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OBSERVABILITY BIFURCATION VERSUS OBSERVING BIFURCATIONS

L. Boutat-Baddas[1], J.P. Barbot[1], D. Boutat[2] and R. Tauleigne[1] *

* [1] *Equipe Commande des Systèmes (ECS), ENSEA, 6 Av. du Ponceau, 95014 Cergy.*
{boutat,barbot,tauleign}@ensea.fr
[2] *LVR, ENSI-Bourges/ Université d'Orléans, 10, Bd. Lahitolle, 18020 Bourges.*
driss.boutat@ensi-bourges.fr

Abstract: In this paper we highlight the difference between observability bifurcation and observing bifurcation. From this, we deduce that one way to improve transmission by synchronization of chaotic systems may be chaotic transmitter with also observability bifurcation. We end the paper, with an example of chaotic system with also an observability bifurcation. Moreover this example point out some benefits of the step by step sliding mode observer.

Keywords: Observability bifurcation, Chaotic system, Poincaré's normal form, Sliding mode observer, information transmission.

1. INTRODUCTION

In (Nijmeijer and Mareels, 1997), H. Nijmeijer and I. Mareels have pointed out that classical synchronization of two chaotic systems is, in fact, an observation problem. From their paper, many techniques have been developed to design an observer for chaotic system (Observing bifurcation or chaos). Under, some appropriate assumptions, as for example conditions of linearization by output injection form (Krener and Isidori, 1983)(Krener and Q Xiao, 2001) or conditions for obtaining a generalized Hamiltonian form (Sira-Ramirez and Cruz-Hernandez, 2001), this design may be done with out any difficulty. In other hand, in (Kang, 1998), W. Kang has introduced the concept of controllability bifurcation. This bifurcation characterized loss of linear controllability with respect to small parameter or in a neighborhood of some very particular point. In the same way of thinking, in (Boutat-Baddas and al., 2001) we have introduced the same concept for observability (Observability bifurcation) and we recover naturally some properties as universal inputs, resonant terms and so one. After, we highlight the difference between observability bifurcation and observing bifurcation. From this, we deduce that one way to improve security of transmission by synchronization of chaotic systems may be chaotic transmitter with also observability bifurcation. An example of chaotic system with also an observability bifurcation ends the paper.

2. OBSERVING CHAOS

From the article (Nijmeijer and Mareels, 1997) it is well-known that transmission by synchronization of chaotic systems may be interpreted as an observer design problem to chaotic system (Observing chaos). The linearization by output injection (Krener and Isidori, 1983)(Krener and Q Xiao, 2001) is an usual tool for design an observer and consequently to resolve the synchronization problem. In this section, we use the Chua circuit to show that :

- firstly that linearization by output injection is a very helpful tool to design an observer
- secondly, by considering an other output, it is possible to design a step by step sliding mode observer (Barbot and al., 1996; Perruquetti and Barbot, 2002) in spite of the fact that linearization by output injection is not possible.

Let us consider the Chua circuit:

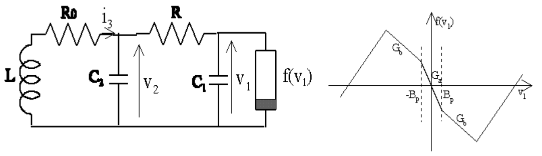

Fig. 1. Chua circuit

The states equations may be :

$$\begin{aligned} \frac{dv_1}{dt} &= \frac{1}{C_1}\left(\frac{v_2-v_1}{R} - f(v_1)\right) \\ \frac{dv_2}{dt} &= \frac{1}{C_2}\left(\frac{v_1-v_2}{R} + i_3\right) \\ \frac{di_3}{dt} &= \frac{1}{L}\left(-v_2 - R_0 i_3\right) \end{aligned} \qquad (1)$$

with $f(v_1) = G_b v_1 + \frac{1}{2}(G_a - G_b)(|v_1+E| - |v_1-E|)$. Setting $x_1 \triangleq v_1$, $x_2 \triangleq v_2$ and $x_3 \triangleq i_3$ and $x \triangleq (x_1, x_2, x_3)^T$, we obtain:

$$\begin{aligned} \frac{dx_1}{dt} &= \frac{-1}{C_1 R}(x_1 - x_2) + \frac{f(x_1)}{C_1} \\ \frac{dx_2}{dt} &= \frac{1}{C_2 R}(x_1 - x_2) + \frac{x_3}{C_2} \\ \frac{dx_3}{dt} &= \frac{-1}{L}(x_2 + R_0 x_3) \\ \dot{x} &\triangleq Ax + F(x_1) \end{aligned} \qquad (2)$$

