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STABILITY AND STABILIZABILITY OF A CLASS OF DYNAMICAL UNCERTAIN SYSTEMS WITH MULTIPLE TIME-VARYING DELAYS

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Abstract: This paper deals with a class of uncertain systems with multiple time-varying delays. The stability and stabilizability of this class of systems are considered. Their robustness are also studied when the norm bounded uncertainties are considered. LMIs Delay-dependent sufficient conditions for both stability and stabilizability and their robustness are established to check whether a system of this class is stable or not and/or is stabilizable or not. Some numerical examples are provided to show the usefulness of the proposed results. *Copyright ©2002 IFAC*

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1. INTRODUCTION

It was shown in different studies that the presence of the time-delay in the systems dynamics is the primary cause of instability and performance degradation. The class of dynamical systems with time-delay has in fact attracted a lot of researchers mainly from the control community. Many results on this class of systems have been reported to the literature. We refer the reader to (Boukas and Liu, 2000; Mahmoud, 2000) and the references therein for more information.

In the present literature there exist two techniques that can be used to study the stability and the stabilizability. The first one is based on the Lyapunov-Razumikhin technique and it consists of considering a Lyapunov function of the form, $V(x_t) = x_t^\top P x_t$, with P a symmetric and positive-definite matrix with appropriate dimension and x_t is the state vector of the system, to develop the conditions that can be used used to check whether the system under study is stable or not; and/or stabilizable or not. This technique gives a condition that depends on the maximum value of the delay. The reader can consult the work of (Hale, 1977; Wang *et al.*, 1987; Su and Huang, 1992; Niculescu *et al.*, 1994; Su, 1994; Xu and Liu, 1994; Mao, 1997; Xu, 1995; Li and de Souza, 1996; Li and de Souza, 1997; Hmamed, 1997; Sun *et al.*, 1997; Mahmoud, 2000) and the references for more information.

The second technique is based on the Lyapunov-Krasovskii approach and it consists of considering a more complicated Lyapunov functional to determine the appropriate delay-dependent condition that in general depends on the upper bound of the first derivative of the delay when it is time-varying. This technique has been extensively used and the large number of references using it confirms this. See for example the works done by (Boukas and Liu, 2000; Mahmoud, 2000; Fridman, 2001) and the references therein for more information;

But from the practical point of view we are interested by conditions that depend on both, i.e:

the upper bound of the delay and the lower and the upper bounds of the first derivative of the time-varying delay. Since in practice the delay is in fact always time-varying, that can be usually represented by a function $h(t)$, and bounded by a constant $\bar{h}$, it is therefore desirable to have conditions that depend on the upper bound of the time-varying delay and on the lower and the upper bounds of the first derivative of the time-varying delay.

The goal of this paper consists of considering the class of uncertain linear systems with multiple time-varying delays and develop sufficient conditions for stability and stabilizability and their robustness that depend on the upper bounds of the delays and on the lower and upper bounds of the first derivative of the time-varying delays. The Lyapunov-Krasovskii approach will be used in this paper.

The paper is organized as follows. In section 2, the problem is stated and the required assumptions are formulated. Section 3 deals with the stability and the robust stability. Section 4 covers the stabilizability and the robust stabilizability of the class of systems under study. Section 5 presents some numerical examples to show the usefulness of the proposed results.

2. PROBLEM STATEMENT

Let us consider the following class of systems with multiple time-varying delays:

$$\dot{x}_t = A(t)x_t + \sum_{j=1}^{p} A_{dj}(t)x_{t-h_j(t)} + B(t)u_t \quad (1)$$

where x_t is the state vector, u_t is the control input, $h_j(t)$; $j = 1, 2, \ldots, p$, is the time-varying delay of the system and the matrices $A(t)$, $A_{dj}(t)$ and $B(t)$ are given by:

$$\begin{aligned} A(t) &= A + DF(t)E \\ A_{dj}(t) &= A_{dj} + D_jF_j(t)E_j, \forall j = 1, 2, \ldots, p \\ B(t) &= B + D_bF_b(t)E_b \end{aligned}$$

with A, A_{dj}, $j = 1, 2, \ldots, p$, B, D, E, D_j, E_j; $j = 1, 2, \ldots, p$, D_b and E_b are given matrices with appropriate dimensions and $F(t)$, $F_j(t)$; $j = 1, 2, \ldots, p$ and $F_b(t)$ represent the system uncertainties satisfying the following assumption.

