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FAULT DETECTION AND ISOLATION VIA A NONLINEAR FILTER

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Abstract: This paper deals with the problem of fault detection and isolation in nonlinear systems. A new design method for residual generation is developed, based on geometric approach. In this work, existence conditions given by De Persis and Isidori (C. De Persis and A. Isidori, 1999 and C. De Persis and A. Isidori, 2000) are considered. In order to illustrate the proposed results, a simulation example is carried out. Copyright© 2002 IFAC

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1. INTRODUCTION

Modern systems are increasingly complex and require more and more sophisticated control algorithms. Fault diagnosis consists in carrying out 3 main tasks: fault detection, fault localization and fault estimation.
In the literature, the fault detection and isolation stages are referred as F.D.I. Generally, performances of a detection method are evaluated by considering:

- faults sensitivity: capacity of the method to detect relatively small faults magnitude,
- detection delay: ability of the method to detect faults with a relatively short delay,
- robustness: skill of the method to operate in the presence of noises, disturbances and other faults with few false alarm as possible.

The isolation performances of the method are closely related to the structural properties of the system, in particular fault distinguability according to their sizes, and models, disturbances and noises.
In this paper, we focuss on methods based on an explicit mathematical system model discribed by differential equations (or equivalent representations), but the main point is that nonlinear systems are considered. Fault diagnosis in nonlinear systems, can be carried out using analytical redundancy, allow residual generation for wich several approaches exist: Kalman filter (D. Aubry, 1999, M.S. Grewal and A. P. Andrews, 1993), observer (G. Besançon, 1999, D. G. Luenberger, 1971, E. A. Misawa and J. K. Hedrick, 1989), parity relations (V. Krishnaswami, 1995, R. J. Patton and J. Chen, 1991, M. Staroswiecki, G. Comtet-Varga, 2001) and parameters estimation (M. Basseville and Q. Zhang, 1999, R. Isermann, 1993). A detection filter approach (C. De Persis and A. Isidori, 2000, J.J. Gertler, 1991) is considered in this work because, under some conditions, it allows fault isolation using two distinct ways (J.J. Gertler, 1991):

- structured residual set: each component of the residual vector (i.e. each residual) is designed to be sensitive to a subset of faults, while remaining insensitive to the others,
- directional residual set: the residual vector lies in a fixed and fault-dependent direction (or subspace) in the residual space.

In the case of nonlinear systems, analytical approaches (E. A. Garcia and P.M. Frank, 1997) are not much developed because of complexity of calculations. In addition, these methods are not generic because they depend on the particular kind of nonlinearities considered. In this article, we proposed a residual generation method based on a geometrical approach. We also believe that such an approach is more generic because it simply consists in partitioning the state space in subspace characteristic of the different faults.
In this paper, our work is based on the results of C. De Persi and A. Isidori to propose a method according to fault isolation filter synthesis. But we choose to design only one filter being able to detec and isolate all faults. That is to say that it is not necessary any more to synthesize a bank of several filters for all faults isolation (C. De Persis and A. Isidori, 2001).
This paper is organized as follows. Section 2 presents a method to verify that the isolation fault filter existence conditions are satisfied by the nonlinear system considered. In section 3, assuming that these conditions are satisfied, the filter synthesis is presented. Finally, the application of method (existence conditions, filter design) is illustrated on two academic examples.

2. EXISTENCE CONDITIONS OF AN ISOLATED FAULT FILTER

The objective of this paper is the design of a nonlinear filter for the detection and the isolation of faults. This type of filter cannot always be feasible. For this reason, existence conditions developed in (C. De Persis and A. Isidori, 1999) and (C. De Persis and A. Isidori, 2000) are recalled in this section. Synthesis of an isolated fault filter and the resolution of some associated problems will be based on these results. Consider the system described by:

$$\begin{cases} \dot{x} = f_0(x) + \sum_{i=1}^{m} f_i(x)u_i + \sum_{j=1}^{l} p_j(x)w_j \\ y = h(x) \end{cases} \quad (1)$$

where input $u(t)$, state $x(t)$, fault $w(t)$ and output $y(t)$ are respectively vectors of dimensions m, n, l, p.