Let choose as output y the state x_1. It is clear that; the system is globally weakly observable (Hermann and Krener, 1977) and linearizable by output injection. Then, there exist many observators for this system. At our knowledge the first classical one was proposed in (Parlitz and al., 1992):

$$\begin{aligned} \frac{d\hat{x}_1}{dt} &= \frac{1}{C_1}\left(\frac{\hat{x}_2-\hat{y}}{R} - f(\hat{y})\right) \\ \frac{d\hat{x}_2}{dt} &= \frac{1}{C_2}\left(\frac{y-\hat{x}_2}{R} + \hat{x}_3\right) \\ \frac{d\hat{x}_3}{dt} &= \frac{1}{L}\left(-\hat{x}_2 - R_0\hat{x}_3\right) \end{aligned} \qquad (3)$$

where, $\hat{x} = (\hat{x}_1, \hat{x}_2, \hat{x}_3)^T$ is the estimate state of x and $\hat{y}$ is the estimate output. Since the article (Nijmeijer and Mareels, 1997) the receiver design was changed and was more closed to observer design. As throughout the paper we will use a step by step sliding mode observer, we give hereafter such kind of observer for system (2) with $y = x_1$ as output.

$$\begin{aligned} \frac{d\hat{x}_1}{dt} &= \frac{1}{C_1}\left(\frac{\hat{x}_2-y}{R} - f(y)\right) + \lambda_1 sign(y - \hat{y}) \\ \frac{d\hat{x}_2}{dt} &= \frac{1}{C_2}\left(\frac{y-\tilde{x}_2}{R} + \hat{x}_3\right) + E_1\lambda_2 sign(\tilde{x}_2 - \hat{x}_2) \\ \frac{d\hat{x}_3}{dt} &= \frac{1}{L}(-\tilde{x}_2 - R_0\tilde{x}_3) + E_2\lambda_3 sign(\tilde{x}_3 - \hat{x}_3) \end{aligned} \qquad (4)$$

with the following conditions: if $\hat{x}_1 = x_1$ then $E_1 = 1$ else $E_1 = 0$ and if $[\hat{x}_2 = \tilde{x}_2$ and $E_1 = 1]$ then $E_2 = 1$ else $E_2 = 0$. Moreover, by definition we have the following auxiliary state: $\tilde{x}_2 = \hat{x}_2 + E_1 C_1 R \lambda_1 sign(y - \hat{y})$ and $\tilde{x}_3 = \hat{x}_3 + E_2 C_2 \lambda_2 sign(\tilde{x}_2 - \hat{x}_2)$.

The proof of observation error convergence is a particular case of the proof of the last section.

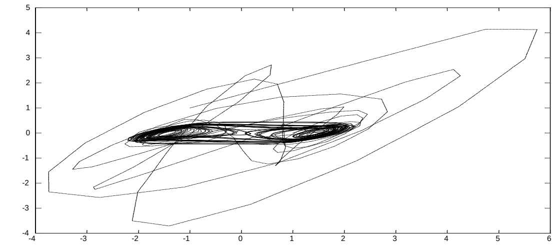

Fig. 2. Generalized phase plane x_1, x_2 for system (2) and '- - -' for system (3)

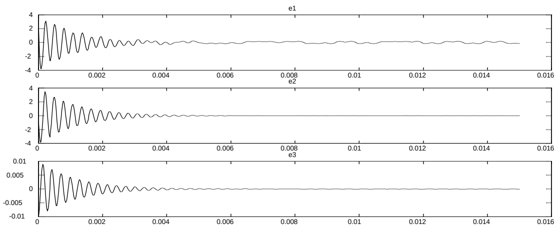

Fig. 3. Observation error for systems (2) and (3)

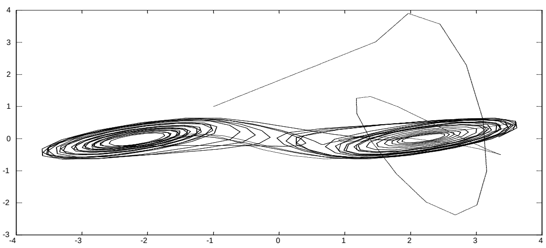

Fig. 4. Generalized phase plane x_1, x_2 for system (2) and '- - - ' for system (4)

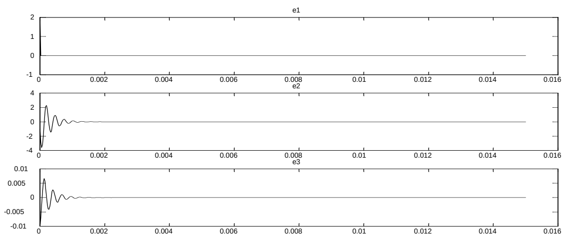

Fig. 5. Observation error for systems (2) and (4)

Comparing the generalized phase plane of x_1, x_2 (system (2)) and $\hat{x}_1, \hat{x}_2$ (system (3) dash line) figure 2, with the generalized phase plane of x_1, x_2 (system (2)) and $\hat{x}_1, \hat{x}_2$ (system (4) dash line) figure 3, we note that, the state of classical observer (system (3)), stays fare from the real state longer than the state of the step by step observer (system (4)). This was confirmed by the figures 4 and 5, where the observation error was shown respectively for the classical observer and the step by step observer.