Assumption 2.1. Let us assume that the following hold:

$$F^\top(t)RF(t) \leq R \quad (2)$$
$$F_d^\top(t)R_dF_d(t) \leq R_d, \quad (3)$$
$$F_b^\top(t)R_bF_b(t) \leq R_b \quad (4)$$

with R, $R_1, \ldots, R_p$ and R_b are given matrices with appropriate dimensions and $R_d = diag(R_1, \ldots, R_p)$, $F_d(t) = diag(F_1(t), \ldots, F_p(t))$

Remark 2.1. The uncertainties that satisfy (2)-(4) will be referred to as admissible uncertainties. Notice that the uncertainties $F(t)$, $F_j(t)$, $j = 1, 2, \ldots, p$ and $F_b(t)$ can be chosen dependent on the system state and the developed results will remain valid. However, in the present paper we will consider only the case of time-varying uncertainties.

Assumption 2.2. The time-varying delay $h_j(t)$, $j = 1, 2, \ldots, p$ is assumed to satisfy the following:

$$0 \leq h_j(t) \leq \bar{h}_j < \infty \quad (5)$$
$$\underline{l}_j \leq \dot{h}_j(t) \leq \bar{l}_j < 1, \forall j = 1, 2, \ldots, p \quad (6)$$

where $\bar{h}_j$, $\underline{l}_j$ and $\bar{l}_j$, $j = 1, \ldots, p$ are given positive constants.

Let us define $\bar{\tau}$ as $\bar{\tau} = max\left(\bar{h}_1, \ldots, \bar{h}_1\right)$ and $\mathbf{x}_t$ as $\mathbf{x}_t(s) = x_{t+s}$, $t - \bar{\tau} \leq s \leq t$. In the rest of the paper we will use $\mathbf{x}_t$ instead of $\mathbf{x}_t(s)$;

In the rest of this paper the notation is standard unless it is specified otherwise. $L > 0$ ($L > 0$) means that the matrix L is symmetric and positive-definite matrix (symmetric and negative-definite).

3. STABILITY AND ROBUST STABILITY

The goal of this section consists of establishing what will be the sufficient conditions that can be used to check whether or not the class of systems under study is stable. We are also interested by the robust stability of this class of systems. These two problems will be discussed in the following subsections.

3.1 *Stability*

Let us now suppose that the control is equal to zero, i.e: $u_t = 0$, $\forall\ t \geq 0$ and that the system doesn't contain uncertainties which gives the following dynamics:

$$\dot{x}_t = Ax_t + \sum_{j=1}^{p} A_{dj}x_{t-h_j(t)} \quad (7)$$

The goal of this subsection consists of developing a condition that can be used to check whether the class of systems under study is stable or not. The condition we are looking for should depend on the upper bound of the delay and on the lower and upper bounds of the first derivative of the time-varying delays given in Assumption 2.2. The following theorem states such result.

Theorem 3.1. Let assume that the assumption 2.2 is satisfied. If there exist $P > 0$, $Q_j > 0$, $W_j > 0$, X_j, Y_j and Z_j for $j = 1, 2, \ldots, p$ such that the following hold:

$$\mathcal{Z}_j = \begin{bmatrix} Z_j & Y_j \\ Y_j^\top & X_j \end{bmatrix} > 0 \tag{8}$$

$$\left(\bar{l}_j - \underline{l}_j\right) X_j + \left(\bar{l}_j - 1\right) W_j < 0, and \tag{9}$$

$$\begin{bmatrix} A^\top P + PA + \Psi_1 & PA_d - \Psi_3 & A^\top \boldsymbol{I}\overline{W} \\ \left(PA_d - \Psi_3\right)^\top & -\Psi_2 & A_d^\top \boldsymbol{I}\overline{W} \\ \overline{W}\boldsymbol{I}^\top A & \overline{W}\boldsymbol{I}^\top A_d & -\overline{W} \end{bmatrix} < 0 \tag{10}$$

