

## THE RBF-ARX MODEL BASED MODELING AND PREDICTIVE CONTROL FOR A CLASS OF NONLINEAR PROCESSES

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**Abstract:** This paper considers modeling and control problems of the non-stationary nonlinear processes whose dynamics depends on the working point. A hybrid RBF-ARX model-based predictive control (MPC) strategy without resorting to on-line parameter estimation for this kind of processes is presented. The RBF-ARX model is composed of the RBF networks and a rather general form of ARX model, which is identified off-line, and whose local linearization may be easily obtained. A quickly-convergent estimation method is applied to optimize the RBF-ARX model parameters. The modeling validity and the MPC performance is illustrated by an application to Nitrogen Oxide (NO<sub>x</sub>) decomposition process in thermal power plants.  
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**Keywords:** Nonlinear systems, non-stationary process; modeling, model based predictive control, radial basis function networks, ARX model.

### 1. INTRODUCTION

Many nonlinear MPC methods and applications have been reported, in which some control schemes (see *e. g.* Mahfouf and Linkens 1998) are based on the direct use of nonlinear models, but resulted in on-line solving a higher order nonlinear optimization problem, which is computationally expensive and may get stuck in a local minimum. Some methods (see *e. g.* Prasad *et al.* 1998) used the piecewise linearization technique to describe the nonlinear behavior of a system, so the model was linearized at each sampling interval, which resulted in the solution of a (or a set of) quadratic programming problem at each such interval, as in the case of linear MPC. However, the estimation of many linear models to be valid only in each small region is not easy in practice. Ayala Botto *et al.* (1999) proposed an affine neural network based

MPC method, in which input-output feedback linearization of the affine nonlinear model was carried out first, and then it used an iterative algorithm to cope with the nonlinear input constraint problem. In that method, at each sampling interval the quadratic programming routines needed to be used iteratively for satisfying some conditions. Liu *et al.* (1998) used a set of neural network based affine nonlinear predictors to design nonlinear MPC, for which the use of nonlinear programming approach was avoided, but the weights of all neural network predictors had to be estimated on-line.

However, the success of MPC is highly dependent on a reliable system model. It is very meaningful work to look for a model which can effectively describe the nonlinear behavior of a system, and may be also easily used to design MPC algorithm. In fact, a large

number of nonlinear processes may be regarded as this kind of systems whose working point changes with time and it can be locally linearized at fixed working point. Lakhdari *et al.* (1995) proposed a MPC scheme for the nonlinear system whose dynamics depends on working point. It used a quasi-linear autoregressive model as internal model of the MPC. The model coefficients were sets of integral rational function with respect to a state variable of describing the working point state, and all the functional coefficients were estimated on-line. At each sampling interval the model was similar to a linear autoregressive model, so the quadratic programming routines could be directly used. However, it still has the problem caused by model parameters on-line estimation.

To avoid on-line estimating the time-varying parameters of internal model or predictor in nonlinear MPC, in this paper, a hybrid RBF-ARX model based on Gaussian radial basis function networks and ARX model is proposed to implement MPC for the non-stationary nonlinear process with working point dependent dynamics. All the model parameters are estimated off-line by using a quickly-convergent structured nonlinear parameter optimization algorithm (Peng *et al.* 2001). The instant form of the RBF-ARX model at any working point may be regarded as a linear ARX model; therefore the quadratic programming routines can be used to solve the optimal control at each sampling interval. The nonlinear MPC design proposed is illustrated with an application to a Nitrogen Oxide (NOx) decomposition (de-NOx) process in thermal power plants.

## 2. RBF-ARX MODEL

### 2.1 Structure of the RBF-ARX model

Consider the non-stationary nonlinear SISO process with working point dependent dynamics, which may be described by a NARX model as follows

$$y(t) = f(\mathbf{X}(t-1)) + \xi(t) \quad (1)$$

$$\mathbf{X}(t-1) = [y(t-1), \dots, y(t-n_y), u(t-1), \dots, u(t-n_u), v(t-1), \dots, v(t-n_v)]^T \quad (2)$$

where  $y(t)$  is the output,  $u(t)$  is the input,  $v(t)$  is the measurable disturbance, and  $\xi(t)$  is the modeling error generally assumed as a white noise. The state  $\mathbf{X}(t-1)$  dependent AR model below is considered as a general description to model (1) (Priestly 1980):

$$y(t) = \varphi_0(\mathbf{X}(t-1)) + \Phi(\mathbf{X}(t-1))^T \mathbf{X}(t-1) + \xi(t) \quad (3)$$

The basic idea of model (3) is to implement the local linearization of the general nonlinear model (1) by introducing the locally linear AR model whose

coefficients are expressed by some functions of the state  $\mathbf{X}(t-1)$ .