In (Besançon, 1999; Plestan and Glumineau, 1997) a generalized output injection form was introduced and following this way of thinking, a very interesting relation between chaotic system

and generalized Hamiltonian system was done in (Sira-Ramirez and Cruz-Hernandez, 2001). Unfortunately, considering equations (2), with x_3 as output, in stead of x_1, the nonlinearity is not an output function and result about output injection (Besançon, 1999; Plestan and Glumineau, 1997; Sira-Ramirez and Cruz-Hernandez, 2001) can not be used to design an observer.
Nevertheless, the observer matching condition (Perruquetti and Barbot, 2002) was verified (i.e. the nonlinearity $F(x_1)$ is in $ker(C, CA)$). Therefore, it is possible to design the following step by step sliding mode observer:

$$\begin{aligned} \frac{d\hat{x}_1}{dt} &= \frac{1}{C_1}\left(\frac{\tilde{x}_2-\hat{x}_1}{R}-f(\tilde{x}_1)\right)+E_2\lambda_1 sign(\tilde{x}_1-\hat{x}_1) \\ \frac{d\hat{x}_2}{dt} &= \frac{1}{C_2}\left(\frac{\hat{x}_1-\tilde{x}_2}{R}+x_3\right)+E_3\lambda_2 sign(\tilde{x}_2-\hat{x}_2) \\ \frac{d\hat{x}_3}{dt} &= \frac{1}{L}\left(-\hat{x}_2-R_0x_3\right)+\lambda_3 sign(x_3-\hat{x}_3) \end{aligned} \quad (5)$$

with the following conditions: if $x_3 = \hat{x}_3$ then $E_3 = 1$ else $E_3 = 0$, and if $[\tilde{x}_2 = \hat{x}_2$ and $E_3 = 1]$ then $E_2 = 0$ else $E_2 = 0$. Moreover by definition $\tilde{x}_2 = \hat{x}_2 - E_3\frac{L}{R_0}\lambda_3 sign(x_3 - \hat{x}_3)$ and $\tilde{x}_1 = \hat{x}_1 - E_2C_2R\lambda_2 sign(\tilde{x}_2 - \hat{x}_2)$, the proof of observation error convergence will be also done in the same way of thinking that the proof of the last section.

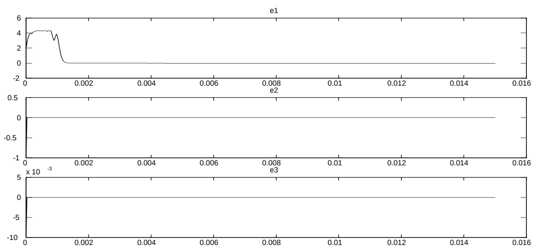

Fig. 6. Observation error for systems (2) and (5)

Figures 6 highlight the efficiency of the step by step observer for the system (2) with x_3 as output. We note that, the simulation results, are very closed to the previous one obtain with x_1 as output.

This section may be sum up in two points :
• Considering x_1 as output, using linearizable by output injection, it is possible to observer the full state of Chua system.
• Thank to a step by step sliding mode observer, it is also possible to design a full state observer for Chua system with a new output x_3. For this output, the system is only observable but is not linearizable by output injection.

In the two next sections, we want to do some thing different from this section and from the classical literature results. We will consider a system with observability bifurcation (i.e. lost of linear observability in one direction). To do this, we will recall, in the next section, some new results on the observability bifurcation (Boutat-Baddas et al 2001)

3. OBSERVABILITY NORMAL FORM

Hereafter, we just recall one result of (Boutat-Baddas et al., 2001) on nonlinear system with one real linear unobservable mode. This result is necessary to study the example of the last section. This example was a transmission by synchronization of chaotic system with observability bifurcation.
Let us consider a nonlinear single input single output (SISO) system:

$$\begin{cases} \dot{\xi} = f(\xi) + g(\xi)u \\ y = C\xi = h(\xi) \end{cases} \quad (6)$$

where, the vector fields $f, g : U \subset R^n \longrightarrow R^n$ are firstly assumed to be real analytic (we relax this assumption later) and such that $f(0) = 0$.