where

$$\boldsymbol{I} = [\, I \quad \ldots \quad I \,] \tag{11}$$
$$A_d = [\, A_{d1} \quad \ldots \quad A_{dp} \,] \tag{12}$$
$$\overline{W} = diag\left(h_1 W_1, \ \ldots \ h_p W_p\right) \tag{13}$$
$$\Psi_1 = \sum_{j=1}^{p} \left[Q_j + \left(\bar{l}_j - \underline{l}_j\right)\left(\bar{h}_j Z_j + Y_j + Y_j^\top\right)\right] \tag{14}$$

$$\Psi_3 = \left[\, \left(\bar{l}_1 - \underline{l}_1\right) Y_1 \ \ldots + \left(\bar{l}_p - \underline{l}_p\right) Y_p \,\right] \tag{15}$$

$$\Psi_2 = diag\left(\left(1 - \bar{l}_1\right) Q_1, \ \ldots \ , \ \left(1 - \bar{l}_p\right) Q_p\right) \tag{16}$$

then the system under study is asymptotically stable.

Proof: Let the Lyapunov functional be defined by:

$$V(\mathbf{x}_t) = V_1(\mathbf{x}_t) + V_2(\mathbf{x}_t) + V_3(\mathbf{x}_t) + V_4(\mathbf{x}_t)$$

where

$$V_1(\mathbf{x}_t) = x_t^\top P x_t$$
$$V_2(\mathbf{x}_t) = \sum_{j=1}^{p} \int_{t-h_j(t)}^{t} \int_{s}^{t} \dot{x}_z^\top W_j \dot{x}_z dz ds$$
$$V_3(\mathbf{x}_t) = \sum_{j=1}^{p} \int_{t-h_j(t)}^{t} x_s^\top Q_j x_s ds$$
$$V_4(\mathbf{x}_t) = \sum_{j=1}^{p} \int_{0}^{t} \left(\bar{l}_j - \dot{h}_j(z)\right) \int_{z-h_j(z)}^{z} \left[\, x_z^\top \ \dot{x}_s \,\right] \begin{bmatrix} Z_j & Y_j \\ Y_j^\top & X_j \end{bmatrix} \begin{bmatrix} x_z \\ \dot{x}_s \end{bmatrix} ds dz$$

After taking the derivative of these functionals and some algebraic manipulations we get

$$\dot{V}(\mathbf{x_t}) = \xi_t^\top M \xi_t + \sum_{j=1}^{p} \int_{t-h_j(t)}^{t} \dot{x}_s^\top \left[\left(\bar{l}_j - \underline{l}_j\right) X_j + \left(\bar{l}_j - 1\right) W_j\right] \dot{x}_s ds$$

with

$$\xi_t^\top = \left[x_t^\top \quad x_{t-h_1(t)}^\top \quad \vdots \quad x_{t-h_p(t)}^\top \right]$$
$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{12}^\top & M_{22} \end{bmatrix}$$

where M_{11}, M_{12} and M_{22} are given by

$$M_{11} = A^\top P + PA + A^\top \boldsymbol{I}\, \overline{W}\, \boldsymbol{I}^\top A + \Psi_1$$

$$M_{12} = PA_d - \Psi_3 + A^\top \boldsymbol{I}\, \overline{W}\, \boldsymbol{I}^\top A_d$$

$$M_{22} = A_d^\top \boldsymbol{I}\, \overline{W}\, \boldsymbol{I}^\top A_d - \Psi_2$$

with Therefore, the system is then asymptotically stable if the following hold:

$$\begin{cases} M < 0 \\ \left[\left(\bar{l}_j - \underline{l}_j\right) X_j + \left(\bar{l}_j - 1\right) W_j\right] < 0, \forall j = 1, \ldots, p \end{cases}$$

Notice that matrix M can be expressed as follows

$$M = \begin{bmatrix} A^\top P + PA + \Psi_1 & PA_d - \Psi_3 \\ \left(PA_d - \Psi_3\right)^\top & -\Psi_2 \end{bmatrix} + \begin{bmatrix} A^\top \boldsymbol{I}\, \overline{W}\, \boldsymbol{I}^\top A & A^\top \boldsymbol{I}\, \overline{W}\, \boldsymbol{I}^\top A_d \\ \left(A^\top \boldsymbol{I}\, \overline{W}\, \boldsymbol{I}^\top A_d\right)^\top & A_d^\top \boldsymbol{I}\, \overline{W}\, \boldsymbol{I}^\top A_d \end{bmatrix}$$