Let  $w(t)$  be the process variable making the system's working point change with time, which have some direct or indirect relations with the input or output of the process; for instance, in nonlinear thermal power plants,  $w(t)$  is the load demand of the plants. Based on the basic structure of the state-dependent ARX model (3), considering the feature of the system to be controlled, and introducing the defined variable  $w(t)$  (which governs system's working point) dependent Gaussian radial basis function (RBF) networks to construct the coefficients of a state-dependent ARX model, the RBF-ARX model can be derived as follows

$$\left\{ \begin{array}{l} y(t) = \phi_0(\mathbf{W}(t-1)) + \sum_{i=1}^{n_y} \phi_{y,i}(\mathbf{W}(t-1))y(t-i) \\ + \sum_{i=1}^{n_u} \phi_{u,i}(\mathbf{W}(t-1))u(t-i) + \sum_{i=1}^{n_v} \phi_{v,i}(\mathbf{W}(t-1))v(t-i) + \xi(t) \\ \phi_0(\mathbf{W}(t-1)) = c_0^0 + \sum_{k=1}^m c_k^0 \exp\{-\lambda_k \|\mathbf{W}(t-1) - \mathbf{Z}_k^y\|_2^2\} \\ \phi_{j,i}(\mathbf{W}(t-1)) = c_{i,0}^j + \sum_{k=1}^m c_{i,k}^j \exp\{-\lambda_k \|\mathbf{W}(t-1) - \mathbf{Z}_k^j\|_2^2\} \\ \mathbf{W}(t-1) = [w(t-1), w(t-2), \dots, w(t-n_w)]^T \\ \mathbf{Z}_k^j = (z_{k,1}^j, z_{k,2}^j, \dots, z_{k,n_w}^j)^T, j = y, u, v \end{array} \right. \quad (4)$$

where  $n_y, n_u, n_v, m$ , and  $n_w$  are the orders;  $\mathbf{Z}_k^j (k=1, 2, \dots, m)$  are the centers of RBF networks;  $\lambda_k (k=1, 2, \dots, m)$  are the scaling parameters;  $c_{i,k}^j (i=1, 2, \dots, n_j; j=y, u, v; k=0, 1, 2, \dots, m)$  and  $c_k^0 (k=0, 1, 2, \dots, m)$  are the scalar weighting coefficients; and  $\|\cdot\|_2$  denotes the vector 2-norm.

The RBF-ARX model (4) is a rather general form of working-point dependent ARX style-model by adding a local mean (offset term)  $\phi_0(\mathbf{W}(t-1))$ , which is necessary to describe a non-stationary process by a global model to be identified off-line. It is clear that the local linearization of the model is a linear ARX model at any working point by fixing  $\mathbf{W}(t-1)$  in (4). It has a natural and appealing interpretation as a locally linear ARX model in which the evolution of the process at time  $(t-1)$  is governed by a set of AR coefficients  $\{\phi_{y,i}, \phi_{u,i}, \phi_{v,i}\}$ , and a local mean  $\phi_0$ , all of which depend on the 'working point' of the process at time  $(t-1)$ . Using the working point dependent functional-coefficients, especially due to the satisfactory properties of RBF networks in function approximation and in learning local variations, makes the RBF-ARX model may effectively represent the behavior of the system at

each working point. The RBF-ARX model has the advantages of the state-dependent ARX model in nonlinear dynamics description and the RBF networks in function approximation. In general, it does not require too many RBF centers compared with a single RBF network model, because the complexity of the model is dispersed into the lags of the autoregressive parts of the model.

In this paper, the RBF-ARX model is used as the internal model of the predictive controller proposed in Section 4. In order to avoid some potential problems caused by on-line parameters estimation, such as parameter divergence and too large computing burden etc., all the parameters of the RBF-ARX model are identified by off-line procedure. For the nonlinear process with working point dependent dynamics, the RBF-ARX model to be identified off-line may exhibit satisfactory fitting precision due to its capability of globally representing the nonlinear dynamics.

