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# ADAPTIVE SPR SPEED/POSITION CONTROL OF INDUCTION MOTOR

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**Abstract**: This paper proposes an adaptive speed/position tracking control of an induction motor subject to unknown load torque via strictly positive real (SPR) analysis. The controller is developed under a special nonlinear coordinate transform such that either speed or position control objective can be fulfilled. The underlying design concepts are to endow the close-loop system while under lack of knowledge of some key system parameters, such as the rotor resistance, motor inertia and motor damping coefficient. The proposed control scheme comes along with a thorough proof derived based on Lyapunov stability theory. The experimental results are also given to validate the effectiveness of the presented control scheme. *Copyright* © 2002 *IFAC*

**Keywords:** Adaptive control, induction motors, tracking systems, Lyapunov stability, coordinate transformations

## 1. INTRODUCTION

The induction motor control is an important issue in both motion control and servo control applications, because the induction motor can operate in a wide-range of both torque and speed. And, their efficiency and robustness are useful features in industry. The control schemes based on indirect FOC are much more popular due to the advantages in applications (Marino, *et al.*, 1999; Tajima and Hori, 1993; Espinosa, *et al.*, 1998). In general applications, indirect vector control of induction motor is widely applied, where the rotor flux is estimated rather than being measured. This requires *a priori* knowledge of the machine parameters, which makes the indirect vector control scheme machine dependent. Given the fact that parameters may change significantly with temperature and there are some states that are not easily acquired, design of appropriate observers becomes crucially important to the success of the control (Krishnan and Bharadwaj, 1990; Shin, *et al.*, 2000). Recently, the sensorless field oriented control scheme gradually appears as a popular control method for induction motor (Marino, *et al.*, 1996; Lin and Fu, 2000). On the other hand, the load torque structure is also a very important knowledge for controller design to achieve high performance control. There have been many research results in the literature about the torque control of induction motor so far (Lee and Fu, 2001; Lascu, *et al.*, 2000; Ortega and Espinosa, 1993).

Given the above observation, we propose a speed/position tracking control scheme based on the indirect FOC with the strictly positive real (SPR) property (Narendra and Annaswamy, 1989). Moreover, the proposed control scheme handles the problems with both uncertainties of rotor resistance and load torque, respectively. The system parameters of the induction motor, except its rotor resistance, and mechanical parameters, are known as mentioned previously. For rigorousness, the developed control scheme is thoroughly analyzed via Lyapunov stability theory, and the asymptotic convergence property is soundly proved. The experimental results are given to validate the performances.

## 2. PRELIMINARIES

In this section, we will first review the mathematical description of the operational principle of an induction motor in the following sections. Before we continue the control of speed/position of the induction motors, we first make some basic assumptions as shown below:

(A.1) The induction motor is assumed without saturation, hysteresis, eddy currents, and spatial flux harmonics.

(A.2) All the states are measurable except the rotor flux. Parameters including rotor resistance, rotor inertia, damping coefficient, and the payload coefficient are assumed unknown.

**Proposition 1.** *Under the assumptions (A.1), if the input voltages in d-q frame of the voltage-fed induction motor are defined as*

$$cV_{qs} = \frac{\lambda_{dr}}{\sqrt{\lambda_{qr}^2+\lambda_{dr}^2}}V \quad \textit{and} \quad cV_{ds} = \frac{-\lambda_{qr}}{\sqrt{\lambda_{qr}^2+\lambda_{dr}^2}}V \tag{1}$$

*then the power transferred to the rotor of induction motor is maximal subject to the constraint* $(V_{qs}^2+V_{ds}^2) = (V/c)^2$ *at any time* (Lee and Fu, 2001; Lee, *et al.*, 2000a; Lee *et al.*, 2000b).

Of course, $V$ does not have to be a constant. Instead, it offers one D.O.F. (Degree of freedom) control to the system. For general mechanical systems, the load torque is a function of the rotor speed $\omega_r$, which normally has the form

$$T_L = J_L\dot{\omega}_r + \mathrm{sgn}(\omega_r)b_0 + b_1\omega_r + b_2\,\mathrm{sgn}(\omega_r)\omega_r^2 = J_L\dot{\omega}_r + f_L(\omega_r).$$

This assumption is more realistic than a constant load torque. Therefore, the mechanical load in the form aforementioned can be rearranged as $T_L = W_r^T\Theta$ with the constant parameter vector $\Theta$, and the known function vector $W_r^T$. In the sequel, we will assume that $\Theta$ is unknown.