$$= \begin{bmatrix} A^\top P + PA + \Psi_1 & PA_d - \Psi_3 \\ \left(PA_d - \Psi_3\right)^\top & -\Psi_2 \end{bmatrix} + \begin{bmatrix} A^\top \boldsymbol{I}\, \overline{W} \\ A_d^\top \boldsymbol{I}\, \overline{W} \end{bmatrix} \left(\overline{W}\right)^{-1} \begin{bmatrix} A^\top \boldsymbol{I}\, \overline{W} \\ A_d^\top \boldsymbol{I}\, \overline{W} \end{bmatrix}^\top$$

Using the Schur complement, we conclude that M is negative definite if and only if (10) is satisfied which is verified by assumption and therefore, since (9) and (8) are assumed to be satisfied the system under study is asymptotically stable. This ends the proof of the theorem. $\nabla\nabla\nabla$

Remark 3.1. The results of Theorem 3.1 are only sufficient and therefore if these conditions are not verified we can't claim that the system under study is not stable.

3.2 *Robust stability*

Let us now assume that the control is still equal to zero for all time and assume that the system has uncertainties on all the matrices, i.e:

$$\dot{x}_t = [A + DF(t)E]\, x_t + \sum_{j=1}^{p} [A_{dj} + D_j F_j(t) E_j]\, x_{t-h_j(t)} \quad (17)$$

where all the terms keep the same meaning as before.

We introduce the following notations

$$\tilde{A} = A + DF(t)E$$
$$\tilde{A}_d = [\, A_{d1} + D_1F_1(t)E_1 \quad \dots \quad A_{dp} + D_pF_p(t)E_p\,] = A_d + D_dF_dE_d$$

where E_d and D_d are given by

$$D_d = [\, D_1 \quad \dots \quad D_p\,] \qquad E_d = diag\big(E_1,\ \dots E_p\big)$$

Note that conditions (8) and (9) do not depend on the system matrices so they do not need to be adapted to the uncertain case. Besides, we have to replace A and A_d respectively by $\tilde{A}$ and $\tilde{A}_d$ in condition (10) to get a condition for the robust case.

Thus after some algebraic manipulation we have the following result.

Theorem 3.2. Let assume that the assumptions 2.1-2.2 are satisfied. If there exist $P > 0$, $Q_j > 0$, $W_j > 0$, X_j, Y_j, Z_j for $j = 1, 2, \dots, p$ and λ such that conditions (8), (9) and (26) hold. then the uncertain system under study is asymptotically stable for all admissible uncertainties.

3.3 *Stabilizability*

This section deals with the stabilizability problem, and we will try to design a controller that stabilizes the closed-loop system. We will restrict our self to the class of memoryless state feedback controller.

Thus the state feedback controller is of the form:

$$u(t) = Kx(t) \quad (18)$$

Substituting (18) in the plant model and taking $A^{cl} = (A+BK)$ we get the closed-loop dynamics:

$$\dot{x}_t = A^{cl}x_t + \sum_{j=1}^{p} A_{dj}(t)x_{t-h_j(t)} \quad (19)$$

We note that only condition (10) must be adapted to the stabilizability case. We replace A by A^{cl} in (10) and we multiply both sides by $diag\big(P^{-1}, I_p \otimes P^{-1}, (\overline{W})^{-1}\big)$ with I_p the unity matrix in $\mathbb{R}^p$ and $\otimes$ stands for the Kronecker product. Further, assuming that

$$\overline{W} \;<\; \Gamma \otimes P$$

with $\Gamma = diag\big(\gamma_1,\ \dots\ ,\ \gamma_p\big)$ and $T_p = \big(I_p \otimes P^{-1}\big)$ we get

$$\begin{bmatrix} \alpha & A_dT_p - \bar{\Psi}_3 & P^{-1}\big(A^{cl}\big)^\top \boldsymbol{I} \\ \Big(A_dT_p - \bar{\Psi}_3\Big)^\top & -\bar{\Psi}_2 & T_pA_d^\top \boldsymbol{I} \\ \boldsymbol{I}^\top\big(A^{cl}P^{-1}\big) & \boldsymbol{I}^\top A_dT_p & -\big(\Gamma \otimes P\big)^{-1} \end{bmatrix} < 0 \quad (20)$$

with $\alpha = P^{-1}\big(A^{cl}\big)^\top + \big(A^{cl}\big)P^{-1} + \bar{\Psi}_1$ and $\bar{\Psi}_i = P^{-1}\Psi_iP^{-1}$, $i = 1, 2, 3$ (this notation applies in the sequel).