## 2.2 Identification of the RBF-ARX model

Identification of RBF-ARX model (4) includes the order selection and the estimation of all parameters. The orders ( $n_y$ ,  $n_u$ ,  $n_v$ ,  $m$ , and  $n_x$ ) can be selected by comparing the AIC (Akaike Information Criterion) (Akaike 1974) values under different orders and the model dynamics. First we must have a good parameter estimation method, and then we repeat the method for the models under different orders to select final model. Here main concern is focused on the parameter estimation, which is an off-line nonlinear parameter optimization problem. In general cases, the number of the linear weights is larger than that of the nonlinear centers and scaling parameters in a RBF-ARX model, so applying some classic methods to estimate all parameters simultaneously regardless of the feature of them, such as Gauss-Newton method (GNM) and Levenburg-Marquardt method (LMM) etc., may spend many computational time and may not obtain a satisfactory result.

In this paper, the structured nonlinear parameter optimization method (SNPOM) (Peng *et al.* 2001) is used to estimate the RBF-ARX model. This is a hybrid method composed of the LMM for nonlinear parameter estimation and the Least Squares method (LSM) for linear parameter estimation, but it is not a Variable Rotation method (VRM) (*i.e.* rotationally fix partial variables to optimize other variables). Therefore the SNPOM could largely accelerate the computational convergence of parameter optimization search process, especially for the RBF-ARX model with more linear weights and less nonlinear parameters.

## 3. PREDICTIVE CONTROL BASED ON RBF-ARX MODEL

### 3.1 Multi-step-ahead predictor

Rewrite the RBF-ARX model (4) as

$$\begin{cases} A_i(q^{-1})y(t) = a_{0,t} + B_i(q^{-1})u(t-1) + D_i(q^{-1})v(t-1) + \xi(t) \\ a_{0,t} = c_0^0 + \sum_{k=1}^m c_k^0 \exp\{-\lambda_k \|\mathbf{W}(t-1) - \mathbf{Z}_k^y\|_2^2\} \\ A_i(q^{-1}) = 1 + \sum_{i=1}^{n_y} \{c_{i,0}^y + \sum_{k=1}^m c_{i,k}^y \exp[-\lambda_k \|\mathbf{W}(t-1) - \mathbf{Z}_k^y\|_2^2]\} q^{-i} \\ B_i(q^{-1}) = \sum_{i=1}^{n_u} \{c_{i,0}^u + \sum_{k=1}^m c_{i,k}^u \exp[-\lambda_k \|\mathbf{W}(t-1) - \mathbf{Z}_k^u\|_2^2]\} q^{-i+1} \\ D_i(q^{-1}) = \sum_{i=1}^{n_v} \{c_{i,0}^v + \sum_{k=1}^m c_{i,k}^v \exp[-\lambda_k \|\mathbf{W}(t-1) - \mathbf{Z}_k^v\|_2^2]\} q^{-i+1} \\ \mathbf{W}(t-1) = [w(t-1), w(t-2), \dots, w(t-n_w)]^T \\ \mathbf{Z}_k^j = (z_{k,1}^j, z_{k,2}^j, \dots, z_{k,n_w}^j)^T, j = y, u, v \end{cases} \quad (5)$$

where  $q^{-1}$  is the unit delay operator. Assume that the noise sequence  $\{\xi(t) \in \mathfrak{R}\}$  in (5) satisfy

$$E\{\xi(t) | F_{t-1}\} = 0, \quad E\{\xi(t) \xi(t)^T\} = \Omega$$

here  $F_t$  denotes the  $\sigma$ -algebra generated by the data up to and including time  $t$ , and  $\Omega$  is a positive definite matrix.