Setting: $A = \frac{\partial f}{\partial \xi}(0)$ and $B = g(0)$ around the equilibrium point $\xi_e = 0$, the system can be rewritten in the following form:

$$\begin{cases} \dot{z} = Az + Bu + f^{[2]}(z) + g^{[1]}(z)u + O^{[3]} \\ y = Cz \end{cases} \quad (7)$$

$$f^{[2]}(z) = \begin{bmatrix} f_1^{[2]}(z) \\ f_2^{[2]}(z) \\ \vdots \\ f_n^{[2]}(z) \end{bmatrix} \text{ and } g^{[1]}(z) = \begin{bmatrix} g_1^{[1]}(z) \\ g_2^{[1]}(z) \\ \vdots \\ g_n^{[1]}(z) \end{bmatrix}$$

with for all $1 \leq i \leq n$ the $f_i^{[2]}(z)$ and $g_i^{[1]}(z)$ are respectively homogeneous polynomials of degree 2 and 1 in z (Kang and Krener, 1992).

Definition 1. Let us consider the two following systems

$$\begin{cases} \dot{z} = Az + Bu + f^{[2]}(z) + g^{[1]}(z)u + O^{[3]} \\ y = Cz \end{cases} \quad (8)$$

and

$$\begin{cases} \dot{x} = Ax + Bu + \bar{f}^{[2]}(x) + \bar{g}^{[1]}(x)u \\ \quad +\beta^{[2]}(y) + \gamma^{[1]}(y)u + O^3(x,u) \\ y = Cx \end{cases} \quad (9)$$

the system (8) is said to be quadratically equivalent to system (9) modulo an output injection $(\beta^{[2]}(y) + \gamma^{[1]}(y)u)$ if there exists a diffeomorphism of the form:

$$x = z - \Phi^{[2]}(z)$$

which transforms the quadratic part of one to the quadratic part of the other one. Where $\Phi^{[2]}(z) = \left[\Phi_1^{[2]}(z), \ldots\ldots, \Phi_n^{[2]}(z)\right]^T$ and $\Phi_i^{[2]}(z)$ is an order two homogeneous polynomial in z .

Remark 2. In this section, we deal with, system with linearly observable part in the Brunovsky form. Moreover the output is always taken equal to the first state component. Consequently, the diffeomorphism ($x = z - \Phi^{[2]}(z)$) is such that $\Phi_1^{[2]}(z) = 0$.

Assumption A.1
The pair $\left(\frac{\partial f}{\partial \xi}(0), C\right)$ of system (6) has one unobservable real mode.

Under this assumption, there is a linear change of coordinates ($z = T\xi$) and a Taylor expansion which transform the system (6) in the following form:

$$\begin{aligned} \dot{\tilde{z}} &= A_{obs}\tilde{z} + B_{obs}u + \tilde{f}^{[2]}(z) + \tilde{g}^{[1]}(z)u \\ &\quad + O^3(z,u) \\ \dot{z}_n &= \alpha z_n + \sum_{i=1}^{n-1} \alpha_i z_i + b_n u + f_n^{[2]}(z) \\ &\quad + g_n^{[1]}(z)u + O^3(z,u) \\ y &= C_{obs}\tilde{z} \end{aligned} \tag{10}$$

where: $\tilde{z} = [z_1, z_2, ..., z_{n-1}]^T$ and $z = [\tilde{z}^T, z_n]^T$

$$A_{obs} = \begin{pmatrix} a_1 & 1 \; 0 \; . \; 0 \\ a_2 & 0 \; 1 \; 0 \; . \\ . & 0 \; . \; . \; 0 \\ a_{n-2} & 0 \; . \; 0 \; 1 \\ a_{n-1} & 0 \; . \; . \; 0 \end{pmatrix},$$

$B_{obs} = [b_1, ..., b_{n-1}]^T$ and $C_{obs} = \begin{bmatrix} 1 & 0 & . & . & 0 \end{bmatrix}$. The quadratic observable normal form is given by:

Theorem 3. (Boutat-Baddas et al., 2001) The normal form with respect to the quadratic equivalence modulo an output injection of the system (10) is:

$$\begin{aligned} \dot{x}_1 &= a_1 x_1 + b_1 u + \sum_{i=2}^{n} k_{1i} x_i u \\ \dot{x}_2 &= a_2 x_1 + b_2 u + \sum_{i=2}^{n} k_{2i} x_i u \\ . &= . \\ \dot{x}_{n-2} &= a_{n-2} x_1 + b_{n-2} u + \sum_{i=2}^{n} k_{(n-2)i} x_i u \\ \dot{x}_{n-1} &= a_{n-1} x_1 + b_{n-1} u + \sum_{j \geq i=2}^{n} h_{ij} x_i x_j \\ &\quad + h_{1n} x_1 x_n + \sum_{i=2}^{n} k_{(n-1)i} x_i u \\ \dot{x}_n &= \alpha_n x_n + \sum_{i=1}^{n-1} \alpha_i x_i + b_n u \\ &\quad + \alpha_n \Phi_n^{[2]} + \sum_{i=1}^{n-1} \alpha_i \Phi_i^{[2]} - \frac{\partial \Phi_n^{[2]}}{\partial \tilde{z}} A_{obs}\tilde{z} \\ &\quad + f_n^{[2]}(z) + \sum_{i=2}^{n} k_{(n-1)i} x_i u \end{aligned}$$

and for the last equation, $\Phi_n^{[2]}(z)$ must verified

$$\alpha_n \Phi_n^{[2]} + \sum_{i=1}^{n-1} \alpha_i \Phi_i^{[2]} = \frac{\partial \Phi_n^{[2]}}{\partial \tilde{z}} A_{obs}\tilde{z} - f_n^{[2]}(z) + \beta_n^{[2]}(z_1)$$

From classical results an normal forms and previous theorem, we obtain:

Corollary 4. Suppose $A = \frac{\partial f}{\partial \xi}(0)$ is diagonalizable with $(\lambda_1, \lambda_2, ..., \lambda_{n-1}, \alpha_n)$ as eigenvalues.Then, if (See "resonance terms" for example in Wiggins (Wiggins, 1990)),

$$\underbrace{\alpha_n \prod_{i=1}^{n-1} \lambda_i \prod_{i=1}^{n-1} (\alpha_n - 2\lambda_i) \prod_{i=1>j}^{n-1} (\alpha_n - \lambda_i - \lambda_j)}_{\neq 0} \tag{11}$$

we obtain:

$$\dot{x}_n = \alpha_n x_n + \sum_{i=1}^{n-1} \alpha_i x_i + b_n u + \sum_{i=2}^{n} k_{(n-1)i} x_i u$$

and, moreover, if (11) is no verified, we have, on **eigenvectors basis:**

a resonant term in x_n^2**for**, $\alpha_n = 0$.
a resonant term in $x_i x_n$**for**, $\lambda_i = \alpha_n = 0$.
a resonant term in x_i^2**for**, $\alpha_n - 2\lambda_i = 0$.
a resonant term in $x_i x_j$**for**, $\alpha_n - \lambda_i - \lambda_j = 0$

Remark 5. In fact, the restriction that $A = \frac{\partial f}{\partial \xi}(0)$ should be diagonal is not necessary; it is well know that works perfectly well in the cas of degenerete eigenvalues, but with tedious calculus.

Remark 6. Thank to the $k_{jn} x_n u$ terms in the normal form, it is possible with a well chosen input u (universal input (Gauthier and Bornard, 1981)) to preserve the observability. Moreover, for $\alpha_n < 0$ it is always possible to design a partial estimator and the most interesting case is $\alpha_n = 0$ which will be studied in more detail in a for coming paper.

Remark 7. Thank to the $h_{in} x_i x_n$ terms in the normal form it is also possible to recover the full state observability locally all most everywhere.

In the next section, we will consider a system without input, then we only used the remark 7. Moreover, the analyticity assumption of the vector fields $f(\xi)$ and $g(\xi)$ in (6) may be relaxed, by considering the system

$$\begin{cases} \dot{\xi} = f(\xi) + g(\xi)u + d(y,u) + p(\xi)w \\ y = C\xi = h(\xi) \end{cases} \tag{12}$$

where $d(y,u)$ is discontinuous and $p(\xi)w$ is a perturbation which verifies the observer matching condition (Perruquetti and Barbot, 2002) [1]. In this case we are obliged to consider discontinuous output injection and step by step observer as it is shown in the next section.

4. OBSERVING CHAOS WITH OBSERVABILITY BIFURCATION

Now in order to increase the security of transmission, we propose to add at the transmission by synchronization of chaotic system some observability bifurcations. Here we just give an illustrative example, so let us consider again the system (2) with x_1 as output but with $x_4 = \frac{1}{L}$ as a new state. The variation of L is the information to pass on the receiver. Moreover, we assume that there exist K_1 and K_2 such that $|x_4| < K_1$ and $|\frac{dx_4}{dt}| < K_2$, this means that the information signal and its variation are bounded. Thus, from these assumptions, we obtain the following systems:

$$\begin{aligned} \frac{dx_1}{dt} &= \frac{-1}{C_1R}(x_1 - x_2) + \frac{f(x_1)}{C_1} \\ \frac{dx_2}{dt} &= \frac{1}{C_2R}(x_1 - x_2) + \frac{1}{C_2}x_3 \\ \frac{dx_3}{dt} &= -(x_2 + R_0x_3)x_4 \\ \frac{dx_4}{dt} &= \sigma \end{aligned} \tag{13}$$

with σ an unknown bounded function (i.e. $|\sigma| < K_2$).