If we use the new variables $T = P^{-1}$ and $S = KT$ and after some algebraic manipulation we get the result summarized in Theorem 3.3

Theorem 3.3. Let assume that the assumption 2.2 is satisfied. If there exist $T = P^{-1} > 0$, $\bar{Q}_i > 0$, $\bar{W}_i > 0$, $\bar{X}_i$, $\bar{Y}_i$, $\bar{Z}_i$ for $i = 1,\ \dots,\ p$ and $S = KP^{-1}$ such that the following hold for $i = 1,\ \dots,\ p$

$$\bar{h}_i\bar{W}_i < \gamma_i^{-1}T \quad (21)$$

$$\begin{bmatrix} \bar{Z}_i & \bar{Y}_i \\ \bar{Y}_i^\top & \bar{X}_i \end{bmatrix} > 0 \quad (22)$$

$$\big(\bar{l}_i - \underline{l}_i\big)\,\bar{X}_i + \big(\bar{l}_i - 1\big)\,\bar{W}_i < 0 \quad (23)$$

$$\begin{bmatrix} A_o^\top + A_o + \bar{\Psi}_1 & A_dT_p - \bar{\Psi}_3 & A_o^\top \boldsymbol{I} \\ \Big(A_dT_p - \bar{\Psi}_3\Big)^\top & -\bar{\Psi}_2 & T_pA_d^\top \boldsymbol{I} \\ \boldsymbol{I}^\top A_o & \boldsymbol{I}^\top A_dT_p & -\big(\Gamma\big)^{-1} \otimes T \end{bmatrix} < 0 \quad (24)$$

with $A_o = \big(AT + BS\big)$ then the closed loop system is asymptotically stable.

3.4 *Robust stabilizability*

In this subsection, we are concerned by robust stabilizability of the uncertain system under the control law (18). The closed loop system is then given by

$$\dot{x}_t = [A + BK + DF(t)E + D_bF_b(t)E_bK]\, x_t + \sum_{j=1}^{p} [A_{dj} + D_jF_j(t)E_j]\, x_{t-h_j(t)} \quad (25)$$

where all the terms keep the same meaning as previously. Taking account of the uncertainties in (24), we get

$$\begin{bmatrix} A_o^\top + A_o + \bar{\Psi}_1 & A_dT_p - \bar{\Psi}_3 & A_o^\top \boldsymbol{I} \\ \left(A_dT_p - \bar{\Psi}_3\right)^\top & -\bar{\Psi}_2 & T_pA_d^\top \boldsymbol{I} \\ \boldsymbol{I}^\top A_o & \boldsymbol{I}^\top A_dT_p & -\left(\Gamma\right)^{-1} \otimes T \end{bmatrix}$$
$$+\Delta < 0$$

By considering non null vectors η_1, η_2 and η_3 we get

$$[\eta_1^\top \quad \eta_2^\top \quad \eta_3^\top]\,\Delta \begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{bmatrix} = 2\eta_1^\top DF(t)ET\eta_1$$
$$+2\eta_1^\top D_bF_b(t)E_bS\eta_1 + 2\eta_1^\top\left(D_dF_d(t)E_d \otimes T\right)\eta_2$$
$$+2\eta_3^\top \boldsymbol{I}^\top\left(DF(t)ET + D_bF_b(t)E_bS\right)\eta_1$$
$$+2\eta_3^\top \boldsymbol{I}^\top\left(D_dF_d(t)E_d \otimes T\right)\eta_2$$