**Theorem 1:** Consider the system described by the RBF-ARX model (5). Assume  $\{u(t+j-1) | j=1,2,\dots,N\}$  to be  $F_t$ -measurable, then based on the model (5) at time  $t$ , which is obtained by fixing the model parameters at instant  $t$ , the  $j(j=1,2,\dots,N)$  step ahead optimal predictive output is

$$\begin{aligned} \hat{y}(t+j|t) &= E\{y(t+j|t) | F_t\} \\ &= G'_{t,j}(q^{-1})u(t+j-1) + y_0(t+j|t) \end{aligned} \quad (6)$$

here

$$\begin{aligned} y_0(t+j|t) &= E'_{t,j}(1)a_{0,t} + F_{t,j}(q^{-1})y(t) \\ &+ H_{t,j}(q^{-1})u(t-1) + E'_{t,j}(q^{-1})D_i(q^{-1})v(t+j-1) \end{aligned} \quad (7)$$

where the polynomials  $E'_{t,j}(q^{-1})$ ,  $F_{t,j}(q^{-1})$ ,  $G'_{t,j}(q^{-1})$ , and  $H_{t,j}(q^{-1})$  are the solutions of two Diophantine equations below

$$1 = E'_{t,j}(q^{-1})A_i(q^{-1}) + q^{-j}F_{t,j}(q^{-1}) \quad (8)$$

$$E'_{t,j}(q^{-1})B_i(q^{-1}) = G'_{t,j}(q^{-1}) + q^{-j}H_{t,j}(q^{-1}) \quad (9)$$

here

$$\deg(E'_{t,j}) = j-1, \quad \deg(F_{t,j}) = n_y - 1,$$

$$\deg(G'_{t,j}) = j-1, \quad \deg(H_{t,j}) = n_u - 2.$$

**Proof:** After fixing all the coefficients of the

polynomials  $A_t(q^{-1})$ ,  $B_t(q^{-1})$ ,  $D_t(q^{-1})$ , and the local mean (offset term)  $a_{0,t}$  at instant  $t$ , model (5) at time  $t$  is a locally linearized ARX model with constant term. If taking instant  $t$  as a starting-point, based on model (5) to compute the multi-step-ahead prediction of the output, yields

at  $t+0$  step:

$$A_t(q^{-1})y(t) = a_{0,t} + B_t(q^{-1})u(t-1) + D_t(q^{-1})v(t-1) + \xi(t)$$

at  $t+1$  step:

$$A_t(q^{-1})y(t+1|t) = a_{0,t} + B_t(q^{-1})u(t) + D_t(q^{-1})v(t) + \xi(t+1)$$

⋮

at  $t+j$  step:

$$A_t(q^{-1})y(t+j|t) = a_{0,t} + B_t(q^{-1})u(t+j-1) + D_t(q^{-1})v(t+j-1) + \xi(t+j) \quad (10)$$

Introducing the unit function below

$$1(j) = \begin{cases} 1, & \text{if } j \geq 0 \\ 0, & \text{if } j < 0 \end{cases} \quad (11)$$

then the prediction function (10) can be rewritten as

$$\begin{cases} A_t(q^{-1})y(t+j|t) = a_{0,t}1(j) + B_t(q^{-1})u(t+j-1) \\ + D_t(q^{-1})v(t+j-1) + \xi(t+j), \quad j=1,2,\dots,N \end{cases} \quad (12)$$

It is an autoregressive representation. Multiply (8) by  $y(t+j|t)$  and introduce (7), (9) and (12) into the resulting expression. Noting that  $E'_{t,j}(q^{-1})1(j) = E'_{t,j}(1)$ , this yields

$$y(t+j|t) = G'_{t,j}(q^{-1})u(t+j-1) + y_0(t+j|t) + E'_{t,j}(q^{-1})\xi(t+j) \quad (13)$$

If one requires  $\{u(t+j-1) | j=1,2,\dots,N\}$  to be  $F_t$ -measurable, then equation (13) implies equation (6).  $\square$

**Remark 1:** The RBF-ARX model-based multi-step-ahead predictor (6) is different to general linear ARX model-based predictor (Clarke *et al.* 1987), because it considered the effect of local mean of the model. For a global model of describing a non-stationary nonlinear process, the offset term is necessary to the model, so the predictor (6) may be considered a more general version of multi-step-ahead predictor based on a locally linearized ARX model.

**Remark 2:** The method of solving Diophantine equations (8-9) is similar to that presented by Clarke *et al.* (1987), and hence it is omitted here. In actual

application one has to replace  $v(t+j-1)$  ( $j \geq 2$ ) by  $v(t)$  in formula (7), because the future values of disturbance can not be gotten.