This system has one unobservable real mode and using the linear change of coordinate $z_1 = x_1$, $z_2 = \frac{x_1}{C_2R} + \frac{x_2}{C_1R}$, $z_3 = \frac{x_3}{C_1C_2R}$ and $z_4 = x_4$ we obtain:

$$\begin{aligned} \frac{dz_1}{dt} &= \frac{-(C_1 + C_2)}{C_1C_2R}z_1 + z_2 + \frac{f(x_1)}{C_1} \\ \frac{dz_2}{dt} &= z_3 + \frac{f(x_1)}{C_1C_2R} \\ \frac{dz_3}{dt} &= \frac{z_1z_4}{C_2^2R} - \frac{z_2z_4}{C_2} - R_0z_3z_4 \\ \frac{dz_4}{dt} &= \sigma \end{aligned} \tag{14}$$

Equations (14) are on observability normal form with $\alpha = 0$ and resonant terms $h_{22} = h_{23} = 0$, $h_{14} = \frac{1}{C_2^2R}$, $h_{24} = \frac{1}{C_2}$ and $h_{34} = -R_0$, but with σ as a perturbation and a non smooth output injection $(\frac{f(x_1)}{C_1}, \frac{f(x_1)}{C_1C_2R}, 0, 0)^T$. From the remark 7 we conclude that the resonant terms $h_{i4}x_ix_4$ ensure the full state observability locally all most everywhere. The observability singularity is for $\frac{z_1}{C_2^2R} - \frac{z_2}{C_2} - R_0z_3 = 0$, and taking into account this singularity we can design an observer. Nevertheless, as the system (13) has also a particular structure with $x_4 = z_4$ and $x_3 = C_2C_1R_0z_3$ we can design an observer directly on the original state (the physical one). Obviously, the observability singularity is the same, the equation $-x_2 - R_0x_3 = 0$ is equivalent to $\frac{z_1}{C_2^2R} - \frac{z_2}{C_2} - R_0z_3 = 0$. So, we will use information contain in the terms $-x_4x_2 - R_0x_4x_3$ in order to design a full state observer and recover information on x_4 contain in the equation of $\frac{dx_3}{dt}$.

For this, we use the following sliding mode observer:

$$\begin{aligned} \frac{d\hat{x}_1}{dt} &= \frac{1}{C_1}\left(\frac{\hat{x}_2 - y}{R} - f(y)\right) + \lambda_1 sign(y - \hat{x}_1) \\ \frac{d\hat{x}_2}{dt} &= \frac{1}{C_2}\left(\frac{y - \tilde{x}_2}{R} + \hat{x}_3\right) + E_1\lambda_2 sign(\tilde{x}_2 - \hat{x}_2) \\ \frac{d\hat{x}_3}{dt} &= \hat{x}_4(-\tilde{x}_2 - R_0\tilde{x}_3) + E_2\lambda_3 sign(\tilde{x}_3 - \hat{x}_3) \\ \frac{d\hat{x}_4}{dt} &= E_3\lambda_4 sign(\tilde{x}_4 - \hat{x}_4) \end{aligned} \tag{15}$$

with the following conditions:

if $\hat{x}_1 = x_1$ then $E_1 = 1$ else $E_1 = 0$, similarly if $[\hat{x}_2 = \tilde{x}_2$ and $E_1 = 1]$ then $E_2 = 1$ else $E_2 = 0$ and finally if $[\hat{x}_3 = \tilde{x}_3$ and $E_2 = 1]$ then $E_3 = 1$ else $E_3 = 0$. Moreover, in order to take into account the observability singularity $(\tilde{x}_2 + R_0\tilde{x}_3 = 0)$, we set $E_s = 1$ if $\tilde{x}_2 + R_0\tilde{x}_3 \neq 0$ else $E_s = 0$. By definition we take:

$$\begin{aligned} \tilde{x}_2 &= \hat{x}_2 + E_1C_1R\lambda_1 sign(y - \hat{x}_1) \\ \tilde{x}_3 &= \hat{x}_3 + E_2C_2\lambda_2 sign(\tilde{x}_2 - \hat{x}_2) \\ \tilde{x}_4 &= \hat{x}_4 - \frac{E_3E_s}{(\tilde{x}_2 + R_0\tilde{x}_3 - 1 + E_S))}\lambda_3 sign(\tilde{x}_3 - \hat{x}_3) \end{aligned}$$

Sketch of proof: In this sketch of proof we implicitly assume that the system (13) has bounded state (i.e. obvious due to energy consideration). Consequently, in the observer we add saturation on the integrator in order to also have a bounded state observer. From these two boundness considerations all λ_i may be easily chosen as constants (Utkin, 1992).