The writing of fault contribution only on state vector is not restrictive. Indeed, using a state augmentation, the previous system (1) can always be obtained if sensor faults are considered (for more precision, see the paper J. Park and al, 1994). In the next, it is supposed that the number of faults is less than the number of sensors ($l \le p$). In the contrary case, there would be no solution with the problem arising. Problem is to design a filter taking the following form:

$$\begin{cases} \dot{z} = \left(f_0(z) - \Psi_1(z, y_z)\right) + \left(\sum_{i=1}^{m} f_i(z)u_i - \Psi_2(z, y_z)\right)u \\ + \left(\Psi_1(z, y) + \Psi_2(z, y)u\right) \\ y_z = h(z) \end{cases} \quad (2)$$

with the output injection noted $\Psi_\bullet(\bullet)$.

The objective is to solve the Fundamental Problem of Residual Generation i.e. this filter must allow the detection and the isolation of all faults. To this end, residuals will be generated as follows:

$$r = \Theta(y) - \Theta(y_z) = \begin{bmatrix} r_1 & \cdots & r_j & \cdots & r_l \end{bmatrix}^T \quad (3)$$

where each component r_j of r is designed to be sensitive to only one fault w_j, which must be sufficient to take decisions.

The table of signature is particular and can be described as a diagonal structure (J.J. Gertler, 1998). Moreover, this type of residual's structure allows the detection of several faults appearing simultaneously.

The existence conditions of such a filter (2) will be determine by referring to work of A. Isidori (A. Isidori, 1995). Two steps will be distinguish, related to:

- the existence of an output injection $\Psi_\bullet(\bullet)$ to obtain a diagonal residual structure,
- the existence of an indicator $\Theta(\bullet)$.

The first step is the determination of the smallest invariant distribution for the dynamics of the system (via an output injection) enclosing $Span\{p_j\}$. They are the state variables (or a state variables combination) not being able to be written without the knowledge of the fault. This distribution is determined using the following nondecreasing series:

$$\begin{aligned} S_0^{p_j} &= Span\{p_j\} \\ S_{i+1}^{p_j} &= \bar{S}_i^{p_j} + \sum_{i=0}^{m}\left[f_i, \bar{S}_i^{p_j} \cap Ker\{dh\}\right] \end{aligned} \quad (4)$$

$\bar{S}_i^{p_j}$ corresponds to the smallest involutive distribution containing $S_i^{p_j}$. The stopping conditions of this serie are:

$$if \left.\begin{aligned} \bar{S}_i^{p_j} &= S_{i+1}^{p_j} \\ dim(\bar{S}_i^{p_j}) &= n \quad \forall x \end{aligned}\right\} \Rightarrow \quad S_*^{p_j} = \bar{S}_i^{p_j} \quad (5)$$

We define $Ker\{dh(x)\} = \bigcap_{i=1}^{p} Ker\{dh_i(x)\}$ with $dh(x)$, the *Jacobian Matrix* of $h(x)$.

Thus, to the distribution noted $(S_*^{p_j})^\perp$, corresponds the state combination being able to be observed (using the filter) without the knowledge of the unknown input w_j. The variation of these distributions, associated to each fault, will allow the deduction of the output injection $\Psi_\bullet(\bullet)$.

Note that if all states variable are measured, the existence condition of the output injection is reduced to: $(S_*^{\overline{p_j}}) \cap Span\{(p_j)^T\} = \{0\}$ with $\forall j \in [1 \cdots l]$ and $\overline{p_j} = p_1, \cdots, p_{j-1}, p_{j+1}, \cdots p_l$.

However, in the general case, all the state variables are not measured. Moreover, the residuals are generated considering the difference between outputs combinations of system (1) and filters (2). As in (C. De Persis and A. Isidori, 2001), a bank of residual generators solves the problem of detection and isolating each individual faults. The aim of the series, of nondecreasing co-vectors (6), is to take into account the additional constraint due to the output vector

$$\begin{aligned} Q_0^{p_j} &= (S_*^{p_j})^\perp \cap Span\{dh\} \\ Q_i^{p_j} &= (S_*^{p_j})^\perp \cap (\sum_{i=0}^{m} L_{p_i} Q_i + Span\{dh\}) \end{aligned} \quad (6)$$

with the same stopping conditions (5) and $Q_*^{p_j}$ is the result so obtained. This serie is called in (C. De Persis and A. Isidori, 1999) and (C. De Persis and A. Isidori, 2000), *Observability Co-distribution Algorithm* of $(S_*^{p_j})^\perp$, or *o.c.a.*$((S_*^{p_j})^\perp)$. This algorithm allow to determine the output combinations which can be written without the knowledge of the input w_j. We can build an output combination allowing the residual generation insensitive to w_j if, with $Q_*^{p_j} \ne \{0\}$, it exists a vector function $\Theta^{p_j}(\bullet) \ne \{0\}$ such as:

$Q_*^{p_j} \cap Span\{dh\} = Span\{d(\Theta^{p_j} \circ h(x))\}$. Note that existence conditions in the simple case $l = 2$ are reduced to:

$$\left(Span\{d(\Theta^{p_1} \circ h(x))\}\right)^\perp \cap Span\{(p_2)^T\} = \{0\} \text{ and}$$

$$\left(Span\{d(\Theta^{p_2} \circ h(x))\}\right)^\perp \cap Span\{(p_1)^T\} = \{0\} \quad (7)$$

We point out that the objective is the design of a filter making possible faults detection and isolation using a residual vector. But for a diagonal structure the synthesis requires relatively restrictive conditions of construction:

- $\left(\boldsymbol{Span}\left\{\boldsymbol{d}\left(\Theta^{\overline{p_j}}\,\boldsymbol{o}\,\boldsymbol{h}(\boldsymbol{x})\right)\right\}\right)^{\perp} \cap \boldsymbol{Span}\left\{(\boldsymbol{p}_j)^T\right\} = \{\boldsymbol{0}\}$, $\forall \boldsymbol{j} \in [\boldsymbol{1}\cdots \boldsymbol{l}]$,
- **with each fault** w_j **it must be possible to associate the vector function** $\Theta^{\overline{p_j}}(\bullet)$ **such as**
- $\boldsymbol{Q}_*^{\overline{p_j}} \cap \boldsymbol{Span}\{\boldsymbol{dh}\} = \boldsymbol{Span}\left\{\boldsymbol{d}\left(\Theta^{\overline{p_j}}\,\boldsymbol{o}\,\boldsymbol{h}(\boldsymbol{x})\right)\right\}$ (8) $\forall \boldsymbol{j} \in [\boldsymbol{1}\cdots \boldsymbol{l}]$.

In this section, a method, allowing to ensure the existence of a solution to the problem of the residual generation (with a diagonal structure signature) in the multivariable case, has been recalled. However, that is only the first step of the reasoning. Indeed, the filter must be now built. That is the subject of the following paragraph.

3. SET OF PROBLEMS RELATED TO THE FILTER GENERATION

Assuming that the previous conditions are satisfied, the detection and isolation of all faults are achieved by the design of only one filter instead of a bank of filters, in this section.

This filter synthesis is made up of two stages:

- determination of the output injection $\boldsymbol{\Psi}_1(\mathbf{z},\mathbf{y})+\boldsymbol{\Psi}_2(\mathbf{z},\mathbf{y})\mathbf{u} = \boldsymbol{\Psi}(\mathbf{z},\mathbf{y},\mathbf{u})$ making possible the decoupling of the various components of the faults vector $\boldsymbol{w}$,
- determination of the output combination $\Theta\,\mathbf{o}\,\mathbf{h}(\bullet)$ which allows residual generation according to the isolation properties given by equation (3).

The first step is detailed in the following section.

3.1 Calculation of the output injection

Associated to the fault w_j, the distribution $S_*^{p_j}$ can be defined. The common parts (in a strict sense), between two distributions, noted $\left(\phi^{p_i,p_j}\right)$, is defined by: $(\phi^{p_i,p_j}) = (S_*^{p_i}) \cap (S_*^{p_j}) \neq \{0\}$ with $i \neq j$, and $(\phi^{p_i,p_j}) \cap (S_*^{p_k})^{\perp} = \{0\}$ $\forall k \neq (i,j)$. Consequently to this definition, (ϕ^{p_j}) represents the sub-part of $(S_*^{p_j})$ and is orthogonal with the other distributions, i.e. $(\phi^{p_j}) \subseteq (S_*^{p_j})$; $(\phi^{p_j}) \subseteq (S_*^{p_i})^{\perp}, \forall (i, with\ i \neq j)$

We assume that $S_*^{p_j} = \phi^{p_j} \oplus \phi^{p_j,p_i} \oplus \cdots \oplus \phi^{\cdots,p_j,\cdots}$ (with $\oplus$ direct sum). Thus the diffeomorphism $\overline{x} = \Phi(x)$ can be used, such as:

$$\frac{\partial \Phi(x)}{\partial x} = \begin{pmatrix}\phi^{p_1} & \cdots & \phi^{p_l} & \phi^{p_i,p_j} & \cdots & \phi^{p_k,\cdots,p_q} & \gamma\end{pmatrix} \quad (9)$$

with $j \neq i$ and $k \neq q \leq l$. γ defines a distribution, orthogonal with the others distributions, such as $\Phi(x)$ is a diffeomorphism. The state subspace insensitive to all faults, via an output injection, is included in γ.