### 3.2 The RBF-ARX model based predictive control strategy

Using vector and matrix version to represent the strategy, first define

$$\begin{aligned} \hat{\mathbf{Y}}(t) &= [\hat{y}(t+1|t), \hat{y}(t+2|t), \dots, \hat{y}(t+N|t)]^T \\ \mathbf{Y}_0(t) &= [y_0(t+1|t), y_0(t+2|t), \dots, y_0(t+N|t)]^T \\ \mathbf{U}(t) &= [u(t), u(t+1), \dots, u(t+N_u-1)]^T \\ \mathbf{Y}_r(t) &= [y_r(t+1), y_r(t+2), \dots, y_r(t+N)]^T \end{aligned}$$

where  $N$  is the prediction horizon, whereas  $N_u$  is the control horizon after which control is assumed to have no change, *i.e.*  $u(t+j) = u(t+N_u-1)$  ( $j \geq N_u$ ), and  $\mathbf{Y}_r(t)$  is the desired set-output sequence. From formula (6), yields

$$\hat{\mathbf{Y}}(t) = \mathbf{G}_t \mathbf{U}(t) + \mathbf{Y}_0(t) \quad (14)$$

$$\text{where } \mathbf{G}_t = \begin{bmatrix} g_{t,0} & & & \mathbf{0} \\ g_{t,1} & g_{t,0} & & \\ \vdots & \vdots & \ddots & \\ g_{t,N_u-1} & g_{t,N_u-2} & \cdots & g_{t,0} \\ \vdots & \vdots & \vdots & \vdots \\ g_{t,N-1} & g_{t,N-2} & \cdots & \sum_{i=0}^{N-N_u} g_{t,i} \end{bmatrix}_{N \times N_u}$$

$$G'_{t,j}(q^{-1}) = g_{t,0} + g_{t,1}q^{-1} + \dots + g_{t,j-1}q^{-j+1}$$

Consider the following optimization problem:

$$\begin{aligned} \min_{\mathbf{U}(t)} J &= \|\hat{\mathbf{Y}}(t) - \mathbf{Y}_r(t)\|_{\mathbf{R}}^2 + \|\mathbf{U}(t)\|_{\mathbf{R}}^2 \\ \text{s. t. } \mathbf{Y}_{\min} &\leq \hat{\mathbf{Y}}(t) \leq \mathbf{Y}_{\max}, \quad \mathbf{U}_{\min} \leq \mathbf{U}(t) \leq \mathbf{U}_{\max} \end{aligned} \quad (15)$$

here  $\mathbf{R} = \text{diag}\{r_1, \dots, r_{N_u}\}$  is the weighting matrix. Introduce (14) into (15), after removing the constant terms, the quadratic form of the above optimization problem could be obtained as

$$\begin{aligned} \min_{\mathbf{U}(t)} \tilde{J} &= \frac{1}{2} \mathbf{U}(t)^T (\mathbf{G}_t^T \mathbf{G}_t + \mathbf{R}) \mathbf{U}(t) + [\mathbf{Y}_0(t) - \mathbf{Y}_r(t)]^T \mathbf{G}_t \mathbf{U}(t) \\ \text{s. t. } \begin{bmatrix} \mathbf{G}_t \\ -\mathbf{G}_t \end{bmatrix} \mathbf{U}(t) &\leq \begin{bmatrix} \mathbf{Y}_{\max} - \mathbf{Y}_0(t) \\ -\mathbf{Y}_{\min} + \mathbf{Y}_0(t) \end{bmatrix}, \quad \mathbf{U}_{\min} \leq \mathbf{U}(t) \leq \mathbf{U}_{\max} \end{aligned} \quad (16)$$

The on-line optimization problem (16) may be solved by the quadratic programming routines. In the solved optimal control, just first component  $u(t)$  is used as control input. Note that this RBF-ARX model based predictive controller does not require on-line parameter estimation, because its internal model is a

global off-line estimated model.