On the other hand, we would like to show that, given a desired speed command $\omega_d$, there exists a proper input signal $V$ such that the steady state of the system exactly achieves the purpose of speed tracking, i.e., $\omega_r = \omega_d$, and the objective of maximal power transfer (Proposition 1). To this end, we first introduce a reasonable result $x_2 = \lambda_{qr}^2 + \lambda_{dr}^2 > 0$, and then further simplify the dynamics model of an induction motor with Proposition 1, as shown in (Ortega and Espinosa, 1993; Lin and Fu, 2000):

$$\begin{aligned}
\dot{x}_1 &= -2a_1x_1 + 2a_2x_3 + \frac{2hx_4}{\sqrt{x_2}}V \\
\dot{x}_2 &= -2a_4x_2 + 2a_3x_3 \\
\dot{x}_3 &= a_3x_1 + a_2x_2 - (a_1+a_4)x_3 + px_5x_4 \\
\dot{x}_4 &= -px_5x_3 - (a_1+a_4)x_4 + \sqrt{x_2}V \\
J\dot{x}_5 &= a_5x_4 - f_L(x_5),
\end{aligned} \tag{2}$$

where the parameters $a_1, a_2, a_3, a_4$, and $a_5$ are defined in the nomenclature.

## 3. OBSERVERS AND CONTROLLERS

To proceed with the controller design, we first introduce the observers to estimate the unmeasurable rotor flux, and the unknown rotor resistance.

### *3.1 Observer Design*

Due to Assumption (A.2), we have to build an observer and a parameter estimator to estimate the rotor flux as well as the rotor resistance. There exist various types of flux observers and parameter estimators in the literature, which have been described in (Lin, and Fu, 2000), which we omitted here.

### *3.2 Speed Tracking Controller*

Before we introduce the design of the controller, in order to avoid dealing with the discontinuous function sgn(x), we approximate it by the so-called sigmoid function smod(x) defined below:

$$\mathrm{sgn}(x) \approx \mathrm{smod}(x) = \frac{e^{\gamma x} - e^{-\gamma x}}{e^{\gamma x} + e^{-\gamma x}}, \tag{3}$$

where $\gamma > 1$ determines the slope of the function. By taking such approximation, we will be able to differentiate the payload function $f_L(\omega_r)$ for the subsequent purpose of controller design. To proceed with the design, we first write down the speed tracking error equation from (2) as:

$$J\dot{e}_5 = a_5 x_4 - \Theta^T W, \tag{4}$$

where $e_5 = x_5 - \omega_d$, $\Theta = [J\ b_0\ b_1\ b_2]^T$, and $W = [\dot{\omega}_d\ \ \mathrm{smod}(x_5)\ \ x_5\ \ \mathrm{smod}(x_5)x_5^2]^T$. After differentiating both sides of (4), we obtain the following:

$$J\ddot{e}_5 = -a_5(a_1 + a_4)x_4 - a_5 p x_3 x_5 + a_5\sqrt{x_2}V - \Theta^T \dot{W}, \tag{5}$$

which clearly relates the voltage input $V$ to the tracking error $e_5$, and lays down a ground for constructing an adaptive controller. However, there remain two difficulties before the controller design. One is we need to establish a S.P.R. (strictly positive real) transfer function from the parametric error term to the tracking error $e_5$, and another is to decompose the uncertainty terms into a product of unknown parametric vector and known function vector. To solve the above difficulties, we apply some algebraic manipulation on both Equation (4) and (5) to obtain

$$J(\ddot{e}_5 + \kappa_1\dot{e}_5 + \kappa_2 e_5) = [-a_5(a_1 + a_4)x_4 - a_5 p x_3 x_5]$$

$$+ \kappa_1 a_5 x_4 + a_5\sqrt{x_2}V + \kappa_2 J e_5 - \kappa_1\Theta^T W - \Theta'^T W'$$