Moreover, according to the existence conditions, particularly $\left(Span\left\{d\left(\theta^{\overline{p_j}} \circ h(x)\right)\right\}\right)^{\perp} \cap Span\left\{(p_j)^T\right\} = \{0\}$, with $Span\left\{d\left(\theta^{\overline{p_j}} \circ h(x)\right)\right\} \subseteq (Q_*^{\overline{p_j}}) \subseteq (S_*^{\overline{p_j}})^{\perp}$ then $(S_*^{\overline{p_j}}) \cap Span\left\{(p_j)^T\right\} = \{0\}$, it can be ensure that $dim(\phi^{p_j}) \geq 1$, $\forall (j,x)$. However it is not always easy to choose the vector field (making up the distribution γ) so as to facilitate the integration necessary to the deduction of $\Phi(x)$. From a qualitative viewpoint, it is preferable to choose, as much as possible, some constant vector fields, i.e. independent of x. With regard to the other components of $\Phi(x)$ (which are imposed), they are obtained by successive derivations of the system outputs. Their integrations don't pose any problem. By applying the change of coordinates (9), it is possible to determine a new way to write equations of system (1):

$$\begin{cases} \dot{\overline{x}}_1 = \left(\bar{f}_1(x)+\bar{g}_1(x)u+\bar{p}_1(x)w_1\right) \text{o}\ \Phi^{-1}(\overline{x}) \\ \vdots \\ \dot{\overline{x}}_l = \left(\bar{f}_l(x)+\bar{g}_l(x)u+\bar{p}_l(x)w_l\right) \text{o}\ \Phi^{-1}(\overline{x}) \\ \dot{\overline{x}}_{i,j} = \left(\bar{f}_{i,j}(x)+\bar{g}_{i,j}(x)u\right) \text{o}\ \Phi^{-1}(\overline{x}) \\ \vdots \\ \dot{\overline{x}}_{k,\cdots,q} = \left(\bar{f}_{k,\cdots,q}(x)+\bar{g}_{k,\cdots,q}(x)u\right) \text{o}\ \Phi^{-1}(\overline{x}) \\ \dot{\overline{x}}_\gamma = (\bar{f}_\gamma(x)+\bar{g}_\gamma(x)u)\ \text{o}\ \Phi^{-1}(\overline{x}) \\ y = h(x)\ \text{o}\ \Phi^{-1}(\overline{x}) \end{cases} \quad (10)$$

with $\overline{x} = \begin{bmatrix}\overline{x}_1 & \cdots & \overline{x}_l & \overline{x}_{i,j} & \cdots & \overline{x}_{k,\cdots,q} & \overline{x}_\gamma\end{bmatrix}^T$, $\bar{f}_\bullet(x) = (\phi^{p_\bullet})^T \times f(x)$, $\bar{g}_\bullet(x) = (\phi^{p_\bullet})^T \times g(x)$ and $x = \Phi^{-1}(\overline{x})$ since any diffeomorphism is invertible. The components, $\overline{x}_1 \ \cdots \ \overline{x}_l \ \ \overline{x}_{i,j} \ \cdots \ \overline{x}_{k,\cdots,q} \ \ \overline{x}_\gamma$, are respectively sensitive to faults : $[w_1]$, …, $[w_l]$, $\lfloor w_i \quad w_j \rfloor$, …, $\lfloor w_k \ \cdots \ w_q \rfloor$, no fault. The objective is the determination of an output injection (knowing that it exists) enabling us to express, on $\dot{\overline{x}}_\bullet$ equation, the contribution of any state sensitive to the other faults defined previously. It is then possible to write $\left(\bar{f}_\bullet(x)+\bar{g}_\bullet(x)u\right)\text{o}\ \Phi^{-1}(\overline{x})$ as follows:

$$\left(\bar{f}_\bullet(\overline{z})-\overline{\Psi}_1^\bullet(\overline{z},y_z)\right)+\left(\bar{g}_\bullet(\overline{z})-\overline{\Psi}_2^\bullet(\overline{z},y_z)\right) + \left(\overline{\Psi}_1^\bullet(\overline{z},y)+\overline{\Psi}_2^\bullet(\overline{z},y)u\right)$$

where $\overline{z} = \begin{bmatrix}\overline{z}_1 & \cdots & \overline{z}_l & \overline{z}_{i,j} & \cdots & \overline{z}_{k,\cdots,q} & \overline{z}_\gamma\end{bmatrix}^T$ is the state filter (in the base associated with the diffeomorphism (9)). But the most interesting filter form is undoubtedly :