#### 4. CASE STUDY

Fig.1 shows the structure diagram of a Nitrogen Oxide (NO<sub>x</sub>) decomposition (de-NO<sub>x</sub>) process in thermal power plants, which is used as a system to be controlled for illustrating the modeling and control method proposed in this paper. This process is nonlinear non-stationary, which has the dynamics changing with load demand of power plants (Matsumura *et al.* 1997). In fact, the working point of the process is dependent of the load, so it is also a kind of nonlinear system with working-point dependent dynamics. In Fig.1, the gain-scheduling PI-feedback and feedforward controllers are a set of conventional controllers already existing in the system, and the predictive controller to be added is used to improve control performance. The purpose of the de-NO<sub>x</sub> process control is to reduce the NO<sub>x</sub> concentration in exhaust gas in order to protect environments, for which the most commonly used technology is the selective catalytic reduction method, which is to use ammonia (NH<sub>3</sub>) to decompose NO<sub>x</sub>. The demand is to control the NO<sub>x</sub>  $y(t)$  within the environment regulation value and reduce the expensive ammonia (NH<sub>3</sub>) gas consumption as much as possible. The conventional PI controller is hard to achieve a good trade-off between control performance and ammonia consumption.

In this paper, we present a predictive control design for the process, in which the RBF-ARX model based predictive controller (RBF-ARX-MPC) works with the existing gain-scheduling PI controller in parallel as shown in Fig. 1. This may allow one to use the real input/output data of the process governed by only the PI controller to off-line identify the RBF-ARX model describing the behavior of the process. Therefore this could make the RBF-ARX-MPC easily implement in practice. The RBF-ARX model used in the MPC is

$$A_i(q^{-1})y(t) = a_0(t) + B_i(q^{-1})(u(t-1) + d_2(t-1)) + D_i(q^{-1})d_1(t-1) + \xi(t) \quad (17)$$

in which the front  $\tau - 1$  terms of coefficients of  $B_i$  are zeros in the time-delay  $\tau$ , and the RBF-style polynomials in (17) are similar to that in (6). For off-line training the model, a set of real data partially showed in Fig. 2 under only the PI control are used to identify RBF-ARX model (17) by applying the SNPOM (Peng *et al.* 2001). In the training data the varying region of the load  $x(t)$  is widest in all the measured data-set, thus the training data should include the richest process messages.

The one-step predictions of the estimated model (17) in two cases are shown in Fig. 2-3, in which the model order and some parameters are  $n_y = 4$ ,

$n_u = 11$ ,  $n_{d_1} = 6$ ,  $m = 1$ ,  $n_x = 2$ , and  $\tau = 6$  (sampling time is 20sec.). Fig. 2 and Fig. 3 illustrate the estimated RBF-ARX model used as a global dynamic model of describing the non-stationary nonlinear process has satisfactory fitting accuracy.

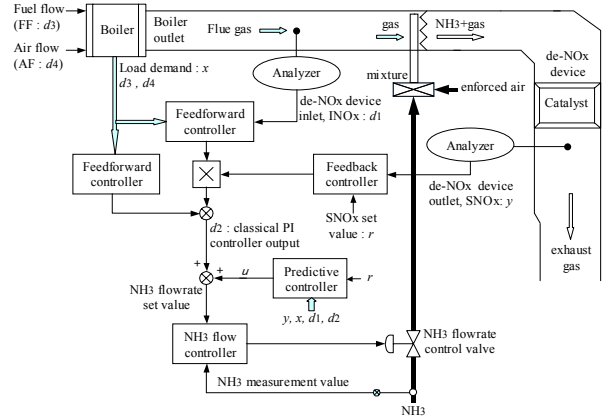


Fig. 1. The de-NO<sub>x</sub> device and the control system.

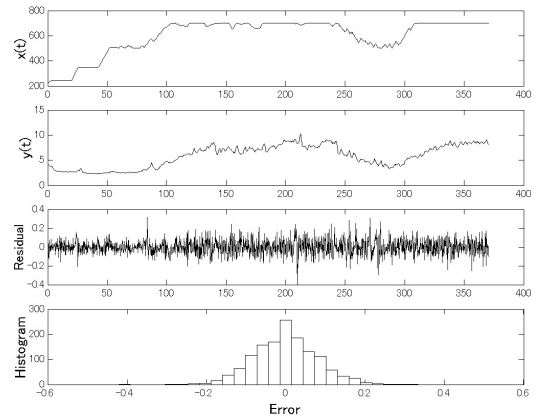


Fig. 2. The residuals and its histogram of the RBF-ARX model (17) estimated by using the training data; The unit:  $x$  /MW (Megawatt),  $y$  /ppm,  $d_1$  /ppm,  $d_2$  /kg/h.