$$= g_1(x) - \Theta_1^T W_1 + a_5\sqrt{x_2}V,$$

for some $\kappa_1,\ \kappa_2 > 0$, where we redefine the unknown parameter vector $\Theta'$ as well as the known function vector $W'$ in the uncertainty term $\Theta^T\dot{W}$, i.e., $\Theta^T\dot{W} = \Theta'^T W'$, $\Theta_1{}^T = [\kappa_1\Theta^T,\ \Theta'^T,\ \kappa_2 J]$ and $W_1 = [W,\ W',\ -e_5]$, and then design the input $V$ as

$$V = \frac{1}{a_5\sqrt{x_2}}[-g_1(x) + L_1(s)(\hat{\Theta}_1^T\Pi_1)], \tag{6}$$

where $L_1(s) = (s + \beta_1)$, for some $\beta_1 > 0$, $\Pi_1 = L_1^{-1}(s)\,W_1$, and $\hat{\Theta}_1$ is the estimate of $\Theta_1$. It then follows that the design (6) will yield

$$e_5 = \frac{(s + \beta_1)}{J(s^2 + \kappa_1 s + \kappa_2)}(\tilde{\Theta}_1^T\Pi_1) = M_1(s)(\tilde{\Theta}_1^T\Pi_1),$$

where $\tilde{\Theta}_1 = \hat{\Theta}_1 - \Theta_1$, and the transfer function $M_1(s)$ can be made S.P.R. by proper choice of $\kappa_1,\ \kappa_2$ and $\beta_1$. It is thus clear from the literature of Narendra and Annaswamy (1989) that if we choose the parameter adaptive law as:

$$\dot{\tilde{\Theta}}_1 = \dot{\hat{\Theta}}_1 = -\Gamma_1 e_5 \Pi_1, \tag{7}$$

for some $\Gamma_1 > 0$, then both $e_5$, $\hat{\Theta}_1$, and hence $x_5$ are bounded. But in fact we can show that the proposed nonlinear adaptive S.P.R. speed controller consisting of control law (6) and adaptation law (7) will guarantee the boundedness of all signals in the closed-loop, and yield convergence of tracking error $e_5$. Such fact is stated in the following theorem.

**Theorem 1.** *Consider an induction motor whose dynamics are governed by system (2) with unknown load torque under the Assumption (A1) and (A2). Given a twice-differentiable smooth desired speed trajectory* $\omega_d$ *with* $\omega_d$, $\dot{\omega}_d$ *and* $\ddot{\omega}_d$ *being all bounded, then the stator voltage designed as Proposition, where V is given by Equation (6) subject to adaptive law (7), will guarantee boundedness of all signals in the closed-loop and convergence of the rotor speed tracking, i.e.,* $\omega_r \to \omega_d$ *as* $t \to \infty$.

**Proof:** The proof is a continuation of the former discussion. First, since the state $x_5$ is bounded and the mechanical subsystem is passive, the electrical torque, which is state $x_4$, is also bounded; otherwise, there will be a contradiction. Then, to show boundedness of the rest of states, we can use the technique as in the previous section and discuss two cases. The first case is trivial since if the stator currents are bounded, then all the states are bounded (Lee and Fu, 2001). The second case is if the stator currents grow unbounded, then $x_2$ will grow unbounded as well due to $x_2$ and $|i_s|^2$ grow at the same rate. On the other hand, we can rearrange the dynamical equations from system (2) as shown below:

$$\dot{X} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -2a_1 & 0 & 2a_2 \\ 0 & -2a_4 & 2a_3 \\ a_3 & a_2 & -(a_1+a_4) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \frac{2hx_4}{\sqrt{x_2}} V \\ 0 \\ px_5x_4 \end{bmatrix}$$
$$= AX + u$$

where $A$ can be shown to be Hurwitz. After reviewing definitions of $x_3$ and $V$, respectively, we found that the first entry of $u$ will be bounded because $x_2$ grows no slower than $x_3$ if $x_3$ does grow unbounded (due to the 2nd equation of (2)). Hence, $u$ is apparently bounded, and hence $X$ will be bounded. But this fact leads to contradiction to the hypothesis in the second case. This then proves the boundedness of all the states. For convergence of tracking error $e_5$, we note that $\dot{e}_5$ is bounded, and thus $e_5 \to 0$ as $t \to \infty$ from Barbalat's Lemma. Q.E.D.

After the design of the proposed speed controller, we then expand it to the position controller as follows.