$$\dot{\overline{z}}_\bullet = \breve{f}_\bullet(\breve{z}_\bullet)+\breve{g}_\bullet(\breve{z}_\bullet)u+\overline{\Psi}^\bullet(\breve{z}_\bullet,y,u) \quad (11)$$

where $\breve{z}_\bullet$ represents if $\bullet = i$, $\bullet = k,\cdots,q$ and $\bullet = \gamma$ respectively $\lfloor \overline{z}_i \quad \overline{z}_\gamma \rfloor$, $\lfloor \overline{z}_k \ \cdots \ \overline{z}_q \ \ \overline{z}_{k,\cdots,q} \ \ \overline{z}_\gamma \rfloor$ and $\lfloor \overline{z}_\gamma \rfloor$. To find the function $\overline{\Psi}^\bullet(\breve{z}_\bullet,y,u)$ (associated with the fault $w_\bullet$) presents a priori no difficulties.

Determination of $\overline{\Psi}^{\bullet}(\breve{z}_{\bullet}, y, u)$

$\overline{\Psi}^{\bullet}(\breve{z}_{\bullet}, y, u)$ must replace (using the measured variables y and u) all nonlinear combinations dependant of an another state that $\breve{z}_{\bullet}$ in its respective state equation $(\dot{\breve{z}}_{\bullet})$.

Note that $\overline{\Psi}^{\bullet}(\breve{z}_{\bullet}, y, u)$ is a vector function made up of $n_{\overline{p_\bullet}}$ lines, with:

$$Span\{p_\bullet\} \le n_{\overline{p_\bullet}} = dim(\phi^{p_\bullet})^T \le dim(S_*^{p_\bullet})^T \le dim((S_*^{\overline{p_\bullet}})^\perp)^T$$

The filter corresponding to system (10) can be written:

$$\begin{cases} \dot{\breve{z}}_1 = \breve{f}_1(\breve{z}_1) + \breve{g}_1(\breve{z}_1)u + \overline{\Psi}^1(\breve{z}_1, y, u) \\ \vdots \\ \dot{\breve{z}}_l = \breve{f}_l(\breve{z}_l) + \breve{g}_l(\breve{z}_l)u + \overline{\Psi}^l(\breve{z}_l, y, u) \\ \dot{\breve{z}}_{i,j} = \breve{f}_{i,j}(\breve{z}_{i,j}) + \breve{g}_{i,j}(\breve{z}_{i,j})u + \overline{\Psi}^{i,j}(\breve{z}_{i,j}, y, u) \\ \vdots \\ \dot{\breve{z}}_{k,\cdots,q} = \breve{f}_{k,\cdots,q}(\breve{z}_{k,\cdots,q}) + \breve{g}_{k,\cdots,q}(\breve{z}_{k,\cdots,q})u \\ + \overline{\Psi}^{k,\cdots,q}(\breve{z}_{k,\cdots,q}, y, u) \\ \dot{\breve{z}}_\gamma = \breve{f}_\gamma(\breve{z}_\gamma) + \breve{g}_\gamma(\breve{z}_\gamma)u + \overline{\Psi}^\gamma(\breve{z}_\gamma, y, u) \\ y_z = \overline{h}(\overline{z}) \end{cases} \quad (12)$$

We note:

$\breve{f}(\overline{z}) = (\breve{f}_1 \ \cdots \ \breve{f}_l \ \ \breve{f}_{i,j} \ \cdots \ \breve{f}_{k,\cdots,q} \ \ \breve{f}_\gamma)^T$,

$\breve{g}(\overline{z}) = (\breve{g}_1 \ \cdots \ \breve{g}_l \ \ \breve{g}_{i,j} \ \cdots \ \breve{g}_{k,\cdots,q} \ \ \breve{g}_\gamma)^T$, and

$\overline{\Psi}(\overline{z}, y, u) = (\overline{\Psi}^1 \ \cdots \ \overline{\Psi}^l \ \ \overline{\Psi}^{i,j} \ \cdots \ \overline{\Psi}^{k,\cdots,q} \ \ \overline{\Psi}^\gamma)^T$