Taking $\zeta_b = F_b(t)E_bS\eta_1$, $\zeta = F(t)ET\eta_1$ and $\zeta_d = \left(F_d(t)E_d \otimes T\right)\eta_2$ then

$$\begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{bmatrix}^\top \Delta \begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{bmatrix} = z^\top \Delta_0 z$$

with $z^\top = [\eta_1^\top \quad \eta_2^\top \quad \eta_3^\top \quad \zeta^\top \quad \zeta_b^\top \quad \zeta_d^\top]$ and Δ_0 given by

$$\Delta_0 = \begin{bmatrix} 0 & 0 & 0 & D & D_b & D_d \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & D & D_b & D_d \\ D^\top & 0 & D^\top & 0 & 0 & 0 \\ D_b^\top & 0 & D_b^\top & 0 & 0 & 0 \\ D_d^\top & 0 & D_d^\top & 0 & 0 & 0 \end{bmatrix}$$

note also that

$$\begin{bmatrix} \zeta \\ \zeta_2 \\ \zeta_3 \end{bmatrix}^\top \begin{bmatrix} R & 0 & 0 \\ 0 & R_b & 0 \\ 0 & 0 & R_d \end{bmatrix} \begin{bmatrix} \zeta \\ \zeta_2 \\ \zeta_3 \end{bmatrix} <$$
$$\eta_1^\top S^\top E_b^\top R_bE_bS\eta_1 + \eta_1^\top TE^\top RET\eta_1$$
$$+\eta_2\left(E_d \otimes T\right)^\top R_d\left(E_d \otimes T\right)\eta_2 then$$

From above and the use of the S-procedure we get condition (27).

The following theorem summarizes the result of robust stability.

Theorem 3.4. Assume that the assumptions 2.1-2.2 are satisfied. If there exist $T = P^{-1} > 0$, $\bar{Q}_j > 0$, $\bar{W}_j > 0$, $\bar{X}_j$, $\bar{Y}_j$, $\bar{Z}_j$ for $j = 1, 2, \ldots, p$, $S = KP^{-1}$ and λ such that conditions (21), (22), (23) and (27), then the closed loop uncertain system under study is asymptotically stable for all admissible uncertainties.

4. EXAMPLE

To show the usefulness of our results, let us consider some numerical examples.

Example 4.1. In this example, we will consider that the system under study has one time-dealy and try to apply the results of Corollary 3.2. Let us assume that the dynamics is described by the following matrices:

$$A = \begin{bmatrix} -3 & 1 \\ 1 & -1 \end{bmatrix} \qquad A_{d1} = \begin{bmatrix} -0.2000 & 0.1000 \\ -0.3000 & -0.1000 \end{bmatrix}$$
$$D = D_1 = 0.2I \qquad E = E_1 = I$$
$$R = R_1 = I \qquad \underline{l} = 0.1 \qquad \bar{l} = 0.4$$

Solving LMIs of Theorem 3.3 for a single time varying delay case we get:

$$X = \begin{bmatrix} 0.7355 & 0.3138 \\ 0.3138 & 0.5133 \end{bmatrix}, \; Y = \begin{bmatrix} 1.7810 & 2.0019 \\ -0.9834 & -1.0850 \end{bmatrix},$$
$$Z = \begin{bmatrix} 8.8733 & -4.7567 \\ -4.7567 & 2.5605 \end{bmatrix}, \; P = \begin{bmatrix} 8.3500 & -2.6960 \\ -2.6960 & 2.2104 \end{bmatrix}$$
$$Q = \begin{bmatrix} 15.3847 & -5.3892 \\ -5.3892 & 2.5053 \end{bmatrix} \qquad W = \begin{bmatrix} 0.5190 & 0.0963 \\ 0.0963 & 0.2809 \end{bmatrix}$$
$$\lambda = 0.2357 \qquad \bar{h} = 4.385$$

The parameter $\bar{h}$ has been found by trial and error method in order to achieve the feasibility of the LMI's. Based on the results of Theorem 3.3, we conclude that the system is robustly stable.

