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EXTENDED DIRECT LEARNING CONTROL FOR MULTI-INPUT MULTI-OUTPUT NONLINEAR SYSTEMS

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Abstract: For a class of nonlinear systems which perform a given task repetitively, an extended type of a *direct learning control*(DLC) is proposed using the information on the (vector) relative degree of the a multi-input multi-output system. *DLC* method can generate the desired control input directly from prestored control input profiles with different time scales without any repetitive learning process. It is shown that existing *DLC* methods can be applied only to a certain limited class of nonlinear systems and the information on the relative degree of a nonlinear system is essential to find the desired control input if the system has higher relative degree. *Copyright © 2002 IFAC*

Keywords: Learning control, nonlinear systems, relative degree, iterative methods, robot manipulators, tracking systems.

1. INTRODUCTION

For precise tracking control of a system which performs a repeated operation over a finite time interval, various types of iterative learning control(ILC) methods have been presented. Since the pioneering work called *Betterment Process* (Arimoto *et al*., 1984), various types of ILC methods have been proposed and applied mainly to the control of robot manipulators and rolling mill processes which perform a given task repetitively (Bondi and Gambardella, 1988; Oh, *et al*., 1988; Ahn, *et al*., 1993; Moore, 1993; Bien and Xu, 1998; Garimella and Srinivasan, 1998). Although ILC has merits in the sense that it does not require an exact mathematical modeling of a controlled system, there are a few problems which prohibit further practical applications of ILC to industrial systems.

Basically, ILC requires a lot of iterations until the tolerance error bound on the output trajectory is satisfied. Moreover, even if a small change occurs in the desired output trajectory due to the variation of control objectives, the previously learned control inputs have not been used for the generation of desired control input and hence, the learning process has to be resumed from the beginning. For example, consider a XY-table drawing several circles in a specified time periods. The first case is assumed to draw all circles with the same radius but different time. And the second case is to draw all circles with the same period but different radii. Obviously, the control inputs in these cases are correlated since they are generated for the same dynamics. The control problem is, for a new output trajectory which is different from all the previous ones in both magnitude scale and time scale, to find the corresponding control input so as to drive the system to follow the given output trajectory and make effective use of pre-obtained control input profiles. This kind of control problem was recently defined as a *nonrepeatable learning control problem* by Xu *et al*. (1999) compared to a *repeatable learning control* like ILC.

Kawamura and Fukao (1995) presented a time-scale interpolation method for the input torque patterns of the robot obtained through learning control. Xu *et al.* (1998, 1999) suggested a DLC method which can find the desired control input profile for a new output trajectory directly from the learned control inputs corresponding to other output trajectories under the assumption of full rankness of control input and output matrices.

For linear systems, however, Ahn (2000) showed that the class of systems to which existing DLC is applicable is limited to a set of systems whose relative degree is only one due to the strict assumption on the system matrices and how the relative degree of a system can be used to find the desired control input for linear systems with relative degree more than one.

In this paper, first, the class of nonlinear systems to which existing DLC can be applied is clarified and an extended type of DLC is proposed using the relative degree of a nonlinear system. The information on the (vector) relative degree of a multi-input multi-output(MIMO) nonlinear system is shown to be essential to find the desired control input through mathematical analysis and simulation results for the tracking control of SCARA robot manipulators are presented to show the validity of the proposed DLC.

2. MOTIVATION AND PRELIMINARIES

Consider a class of single-input single-output(SISO) nonlinear systems described by the following state space equations

$$\begin{aligned}\dot{\mathbf{x}}(t) &= f(\mathbf{x}(t)) + g(\mathbf{x}(t))u(t)\\ y(t) &= \mathbf{c}^T\mathbf{x}(t)\end{aligned} \tag{1}$$

where $\mathbf{x}(t) \in R^n$ is the state vector, $u(t)$ and $y(t)$ are the input and the output, respectively. The functions $f(\cdot) \in R^n$ and $g(\cdot) \in R^n$ are analytic on their domain of definition and **c** is a constant vector. Xu (1999) suggested a DLC law with a strict assumption that the product of control input matrix and the output matrix to be nonsingular as well as these matrices are constant matrices.