Following the determination of the output injection, work in the natural coordinates base of the system is often simpler. This is why, the inverse transformation is used, i.e. the diffeomorphism $z = \Phi^{-1}(\overline{z})$. Thus (12) becomes:

$$\begin{cases} \dot{z} = \tilde{f}(z) + \tilde{g}(z) + \Psi(z, y, u) \\ y_z = h(z) \end{cases} \quad (13)$$

with $\tilde{f}(z) = (\frac{\partial \Phi^{-1}(\overline{z})}{\partial \overline{z}} \times \breve{f}(\overline{z})) \circ \Phi(z)$,

$\tilde{g}(z) = (\frac{\partial \Phi^{-1}(\overline{z})}{\partial \overline{z}} \times \breve{g}(\overline{z})) \circ \Phi(z)$, and

$\Psi(z, y, u) = (\frac{\partial \Phi^{-1}(\overline{z})}{\partial \overline{z}} \times \overline{\Psi}(\overline{z}, y, u)) \circ \Phi(z)$. Consequently, the expression $\Psi_1(z,y) + \Psi_2(z,y)u$ from (2) is identified with $\Psi(z, y, u) = (\frac{\partial \Phi^{-1}(\overline{z})}{\partial \overline{z}} \times \overline{\Psi}(\overline{z}, y, u)) \circ \Phi(z)$. This first stage does not make it possible to detect and isolate faults. For this aim, it is necessary to generate residual from outputs combinations.

3.2 Residual generation

According to equations (4)-(9) and the checking of the existence conditions (8), there are at least l outputs defined by nonlinear vector functions $\Theta^{\overline{p_\bullet}}(\bullet)$. Thus, for each fault it exists a nonlinear function, such as $Q_*^{\overline{p_\bullet}} \cap Span\{dh\} = Span\left\{\frac{\partial \Theta^{\overline{p_\bullet}}(y)}{\partial x}\right\}$. Under this condition $r_{\overline{p_\bullet}} = \Theta^{\overline{p_\bullet}}(y) - \Theta^{\overline{p_\bullet}}(y_z)$, is only sensitive to the occurrence of the fault $w_\bullet$. In this manner the following residual vector is generated:

$$\begin{pmatrix} r_{\overline{p1}} \\ \vdots \\ r_{\overline{pj}} \\ \vdots \\ r_{\overline{pl}} \end{pmatrix} = \begin{pmatrix} \Theta^{\overline{p1}}(y) - \Theta^{\overline{p1}}(y_z) \\ \vdots \\ \Theta^{\overline{pj}}(y) - \Theta^{\overline{pj}}(y_z) \\ \vdots \\ \Theta^{\overline{pl}}(y) - \Theta^{\overline{pl}}(y_z) \end{pmatrix} \quad (14)$$

Determination of $\Theta^{\overline{p_\bullet}}(\bullet)$:

Each function $\Theta^{\overline{p_\bullet}}(\bullet)$ is obtained by integration of $Q_*^{\overline{p_\bullet}} \cap Span\{dh\}$. This step does not constitute a major difficulty but is based on a strong existence condition. In this section, a sufficient condition with the F.P.R.G. to design a residual vector has been described. However, it is possible to increase the dimension of each residual sensitive to a fault. Indeed, $dim(Q_*^{\overline{p_\bullet}} \cap Span\{dh\}) \le dim(Q_*^{\overline{p_\bullet}})$. Meaning it is possible, after derivation of $Q_*^{\overline{p_\bullet}} \cap Span\{dh\}$, to increase the dimension of $r_{\overline{p_\bullet}}$. By this operation, the redundancy is increased and thus the non-detection and false alarm rates can be reduced.

3.3 Comments

Following the step described previously, several points can be clarified. The first difficulty is the choice of the distribution γ completing the diffeomorphism. This distribution must be nonsingular, involutive and the most easily integrable possible. Then, several integration stages are necessary to obtain (13) and (14). To finish, we do not have to speak about stability, but there are a certain number of degree of freedom (the output injection : $\overline{\Psi}^{1,\cdots,l}(\overline{z}, y, u)$ if it exists) allowing to act on the system dynamics and also offering possibilities of stabilization.

4. NUMERICAL EXAMPLES AND SIMULATIONS

In this section, the techniques described previously are illustrated using two examples. For each example, the possible existence of a solution will be sought, and then, the method described in section 3 will be applied to search this solution (if it exists).