Example 4.2. In this example, we consider the robust stabilizability problem. For this purpose let us consider the following data:

$$A = \begin{bmatrix} 2.0 & 0.0 \\ 1.0 & 3.0 \end{bmatrix} \qquad D = 0.2I \qquad E = I$$
$$B = \begin{bmatrix} 1.0 & 2.0 \\ 1.0 & 0.0 \end{bmatrix} \qquad D_b = 0.2I \qquad E_b = I$$
$$A_1 = \begin{bmatrix} -0.1 & 0.0 \\ -0.8 & -1.0 \end{bmatrix} \qquad D_1 = 0.2I \qquad E_1 = I$$
$$R = I, \qquad R_1 = I \qquad R_b = I$$
$$\gamma = 5 \qquad \underline{l} = 0.1 \qquad \bar{l} = 0.4$$

Solving equation (21)-(21) and (27), for a system with a single time varying delay we get:

$$T = 10^{-2}\begin{bmatrix} 1.68 & -.31 \\ -.31 & .52 \end{bmatrix} \; \bar{X} = 10^{-4}\begin{bmatrix} 5 & -1 \\ -1 & 2 \end{bmatrix}$$
$$\bar{Y} = 10^{-4}\begin{bmatrix} -1 & 0. \\ -0. & -0. \end{bmatrix} \; \bar{Z} = 10^{-3}\begin{bmatrix} 7.6 & -1.3 \\ -1.3 & .6 \end{bmatrix}$$
$$\bar{W} = 10^{-3}\begin{bmatrix} 5 & -1 \\ -1 & 2 \end{bmatrix} \; \bar{Q} = 10^{-3}\begin{bmatrix} 5.41 & -.50 \\ -.50 & .55 \end{bmatrix}$$
$$S = 10^{-2}\begin{bmatrix} .55 & -3.33 \\ -2.36 & 1.38 \end{bmatrix} \; \lambda = 10$$

$$K = \begin{bmatrix} -0.9574 & -6.9350 \\ -1.0323 & 2.0171 \end{bmatrix} \qquad \bar{h} = 4.385$$

The parameter $\bar{h}$ has been found by search. Based on the results of the previous theorem, we conclude that the system under study in this example is robustly stable for all admissible uncertainties.

5. CONCLUSION

This paper dealt with class of dynamical linear uncertain systems with multiple time-varying delays in the state. delay-dependent sufficient conditions have been developed to check whether a system of this class of systems is stable or unstable, stabilizable or not stabilizable. A state feedback controller with consequent parameters has been used to stabilize the system. The LMI technique is used in all the development.

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$$\begin{bmatrix} \beta_o & PA_d - \Psi_3 & A^\top \boldsymbol{I}\overline{W} & PD & PD_d \\ \left(PA_d - \Psi_3\right)^\top & -\Psi_2 + \lambda E_d^\top R_d E_d & A_d^\top \boldsymbol{I}\overline{W} & 0 & 0 \\ \overline{W}\boldsymbol{I}^\top A & \overline{W}\boldsymbol{I}^\top A_d & -\overline{W} & \overline{W}\boldsymbol{I}^\top D & \overline{W}\boldsymbol{I}^\top D_d \\ D^\top P & 0 & \left(\overline{W}\boldsymbol{I}^\top D\right)^\top & -\lambda R & 0 \\ D_d^\top P & 0 & \left(\overline{W}\boldsymbol{I}^\top D_d\right)^\top & 0 & -\lambda R_d \end{bmatrix} < 0 \quad (26)$$

$$\beta_o = A^\top P + PA + \Psi_1 + \lambda E^\top RE$$

$$\begin{bmatrix} \beta_1 & A_d T - \bar{\Psi}_3 & A_o^\top \boldsymbol{I} & D & D_b & D_d \\ \left(A_d T - \bar{\Psi}_3\right)^\top & \beta_2 & T A_d^\top \boldsymbol{I} & 0 & 0 & 0 \\ \boldsymbol{I}^\top A_o & \boldsymbol{I}^\top A_d T & -\left(\Gamma\right)^{-1} \otimes T & D & D_b & D_d \\ D^\top & 0 & D^\top & -\lambda R & 0 & 0 \\ D_b^\top & 0 & D_b^\top & 0 & -\lambda R_b & 0 \\ D_d^\top & 0 & D_d^\top & 0 & 0 & -\lambda R_d \end{bmatrix} < 0 \qquad (27)$$

$$\beta_1 = A_o^\top + A_o + \bar{\Psi}_1 + \lambda S^\top E_b^\top R_b E_b S + \lambda T E^\top RET$$

$$\beta_2 = -\bar{\Psi}_2 + \lambda\left(E_d \otimes T\right)^\top R_d\left(E_d \otimes T\right)$$

Condition (27) is linearized with respect to S and T by means of a simple Schur complement which we omit here for lack of space.