For the SISO system in (1), this assumption implies that $g(\mathbf{x}(t))$ should be a constant vector and $\mathbf{c}^T g(\mathbf{x}(t))$ should be nonzero. However, this assumption makes the DLC applicable only to a very limited class of nonlinear systems and it can be shown apparently that a nonlinear system whose relative degree is more than one cannot be controlled by this method.

We recall that the definition of the relative degree of a nonlinear system to characterize the system for which the above assumption is not satisfied. It is necessary to define some notations. The derivative of a scalar function φ along a vector $f = [f_1, .., f_n]^T$ is defined by

$$L_f\varphi(\mathbf{x}) = \frac{\partial\varphi}{\partial\mathbf{x}} f(\mathbf{x}) = \sum_{i=1}^{n} \frac{\partial\varphi}{\partial x_i} f_i(\mathbf{x}) \tag{2}$$

where $\mathbf{x} = [x_1, .., x_n]^T$, and the derivative of φ taken first along f and then along a vector g is

$$L_g L_f \varphi(\mathbf{x}) = \frac{\partial(L_f\varphi)}{\partial\mathbf{x}} g(\mathbf{x}) . \tag{3}$$

If φ is being differentiated j times along f, the notation $L_f^j\varphi(\mathbf{x})$ is used with $L_f^0\varphi(\mathbf{x}) = \varphi(\mathbf{x})$.
Let the nonlinear system in (1) have the relative degree q, then we have (Isidori, 1995)

$$L_g L_f^k(\mathbf{c}^T\mathbf{x}) = 0 ,\ 0 \le k \le q-2 \tag{4}$$

$$L_g L_f^{q-1}(\mathbf{c}^T\mathbf{x}) \ne 0 . \tag{5}$$

If $q \ge 2$ for a nonlinear system to be controlled, we get $L_g L_f^0(\mathbf{c}^T\mathbf{x}) = \mathbf{c}^T g = 0$. However, Xu *et al.* (1998, 1999) required an assumption that CB has full rank where C is the output matrix and B is the control input matrix, respectively. For the case of SISO systems as in (1), this reduces to $\mathbf{c}^T g \ne 0$. Hence, this type of DLC method may not be applied even for the systems of relative degree two. Actually, we may often see such a system in typical electro-mechanical systems such as a DC motor with a load or a robot manipulator. If we let the torque as the input and the position as the output for these systems, we can easily show that the relative degree of such systems is two. In order to find the control input for this problem, a new type of DLC for linear SISO systems was suggested using the relative degree of a system (Ahn, 2000).

In this paper, the DLC method for linear systems is extended to nonlinear MIMO systems to handle practically controlled systems. Before introducing the extended DLC, we need to define the relations between output trajectories which are given with different time scales.

Definition 1: Trajectory $\mathbf{y}_i(t_i)$ $t_i \in [0, T_i]$ is said to be *proportional* to another trajectory $\mathbf{y}(t)$, $t \in [0, T]$ in *time scales* if and only if $\mathbf{y}_i(t_i) = \mathbf{y}(t)$, where $\rho_i(t) = t_i = p_i t$ is the time scaling factor satisfying $\rho_i(0) = 0$ and $\rho_i(T) = T_i$.

Assumption 1: There are $l(l \ge 2)$ prestored trajectories $\mathbf{y}_i(t_i), t_i \in [0, T_i]$. The corresponding control input profiles $\mathbf{u}_i(t_i)$ have already been obtained *a priori* through iterative learning process. For any prestored trajectories $\mathbf{y}_i$ and $\mathbf{y}_j$ $(i \ne j)$, it should be $p_i \neq 0$, $p_j \neq 0$ and $p_i \neq p_j$ for $i, j = 1, \cdots, N$.

Now, the control problem in this paper is, for a new desired output trajectory $\mathbf{y}_d(t_d)$, $t_d \in [0, T_d]$ which is *proportional* to other prestored output trajectories, to find the desired control input profile $\mathbf{u}_d(t_d)$, which yields $\mathbf{y}_d(t_d)$ directly from the prestored $\mathbf{u}_i(t_i)$ $(i = 1, \cdots, l)$.