4.1 Example 1

A nonlinear system, referenced in (A.J. Fossard et D Normand-Cyrot, 1995) and described by (15), is considered. Actuator faults w_1 and w_2 are considered.

$$\begin{cases} \dot{x} = \begin{pmatrix} x_1 x_4 \\ x_3(1 - x_4) \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & x_1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & x_1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} \\ y = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \end{cases} \quad (15)$$

The application of the vector field series (4) allows to determine the smallest distributions $S_*^{\overline{p1}}$ (resp. $S_*^{\overline{p2}}$)

(h, f)_invariant containing w_2 (resp. w_1). In the same way, the application of the co-vector series (6) identifies the output combinations which would solve this problem:

- associated to the fault w_1 and w_2,

$$\left(Q_*^{\overline{p2}}\right)^{\perp} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ x_1 & -x_3 & 0 & 0 \end{pmatrix} \left(Q_*^{\overline{p1}}\right)^{\perp} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

The conditions

$$\left(Q_*^{\overline{p1}}\right)^{\perp} \cap Span\left\{(p_1)^T\right\} = \{0\} \qquad (16)$$

$$\left(Q_*^{\overline{p2}}\right)^{\perp} \cap Span\left\{(p_2)^T\right\} = \{0\} \qquad (17)$$

are satisfied but the condition allowing the $Q_*^{\overline{p2}}$ building from an output combination is not. That is to say it doesn't exist a vector function $\Theta^{\overline{p2}}$ which verify:

$$Q_*^{\overline{p2}} \cap Span\{dh\} = Span\left\{d\left(\Theta^{\overline{p2}} \circ h(x)\right)\right\} \qquad (18)$$

This problem is due to the apparition of the state x_3 within the co-distribution $Q_*^{\overline{p2}}$. Indeed, this state component does not appear in any output equations, so it is not possible to observe it from these equations. It is not possible to build an isolated fault filter with a diagonal type structure. On the other hand, obtaining a such structure is not a necessary condition to the fault isolation (C. Join and al, 2001). Indeed, the isolation will be possible only if the signature associated with each fault is independent with the other fault signatures. It is an example to the contrary of the proposed method and highlights that the isolation conditions are necessary.

4.2 Example 2

In this section, the system (15) will be considered with the following modified output equation: $y=(x_1\ x_3)^T$. As previously, the different distributions are calculated in the following cases:

- associated to the fault w_1 and w_2,

$$\left(Q_*^{\overline{p2}}\right)^{\perp} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \left(Q_*^{\overline{p1}}\right)^{\perp} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

These results are obtained by developing calculations (4) and (6). The three conditions (16), (17) and (18) are satisfied. So, it exists a solution to the residual generation problem with a diagonal structure.

In reference to the paragraph 3, the following change of coordinates is defined as in equation (9):

$$\frac{\partial \Phi(x)}{\partial x} = \begin{pmatrix} \phi^{p1} \\ \phi^{p2} \\ \gamma \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

The diffeomorphism making it possible to return in the original base is defined by:

$$\Phi^{-1}: \begin{cases} x_1 = \bar{x}_2 \\ x_2 = \bar{x}_4 \\ x_3 = \bar{x}_1 \\ x_4 = \bar{x}_3 \end{cases}, \text{ so } \frac{\Phi^{-1}(\bar{x})}{\partial \bar{x}} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

The original system can be written in the new base by:

$$\begin{cases} \dot{\bar{x}} = \begin{pmatrix} 0 \\ \bar{x}_2\bar{x}_3 \\ 0 \\ \bar{x}_1(1-\bar{x}_3) \end{pmatrix} + \begin{pmatrix} 0 & \bar{x}_2 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} + \begin{pmatrix} 0 & \bar{x}_2 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} \\ y = \begin{pmatrix} \bar{x}_2 \\ \bar{x}_1 \end{pmatrix} \end{cases} \qquad (19)$$

With $\overline{\Psi}(\bar{z}, y, u) = (y_1u_2 \quad 0 \quad 0 \quad 0)^T$, the filter is written:

$$\begin{cases} \dot{z} = \begin{pmatrix} z_1z_4 \\ z_3(1-z_4) \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & y_1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \\ y_z = \begin{pmatrix} z_1 \\ z_3 \end{pmatrix} \end{cases} \qquad (20)$$

The residual verifying (14) is defined by:

$$\left(r_{\overline{p1}} \quad r_{\overline{p2}}\right)^T = \left(x_3 - z_3 \quad x_1 - z_1\right)^T \qquad (21)$$

with $\Theta(y) = (y_2 \quad y_1)^T$.

These results are illustrated by simulations.