• **First Step:** assuming that $E_1 = 0$ (if $E_1 = 1$ we directly move to the next step), the observation error dynamics $(e = x - \hat{x})$ is:

$$\begin{aligned} \dot{e}_1 &= \frac{e_2}{C_1R} - \lambda_1 sign(x_1 - \hat{x}_1) \\ \dot{e}_2 &= \frac{e_2}{C_2R} + \frac{e_3}{C_2} \\ \dot{e}_3 &= [x_4(-x_2 - R_0x_3)] - [\hat{x}_4(-\hat{x}_2 - R_0\hat{x}_3)] \\ \dot{e}_4 &= 0 \end{aligned}$$

Due to the finite time convergence of the sliding mode, there exists $\tau_1 \geq 0$ such that $\forall t \geq \tau_1$ $\hat{x}_1 = x_1$ and we pass to the:

• **Second Step:** As $\hat{x}_1 = x_1$ then $E_1 = 1$ and as $e_1 = 0$ for all $t \geq \tau_1$ then $\dot{e}_2 = 0$ and consequently, invoking the equivalent vector (Utkin, 1992), $\tilde{x}_2 = x_2$, and we obtain

$$\begin{aligned} \dot{e}_1 &= \frac{e_2}{C_1R} - \lambda_1 sign(x_1 - \hat{x}_1) = 0 \\ \dot{e}_2 &= \frac{e_3}{C_2} - \lambda_2 sign(x_2 - \hat{x}_2) \end{aligned}$$

[1] For single output linear system the observer matching condition is $p(\xi) \triangleq P \in Ker[C^T, (CA)^T, ..., (CA^{n-2})^T]$ with n the state dimension.

$$\dot{e}_3 = [x_4(-x_2 - R_0x_3)] - [\hat{x}_4(-\hat{x}_2 - R_0\hat{x}_3)]$$
$$\dot{e}_4 = 0$$

Due to the finite time convergence of the sliding mode, there exists $\tau_2 \geq \tau_1 \geq 0$ such that $\forall t \geq \tau_2$, $\hat{x}_2 = \tilde{x}_2 = x_2$ and we pass to the:

• **Third Step:** As $[\hat{x}_2 = x_2$ and $E_1 = 1]$ then $E_2 = 1$ and as $e_2 = 0$ for all $t \geq \tau_2$ then $\dot{e}_3 = 0$ and consequently, invoking the equivalent vector, $\tilde{x}_3 = x_3$, and we obtain

$$\begin{aligned}
\dot{e}_1 &= \tfrac{e_2}{C_1R} - \lambda_1 sign(x_1 - \hat{x}_1) = 0 \\
\dot{e}_2 &= \tfrac{e_3}{C_2} - \lambda_2 sign(x_2 - \hat{x}_2) = 0 \\
\dot{e}_3 &= -(x_2 + R_0x_3)e_4 - \lambda_3 sign(x_3 - \hat{x}_3) \\
\dot{e}_4 &= 0
\end{aligned}$$

Due to the finite time convergence of the sliding mode, there exists $\tau_3 \geq \tau_2 \geq \tau_1 \geq 0$ such that $\forall t \geq \tau_3$, $\hat{x}_3 = \tilde{x}_3 = x_3$ and we pass to the:

• **Last Step:** As $[\hat{x}_3 = x_3$ and $E_3 = 3]$ then $E_3 = 1$ and we obtain :

$$\begin{aligned}
\dot{e}_1 &= \tfrac{e_2}{C_1R} - \lambda_1 sign(x_1 - \hat{x}_1) = 0 \\
\dot{e}_2 &= \tfrac{e_3}{C_2} - \lambda_2 sign(x_2 - \hat{x}_2) = 0 \\
\dot{e}_3 &= -(x_2 + R_0x_3)e_4 - \lambda_3 sign(x_3 - \hat{x}_3) = 0 \\
\dot{e}_4 &= E_s\lambda_4 sign(\tilde{x}_4 - \hat{x}_4)
\end{aligned}$$

Therefore, if $E_s = 1$ then e_4 goes to zero in finite time, else $E_s = 0$ and we frozen the e_4 dynamic (the data acquisition). Nevertheless, the singularity $(x_2 + R_0x_3)$ is local, so as the transmitter is chaotic we never stay enough time on the singularity to alter substantially the data acquisition. $\triangle$

Remark 8. In practice we add some law pass filter on the auxiliary state $\tilde{x}_i$ and we set $E_i = 1$ for $i \in \{1, 2, 3\}$, not exactly when we are on the sliding surface but when we are enough close. Similarly, $E_s = 0$ when we are close to the singularity, not only when we are on.