3. DLC FOR MIMO SYSTEMS

First, it is observed that the nonsingularity assumption mentioned in the above section still limits the application of DLC in MIMO systems. Consider a class of MIMO systems as follows:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= f(\mathbf{x}(t)) + G(\mathbf{x}(t))\mathbf{u}(t) \\ \mathbf{y}(t) &= h(\mathbf{x}(t)) = C^T \mathbf{x}(t) \end{aligned} \tag{6}$$

where $\mathbf{y}(t) = [y_1, \cdots, y_m] \in R^m$, $\mathbf{u}(t) = [u_1, \cdots, u_m] \in R^m$, $G(x) = [g_1, \cdots, g_m] \in R^{n \times m}$, $C = [\mathbf{c}_1, \cdots, \mathbf{c}_m]$, $\in R^{n \times m}$ and $h(\mathbf{x}) = [h_1(\mathbf{x}), \cdots, h_m(\mathbf{x})]^T \in R^m$.

Definition 2(Isidori, 1995): A MIMO nonlinear system of the form (6) has a (vector) relative degree $\{r_1, \cdots, r_m\}$ at $\mathbf{x}_0$ if

(i) $L_{g_j} L_f^k h_i(x) = 0$ for all $1 \le j \le m$, $1 \le i \le m$, $k < r_i - 1$ and for all $\mathbf{x}$ in a neighborhood of $\mathbf{x}_0$

(ii) the decoupling matrix

$$\mathrm{J}(\mathbf{x}) = \begin{bmatrix} L_{g1} L_f^{r_1 - 1} h_1(\mathbf{x}) & \cdots & L_{gm} L_f^{r_1 - 1} h_1(\mathbf{x}) \\ L_{g1} L_f^{r_2 - 1} h_2(\mathbf{x}) & \cdots & L_{gm} L_f^{r_2 - 1} h_2(\mathbf{x}) \\ \vdots & \cdots & \vdots \\ L_{g1} L_f^{r_m - 1} h_m(\mathbf{x}) & \cdots & L_{gm} L_f^{r_m - 1} h_m(\mathbf{x}) \end{bmatrix} \tag{7}$$

is nonsingular at $\mathbf{x} = \mathbf{x}_0$.

There may be points where a relative degree cannot be defined. However, we'll consider the system whose relative degree at $\mathbf{x}(t)$ can be defined for all t in a given finite time interval. Note that each integer r_i is associated with the i th output channel of the system and the i th row of $C^T G$ is $[(\partial h_i / \partial \mathbf{x}) g_1, \cdots, (\partial h_i / \partial \mathbf{x}) g_m]$, i.e. $[\mathbf{c}_i^T g_1, \cdots, \mathbf{c}_i^T g_m]$.

For a nonlinear system whose relative degree is more than one, these row vectors are identically zero for $i = 1, \cdots, m$ from *Definition 2*. Thus, the assumption of the nonsingularity of $C^T G$ cannot be satisfied for these systems as well as for SISO systems.

Let the nonlinear system in (6) have the (vector) relative degree $\mathbf{q} = \{q_1, \cdots, q_m\}$ and $y_i^{(k)}$ be the kth order time derivatives of the output, then high order derivatives of the output are found such as

$$\begin{aligned} y_i^{(j)} &= L_f^i (c_i^T \mathbf{x}),\ j < q_i - 1 \\ y_i^{(q_i)} &= L_f^{q_i} (c_i^T \mathbf{x}) \\ &\quad + \left[L_{g_1} L_f^{q_i - 1} (c_i^T \mathbf{x}), \ldots, L_{g_m} L_f^{q_i - 1} (c_i^T \mathbf{x}) \right] \mathbf{u} \end{aligned} \tag{8}$$

Define $\mathbf{y}^{(\mathbf{q})} = \left[y_1^{(q_1)}, \cdots, y_m^{(q_m)} \right]^T$, then we can rewrite (8) as the following.