4.3 Simulation results

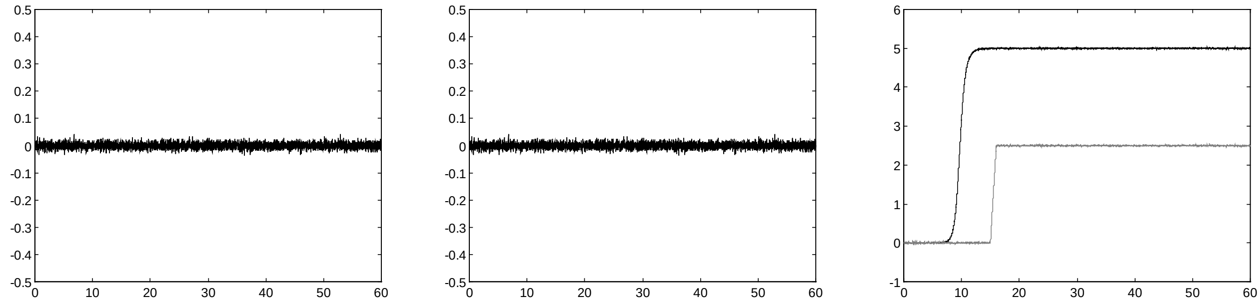
These simulations concern the system studied in the previous paragraph.

First, the fault free case is considered, with w_1=w_2=0 (Figure 1). The outputs (Figure 1c) have a completely normal behavior. A Gaussian white noise (N(0, 0.1)) is added on each output. When there is no fault, the two residuals are identically null (Figures 1a and 1b). Then, in order to show the advantage of a diagonal residual type structure, a simulation has been performed with two simultaneous faults (Figure 2). Two sinusoidal actuator faults of magnitude 10^{-3} and 10^{-2} (0.1 % of input) occur at time 30sec. In this case, outputs deviate (Figure 2c) and the two residuals are not equal to zero (Figures 2a and 2b). Indeed, only the fault w_1 (resp. w_2) has influence on residual r_1 (resp. r_2).

4.4 Comments

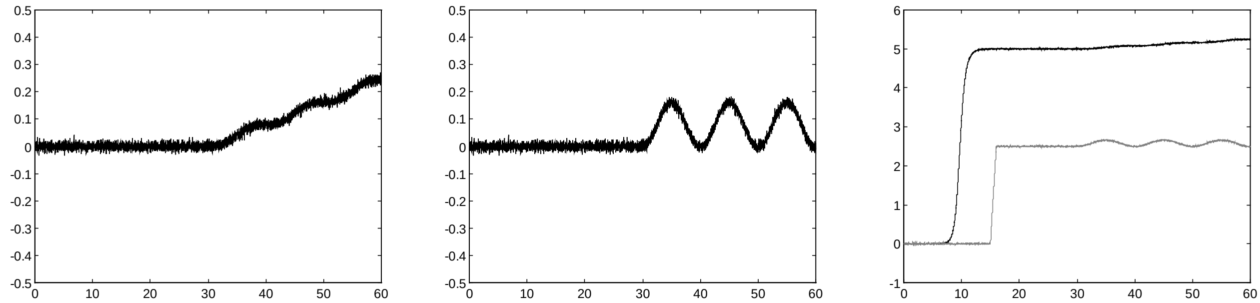
At first, nonlinear system (15) has been considered and the necessary existence condition solution are not satisfied for the diagonal residual type structure problem. However, in case of initial system (15), there is no solution to the F.P.R.G. with a diagonal type structure (3). But since only two faults can appear and it exists a function $\Theta^{\overline{p1}}(\bullet)$, it is possible to detect and localize these faults. But an additional assumption (not very restrictive in practice) is necessary: the faults should not appear simultaneously.

With the modified output vector, it has been proved that it exists a solution to F.P.R.G. Only one isolation filter is synthesized following the proposed method. Results are shown on simulations.



a) Residual y_1-y_{z1} sensitive to w_1 b) Residual y_2-y_{z2} sensitive to w_2 c): System outputs

Figure 1: Fault free case



a) Residual y_1-y_{z1} sensitive to w_1 b) Residual y_2-y_{z2} sensitive to w_2 c): System outputs

Figure 2: Two simultaneous faults case

5. CONCLUSION

In this article, the fundamental problem of residual generation with diagonal type structure has been considered for nonlinear system. A new geometrical method which allow the building of a such filter is developed (in case of it exists), with the determination of an output injection. In reference with works of Massoumnia (M.-A Massoumnia, 1986 and M.-A Massoumnia and al, 1989) (linear theory) and of A. Isidori *et al.* (A. Isidori and al, 1981 and A. Isidori, 1995) (nonlinear theory), the existence conditions of a fault isolated filter are recalled. Simulation results has been done in order to show the ability and performances of such filter.

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