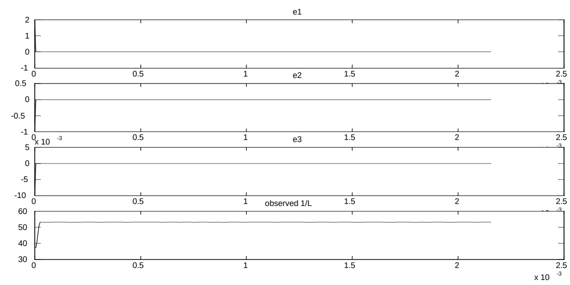

Fig. 7. e_1, e_2, e_3 and $\hat{x}_4$

Figure 7, shows that the observation errors e_i go to zero rapidly. Moreover, the observer state $\hat{x}_4$, goes to a constant value 53.191 which is exactly the value of $\frac{1}{L}$ with $L = 18.8mH$ (inductance of the system (1)). Consequently, for information transmission, we just must find a variation of L which preserves the Chaotic behavior of the system (1).

5. REFERENCES

J-P Barbot, T Boukhobza and M. Djemai,"Sliding mode observer for traingual input form", *Proceeding of the 35th CDC, Kobe, Japan,* 1996.

G. Besançon, "A viewpoint on observability and observer design for nonlinear system", *New directions in Nonlinear Observer Design, Eds H. Nimeijer and T.I. Fossen, Lecture Notes in Control and Information Sciences 244, Springer,pp 1-22,* 1999.

L. Boutat-Baddas, D. Boutat, J-P. Barbot and R. Tauleigne, "Quadratic Observability normal form", *In Proc. of the 41th IEEE CDC 01,* 2001.

J-P. Gauthier and G. Bornard,"Observability for any $u(t)$ of a class of bilinear systems", *IEEE TAC Vo 26, pp 922-926,* 1981.

R. Hermann and A.J. Krener,"Nonlinear contollability and observability", *IEEE Transaction on Automatic Control, Vol. 22, pp. 728-740,* 1977.

W. Kang, "Bifurcation and normal form of nonlinear control system: Part I and II", *SIAM J. Control and Optimization, Vol 36, pp 193-232,* 1998.

W. Kang and A.J. Krener, "Extended quadratic controller normal form and dynamic state feedback linearization of nonlinear systems", *SIAM J. Control and Optimization, Vol 30, No 6, pp 1319-1337,* 1992.

A. Krener and A. Isidori, "Linearization by output injection and nonlinear observer." *Systems & Control Letters, Vo 3, pp 47-52,* 1983.

A. Krener and M. Q Xiao, "Nonlinear observer designin the Siegel domain through coordinate changes" *In Proc of the 5th IFAC Symposium, NOLCOS01, Saint-Petersburg, Russia, pp 557-562,* 2001.

H. Nijmeijer and I.M.Y. Mareels, "An observer looksat synchronization", *IEEE Trans. On Circuits and Systems-1: Fundamental Theory and Applications, Vol. 44, No 11, pp. 882-891,* 1997.

U. Parlitz, L.O. Chua, Lj. Kocarev, K.S. Halle and A. Shang, "Transmission of digital signals by chaotic synchronization", *Inter. Journal of Bifurcation and Chaos, Vol. 2, No. 4, pp 973-997,* 1992.

W. Perruquetti and J-P. Barbot, "Sliding Mode control in Engineering", *M. Dekker, Edition* 2002.

F. Plestan and A. Glumineau,"Linearization by generalized input output injection", *Syst. Contr. Letters, 31, pp 115-128,* 1997.
"Observateurs non-linéaires par plongement", *CNRS, Proc. of Journées Nationales d'Automatique, Autrans, France, CDROM,* 2001.

H. Sira-Ramirez and C. Cruz-Hernandez, "Synchronization of chaotic systems: A generalized Hamiltonian systemsapproach", *Int. Journal of Bifurcation and Chaos, Vol 11, No 5, pp1381-1395,* 2001.

V.I. Utkin,"Sliding modes in control optimization", *Engineering Series, Springer-Verlag,* 1992.

S. Wiggins, "Introduction to Applied Nonlinear Dunamical Systems and Chaos", *Springer, Texts in Applied Mathematics 2,* 1990.