$$\begin{aligned} \mathbf{y}^{(\mathbf{q})}(t) &= \begin{bmatrix} L_f^{q_1} (\mathbf{c}_1^T \mathbf{x}(t)) \\ L_f^{q_2} (\mathbf{c}_2^T \mathbf{x}(t)) \\ \vdots \\ L_f^{q_m} (\mathbf{c}_m^T \mathbf{x}(t)) \end{bmatrix} + J(\mathbf{x}) u \\ &= \Delta_f^{\mathbf{q}} (\mathbf{x}(t)) + J(\mathbf{x}) \mathbf{u} \end{aligned} \tag{9}$$

where $\Delta_f^{\mathbf{q}} (\mathbf{x}(t)) = \begin{bmatrix} L_f^{q_1} (\mathbf{c}_1^T (\mathbf{x}(t)) \\ \vdots \\ L_f^{q_m} (\mathbf{c}_m^T \mathbf{x}(t)) \end{bmatrix}$.

Assume that all the output channel of the system have the same relative degree in this paper. As a practical example which satisfies this assumption for the output channels with the same relative degree, consider an n degree-of-freedom robot manipulator which performs a given task repetitively. If we let the input of the robot be n joint torques and the output of the robot be n joint angles, then the (vector) relative degree of the robot system is {2, 2, …, 2} where the number of elements is n.

In the following theorem, we show that the desired control input, which yields the given desired output trajectory, can be generated from the prestored control input profiles. Note that the prestored input profiles were obtained from other output trajectories with different time scales by iterative learning control methods.

Theorem: For the nonlinear system (6) whose (vector) relative degree is $\mathbf{q} = \{q, \cdots, q\}$, the desired control input profile $\mathbf{u}_d(t_d)$, which yields the $\mathbf{y}_d(t_d)$, $t_d \in [0, T_d]$, can be directly generated from the prestored $\mathbf{u}_i(t_i)$ as follows:

$$\mathbf{u}_d(t_d) = [I \quad I] W^{\#} \bar{\mathbf{u}}_l \tag{10}$$

where

$W^{\#} = (W^T\ W)^{-1} W^T$, $\bar{\mathbf{u}}_l = [\mathbf{u}_1^T(t_1), \cdots, \mathbf{u}_l^T(t_l)]^T$, and

$$W = \begin{bmatrix} p_1^{-q} I & I \\ p_2^{-q} I & I \\ \vdots & \vdots \\ p_l^{-q} I & I \end{bmatrix}$$

Proof: For a nonlinear system with a (vector) relative degree $\mathbf{q} = \{q, \cdots, q\}$ where the number of elements is *m*, the desired control input can be found from (9) as follows:

$$\mathbf{u}(t) = \mathrm{J}(\mathbf{x}(t))^{-1} [\mathbf{y}^{(\mathbf{q})}(t) - \Delta_f^{\mathbf{q}}(\mathbf{x}(t))] \quad t \in [0, T] \quad (11)$$

For the new desired output $\mathbf{y}_d(t_d)$, $t_d \in [0, T_d]$, we can have

$$\mathbf{u}_d(t_d) = \mathrm{J}(\mathbf{x}_d(t_d))^{-1} \left\lfloor \mathbf{y}_d^{(\mathbf{q})}(t_d) - \Delta_f^{\mathbf{q}}(\mathbf{x}_d(t_d)) \right\rfloor$$
$$, \ t_d \in [0, T_d] \quad (12)$$

where $\mathbf{y}_d^{(\mathbf{q})}(t_d) = \left\lfloor y_{d,1}^{(q)}, \ y_{d,2}^{(q)}, \ \cdots \ y_{d,m}^{(q)} \right\rfloor$. Note that $\mathbf{u}_d(t_d)$ is not available directly in terms of above formula due to the existence of system uncertainties in $f(\cdot)$, $g(\cdot)$ and $\mathbf{c}_i$. Now choose prestored trajectories $\mathbf{y}_i(t_i), t_i \in [0, T_i]$, $i = 1,2,...,l$ whose control input profiles have been obtained *a priori* i.e.,