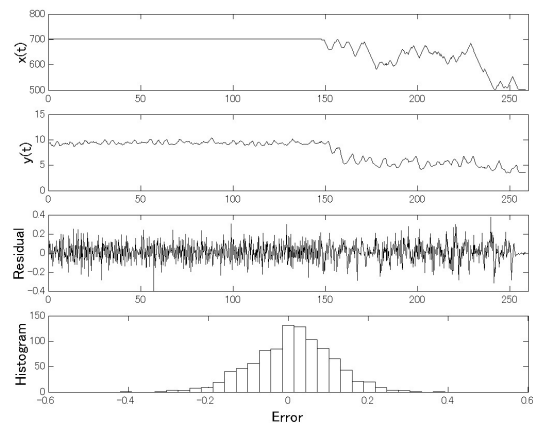


Fig. 3. The prediction precision of the estimated RBF-ARX model (17) for the test data.

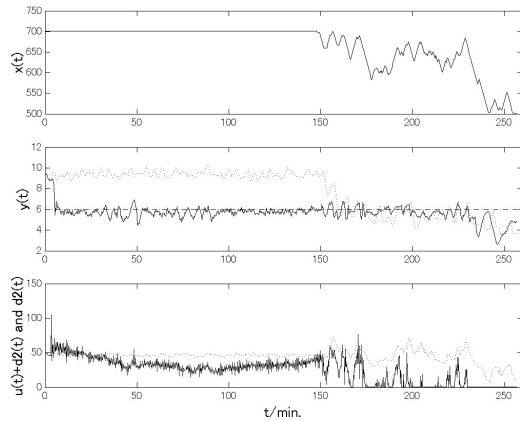


Fig. 4. Control result for the test data; Solid line: the RBF-ARX-MPC; Dotted line: the gain-scheduling-PI control; set-value of  $y(t)$  is 6ppm.

Fig. 4 shows the estimated RBF-ARX model (17) based predictive control (RBF-ARX-MPC) result for the test data. In this result, the internal model of the MPC is identified using the training data, and the simulation plant to be controlled is the identified RBF-ARX model (17) plus the model prediction residual (showed in Fig. 3) for the test data, so the output  $y(t)$  is same to the gain-scheduling PI control result if the predictive controller output  $u(t)$  is zero all over the time. Using only the PI controller, very large regulation errors occurred in the output  $y(t)$ , and overmuch ammonia ( $\text{NH}_3$ ) are consumed especially during the lower load operation due to the process nonlinearity. The RBF-ARX-MPC obviously improved the control performance and largely reduced the ammonia consumption. In Fig. 4 the ammonia consumption under the RBF-ARX-MPC-plus-PI control is 55.1% of that under only the PI control. Notes that in Fig. 4, there are also the cases that sometimes the output  $y(t)$  is very low from the set-value under the RBF-ARX-MPC-plus-PI control during some lower load operation, because the PI controller gave overmuch output ( $d_2(t)$ ) at the time, it led the  $y(t)$  to be too low from set-point, so the MPC makes the ammonia input ( $u(t) + d_2(t)$ ) almost be regulated to zero over the time. However the ammonia input is unable to be negative, so the larger negative overshoot happened.

## 5. CONCLUSIONS

For the non-stationary nonlinear systems with working-point dependent dynamics, an off-line estimated global RBF-ARX model based modeling and predictive control method was presented. The RBF-ARX model could efficiently represent the behavior of the system whose dynamics depend on the signal of deciding system working-point. The structured nonlinear parameter optimization method (SNPOM) could be used to off-line estimate the model parameters.

This paper proposed a nonlinear MPC strategy based on the global RBF-ARX model with an offset term, in which the standard quadratic programming routines could be applied to solve the optimal control problem with constraints, because the local linearization of the RBF-ARX model could be easily obtained by fixing the 'state' variable in the model on the value at instant  $t$ . The proposed MPC based on the global internal model didn't resort to on-line parameter estimation, but the GPC based on an ARX or CARIMA model for a non-stationary process has to require on-line parameter estimation, because general ARX or CARIMA model is only a local model. The simulation study on a nonlinear chemical reaction process illustrated the validity of the proposed modeling and MPC method.

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