*3.3 Position Tracking Controller*

In order to design the position tracking controller, we augment the system (2) with additional previous defined Equation (8) as shown below:

$$\dot{x}_6 = x_5, \tag{8}$$

where $x_6 = \theta_r$ indicates the rotor position of the induction motor with an assumption as follows:

(A.3) The desired position command $\theta_d \in C^3$ is three-time differentiable smooth function with $\theta_d$, $\dot{\theta}_d$, $\ddot{\theta}_d$, and $\dddot{\theta}_d$ being all bounded.

To proceed with the design, we first define the joint tracking error $\varepsilon = e_5 + \kappa_3 e_6$ and then write down the associated differential equation from (2) and (8) as:

$$J\ \dot{\varepsilon} = J\dot{e}_5 + \kappa_3 J\dot{e}_6 = a_5x_4 - \Theta^T W + \kappa_3 Je_5, \tag{9}$$

for $\kappa_3 > 0$,where $e_5 = x_5 - \omega_d$, $e_6 = x_6 - \theta_d$, and $\Theta = [J\ b_0\ b_1\ b_2]^T$, $W = [\dot{\omega}_d\ \ \mathrm{smod}(x_5)\ \ x_5\ \ \mathrm{smod}(x_5)x_5^2]^T$ as defined previously. After differentiating both sides of (9), we obtain the following:

$$\begin{aligned} J\ \ddot{\varepsilon} = &-a_5(a_1+a_4)x_4 - a_5px_3x_5 + a_5\sqrt{x_2}V, \\ &-\Theta^T\dot{W} + \kappa_3 Ja_4x_4 - \kappa_3 J\Theta^T W \end{aligned} \tag{10}$$

We first apply some algebraic manipulation on both (8) and (10) to obtain

$$\Theta_2^T = [\Theta'^T,\ (\kappa_3 J + \kappa_4)\Theta^T,\ \kappa_3\kappa_4 J,\ \kappa_3 a_4 J,\ \kappa_5 J],$$
$$W_2 = [W,\ W',\ -x_4,\ e_5,\ \varepsilon],$$

and then design the input $V$ as

$$V = \frac{1}{a_5\sqrt{x_2}}[-g_2(x) + L_2(s)\ (\hat{\Theta}_2^T\Pi_2)], \tag{11}$$

for some $\kappa_4,\ \kappa_5 > 0$, $\beta_2 > 0$, $\Pi_2 = L_2^{-1}(s)W_2$, and $\hat{\Theta}_2$ is the estimate of $\Theta_2$. Similar to the previous subsection the transfer function $M_2(s)$ can be made SPR by proper choice of $\kappa_4$, $\kappa_5$ and $\beta_2$. If we choose the parameter adaptive law as:

$$\dot{\tilde{\Theta}}_2 = \dot{\hat{\Theta}}_2 = -\Gamma_2\ \varepsilon\ \Pi_2, \tag{12}$$

for some $\Gamma_2 > 0$, then both $\varepsilon$, $\hat{\Theta}_2$, and hence $x_5$, $x_6$ are bounded. The following theorem concluded.

**Theorem 2.** *Consider the induction motor system (2) and the Equation (8) under the Assumption (A1), (A2), and (A3). The stator voltage designs similar to the one suggested in Theorem 1, where V is given by Equation (11) subject to the adaptive law (12). Then, boundedness of all signals in the closed-loop and convergence of the position tracking error, i.e.,* $\theta_r \to \theta_d$ *as* $t \to \infty$*, can both be guaranteed.*

**Proof:** The proof follows that of Theorem 1, but note that it is $\varepsilon$, rather than $e_5$, which is bounded and converges to zero. However, boundedness and convergence of $\varepsilon$ readily leads to boundedness of $e_5$ and convergence of $e_6$. The rest of proof will be the same. Q.E.D.