$$\mathbf{u}_i(t_i) = \mathrm{J}(\mathbf{x}_i(t_i))^{-1} \left\lfloor \mathbf{y}_i^{(\mathbf{q})}(t_i) - \Delta_f^{\mathbf{q}}(\mathbf{x}_i(t_i)) \right\rfloor$$
$$, \ t_i \in [0, T_i] \quad (13)$$

where $\mathbf{y}_i^{(\mathbf{q})}(t_i) = \left\lfloor y_{i,1}^{(q)}, \ y_{i,2}^{(q)}, \ \cdots \ y_{i,m}^{(q)} \right\rfloor$. Noting that $t_i = \rho_i(t_d)$ and differentiating the first element of $\mathbf{y}_d(t_d)$ with respect to t_d,

$$\begin{aligned} \frac{dy_{d,1}(t_d)}{dt_d} &= \frac{d}{dt_i}(y_{i,1}(t_i)) \frac{d\rho_i(t_d)}{dt_d} \\ &= \frac{d}{dt_i} y_{i,1}(t_i) \cdot p_i \end{aligned} \quad (14)$$

where $p_i = \dfrac{d\rho_i(t_d)}{dt_d}$. Continuing the differentiation q times, we get

$$\frac{d^q y_{d,1}(t_d)}{dt_d^q} = \frac{d^q y_{i,1}(t_i)}{dt_i^q} \cdot p_i^q . \quad (15)$$

Thus, we have $\mathbf{y}_d^{(\mathbf{q})}(t_d) = p_i^q \mathbf{y}_i^{(\mathbf{q})}(t_i)$. We also have $\mathbf{x}_i(t_i) = \mathbf{x}_d(t_d)$ from $\mathbf{y}_i(t_i) = \mathbf{y}_d(t_d)$. Hence, (13) becomes

$$\mathbf{u}_i(\rho_i(t_d)) = J(\mathbf{x}_d(t_d))^{-1} \left\lfloor p_i^{-q} \mathbf{y}_d^{(\mathbf{q})}(t_d) - \Delta_f^{\mathbf{q}}(\mathbf{x}_d(t_d)) \right\rfloor$$
$$(16)$$

Let $\mathbf{d}_1(\mathbf{x}_d(t_d)) = J(\mathbf{x}_d(t_d))^{-1} \cdot \mathbf{y}_d^{(\mathbf{q})}(\mathbf{x}_d(t_d))$ and $\mathbf{d}_2(\mathbf{x}_d(t_d)) = -J(\mathbf{x}_d(t_d))^{-1} \cdot \Delta_f^{\mathbf{q}}(\mathbf{x}_d(t_d))$, then we have

$$\begin{bmatrix} p_1^{-q} I & I \\ p_2^{-q} I & I \\ \vdots & \vdots \\ p_l^{-q} I & I \end{bmatrix} \begin{bmatrix} \mathbf{d}_1(\mathbf{x}_d(t_d)) \\ \mathbf{d}_2(\mathbf{x}_d(t_d)) \end{bmatrix} = \begin{bmatrix} \mathbf{u}_1(\rho_1(t_d)) \\ \mathbf{u}_2(\rho_2(t_d)) \\ \vdots \\ \mathbf{u}_l(\rho_l(t_d)) \end{bmatrix} \quad (17)$$

or $W\mathbf{d} = \bar{\mathbf{u}}_l$ where $\mathbf{d} = [\mathbf{d}_1^T(\mathbf{x}_d(t_d)), \ \mathbf{d}_2^T(\mathbf{x}_d(t_d))]^T$. Since $W^T W$ is invertible from the *Assumption* 1, we can solve $\mathbf{d}$ in (17). Recall that, from (12),

$$\begin{aligned} &\mathbf{d}_1(\mathbf{x}_d(t_d)) + \mathbf{d}_2(\mathbf{x}_d(t_d)) \\ &= \mathrm{J}(\mathbf{x}_d(t_d))^{-1} \left\lfloor \mathbf{y}_d^{(\mathbf{q})}(t_d) - \Delta_f^{\mathbf{q}}(\mathbf{x}_d(t_d)) \right\rfloor \\ &= \mathbf{u}_d(t_d) \end{aligned} \quad (18)$$

Combining (17) and (18), we obtain $\mathbf{u}_d(t_d)$ as shown in (10). ∇∇∇

In the *Theorem* above, it is shown that the relative degree q of the system should be included in W which is different from the result in Xu *et al.* (1998, 1999).

