distances between wheels on front and rear axles respectively.

6. CONCLUSIONS AND FUTURE WORK

In this paper a complete model of a road vehicle has been developed and proposed; this model is intended to be the basis for control system design for an innovative concept of "performance-oriented" yaw-controller. The next steps of this research, currently underway, will be the quantitative analysis of the trade-off between stability and performance of a car, and the design and tuning of a performance-oriented control scheme.

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