## 4. EXPERIMENTAL RESULTS

To validate the performances of the proposed controller, we hold a series experiments. Whose parameters are listed as follows: $R_s$ = 0.83Ω, $R_{rn}$ = 0.53Ω, $L_s$ = 0.08601(H), $L_r$ = 0.08601(H), $L_m$ = 0.08259(H), *4 poles*, rated current 8.6 A, 220 V, 60 Hz, AC. $J_m$ and $B_m$ are assumed unknown. The mechanical load torque is $T_L$ = $J_L$ $\dot{\omega}_r$ + $b_0 \mathrm{sgn}(\omega_r) + b_1\omega_r + b_2 \mathrm{sgn}(\omega_r)\omega_r^2$. In the experiment, there is no load applied on the induction motor. The gains are $[\beta_1, \beta_2, \kappa_1, \kappa_2, \kappa_3, \kappa_4, \kappa_5]$ = [50, 80, 300, 50, 10, 500, 80], in the nonlinear adaptive SPR speed/position controller, and all the adaptation gains $\Gamma_{1\sim 2}$ are set to unity of the controller design. Accompany with $k_{d1} = k_{d2} = k_{q1} = k_{q2} = 0.003$, and $k_0 = 0.01$, $k_R = 200$ of the observers' gains. The experimental results of both speed and position tracking are demonstrated in the following cases:

*Case 1.* The speed command is a sort of step-type command to validate the proposed controller. Figure 1 shows the boundeness of estimated parameters and also the performances of the proposed controller with a benchmark speed command. The estimated values of the unknown parameters are truly bounded. And, the errors of both speed tracking and the deviation of rotor resistance are rapidly converged. The control objective is achieved even with the crucial condition.

*Case 2.* The position tracking controller is operated in a critical situation of ramp-type commands. Figure 2 shows the satisfactory performances of the position tracking.

All the experiments are conducted without the information of the deviation of rotor resistance, the motor inertia and the damping coefficient of the induction motor. And, the parameters of load torque are unknown, either.

## 5. CONCLUSION

There are some concluding remarks summarized as below: The proposed control scheme is developed based on a novel dynamical model of induction motor. The proposed speed/position tracking controller of induction motor, which copes with unknown motor parameters ($J_m$ and $B_m$), the uncertainty of rotor resistance, and unknown load torque $T_L$, is a Lyapunov-based design. Although the speed/position command is assumed being a twice/(three-times) differentiable trajectory, but the step type command can be directly applied to the proposed controller with satisfactory performances, i.e., the assumption can be relaxed in reality.

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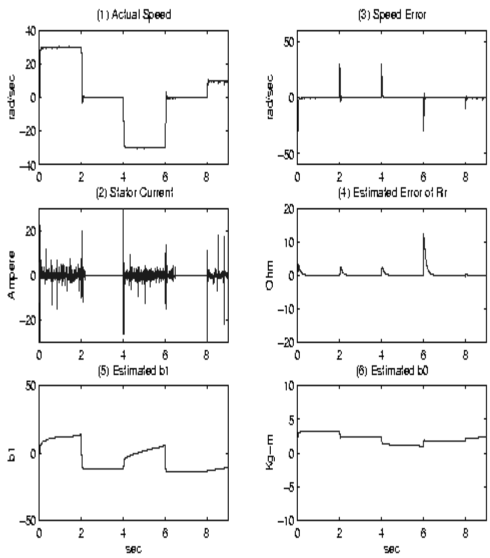


Fig 1. Experimental results of a step-type speed tracking.

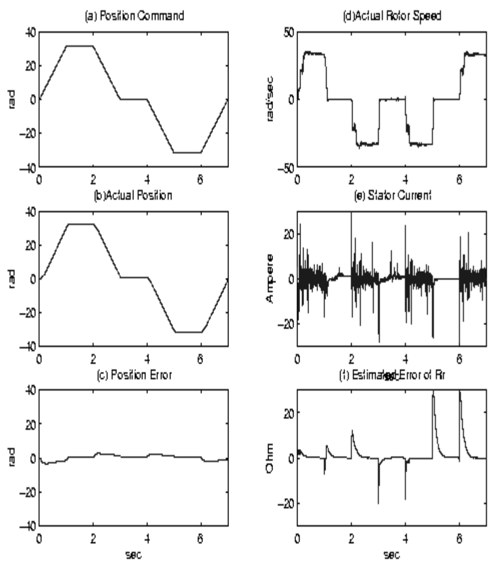


Fig. 2. Experimental results of position tracking (ramp-type).

NOMENCLATURE

$D = (L_s L_r - L_m^2)$ $\beta = L_m / D$

$a_2 = L_m R_s / D$ $a_3 = L_m R_r / D$

$a_4 = L_s R_r / D$ $a_5 = L_r K_t / D$

$L_0 = L_r D / L_m$ $L_1 = L_r^2 R_s / L_m$

$K_t = 3pL_m / 2L_r$ $a_1 = L_r R_s / D$