Remark 1: Now, we consider a linear system as a special case of the system (6). Let a linear SISO system with the relative degree q be represented by

$$\begin{aligned} \dot{\mathbf{x}}(t) &= A\mathbf{x}(t) + \mathbf{b}u(t) \\ y(t) &= \mathbf{c}^T \mathbf{x}(t) \end{aligned} \quad (19)$$

and y_i in (8) be the single output y in (19). For this system, (8) becomes

$$\begin{aligned} y^{(q_i)} &= L_f^{q_i}(\mathbf{c}^T \mathbf{x}) \\ &\quad + \left\lfloor L_{g_1} L_f^{q_i - 1}(\mathbf{c}^T \mathbf{x}), \ \cdots, \ L_{g_m} L_f^{q_i - 1}(\mathbf{c}^T \mathbf{x}) \right\rfloor \mathbf{u} \\ &= \mathbf{c}^T A^q x(t) + \mathbf{c}^T A^{q-1} \mathbf{b} u(t) . \end{aligned} \quad (20)$$

Since (20) is the same as (4) in Ahn (2000), we can find that the result for nonlinear systems obtained in this paper completely includes the previous result for linear systems.

Remark 2: If control input profiles corresponding to output trajectories with different time scales are obtained from iterative learning process, DLC may not yield the exact desired control input profile directly. However, the smaller we set the tolerance error bound in ILC, the better the actual output converges to the desired output. Even in case that the system is slightly changed, the DLC control input can be used as the best initial control input profile for the new ILC process.

4. APPLICATION TO ROBOT MANIPULATORS

To show the validity and the effectiveness of the proposed extended DLC, some simulation results are performed for the tracking control of a SCARA robot manipulator where the characteristics of the given task is repetitive. Let $\mathbf{x}(t)=[\mathbf{x}_1^T(t),\mathbf{x}_2^T(t)]^T$, $\mathbf{x}_1(t)=[\theta_1(t),\theta_2(t)]^T$, $\mathbf{x}_2(t)=[\dot\theta_1(t),\dot\theta_2(t)]$, and $\mathbf{u}(t)=[\tau_1(t),\tau_2(t)]$, then the dynamic equation of a 2-link SCARA robot manipulator in the state- space is described by

$$\dot{\mathbf{x}}(t)=\begin{bmatrix} I_2 & O \\ O & -M^{-1}(\mathbf{x}_1(t))\end{bmatrix}\begin{bmatrix}\mathbf{x}_2(t) \\ V(\mathbf{x}_1(t),\mathbf{x}_2(t))+F\end{bmatrix} + \begin{bmatrix} O \\ M^{-1}(\mathbf{x}_1(t))\end{bmatrix}\mathbf{u}(t)$$

$$\mathbf{y}(t)=\mathbf{x}_1(t) \qquad (21)$$

where $M(\mathbf{x}(t))\in R^{2\times 2}$ is the inertial matrix, $V(\mathbf{x}_1,\mathbf{x}_2)\in R^{2\times 1}$ is the Coriolis and centrifugal force, and $F\in R^{2\times 1}$ is the friction force. The (vector) relative degree of the robot system in (21) is easily found as {2, 2} from the *Definition 2*.
The desired output trajectory is specified as

$$\mathbf{y}_d=\begin{bmatrix} t_d^3(4-3t_d) \\ t_d^3(4-3t_d)\end{bmatrix},\ t_d\in[0,1] \qquad (22)$$

It is assumed that control input profiles concerning the following output trajectories have been obtained *a priori* by using the iterative learning control (Ahn, 1993) with the tolerance error bound on the output error 0.05.

$$\mathbf{y}_1=\begin{bmatrix} \frac{125}{64}t_1^3(4-\frac{15}{4}t_1) \\ \frac{125}{64}t_1^3(4-\frac{15}{4}t_1)\end{bmatrix},\ t_1\in[0,0.8]$$

$$\mathbf{y}_2=\begin{bmatrix} \frac{125}{216}t_2^3(4-\frac{5}{2}t_2) \\ \frac{125}{216}t_2^3(4-\frac{5}{2}t_2)\end{bmatrix},\ t_2\in[0,1.2].$$

The *proportionality* between output trajectories is satisfied since $t_1=(4/5)t_d$ and $t_2=(6/5)t_d$ i.e., $p_1=4/5$ and $p_2=6/5$. The desired control input is calculated using (10) as follows:

$$\mathbf{u}_d=\begin{bmatrix} I_2 & I_2\end{bmatrix}\begin{bmatrix} (\frac{4}{5})^{-2}I_2 & I_2 \\ (\frac{6}{5})^{-2}I_2 & I_2\end{bmatrix}^{-1}\begin{bmatrix}\mathbf{u}_1(t_1) \\ \mathbf{u}_2(t_2)\end{bmatrix}$$

$$=0.352\,\mathbf{u}_1(t_1)+0.648\,\mathbf{u}_2(t_2) \qquad (24)$$

where $\mathbf{u}_1$ and $\mathbf{u}_2$ are the inputs corresponding to $\mathbf{y}_1$ and $\mathbf{y}_2$, respectively. In Fig. 1 and Fig. 2, the actual output trajectories ($\theta_1(t),\theta_2(t)$) are shown to converge well to the given desired output trajectories without iterative process by using the proposed DLC. The output error can be further reduced if we decrease the tolerance error bound in the iterative learning process, moreover, the perfect tracking can be obtained if the exact modeling of the system is known as in Xu (1999).

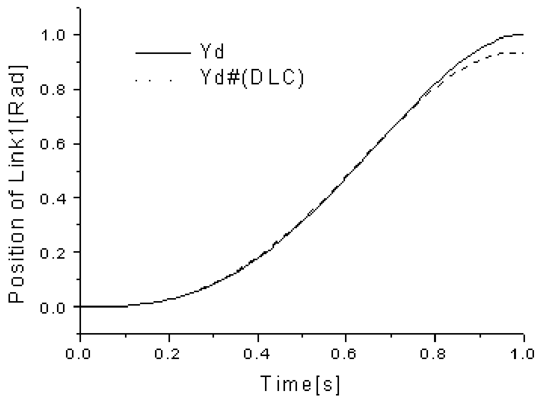

Fig. 1. Output trajectories for the link 1.

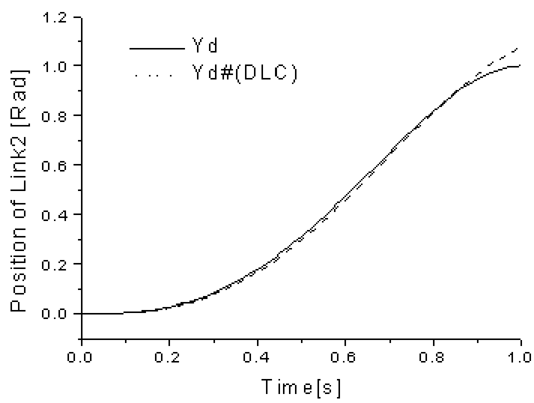

Fig. 2. Output trajectories for the link 2.

5. CONCLUSION

An extended type of DLC has been proposed for a class of MIMO nonlinear systems with the relative degree more than one. It has been first observed that the existing DLC is effective only to nonlinear systems with the relative degree one as well as to linear systems. The information on the relative degree of a system was shown to be essential to find the desired control input directly using DLC through the mathematical analysis. Simulation results for the tracking control of a SCARA robot manipulator have shown the validity of the proposed DLC scheme.

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