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INTERNAL MODEL BASED SENSORLESS CONTROL OF A CLASS OF ELECTRICAL MACHINES

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Abstract: The problem of sensorless (local) speed regulation of a class of electrical machines is addressed and solved using a simple linear-time varying controller. The class, which contains permanent magnet synchronous motors, consists of all Blondel–Parks transformable machines, and of all machines whose magneto motive force can be approximated by a first harmonic Fourier expansion. The controller—which contains an internal model of the steady-state control—is able to asymptotically reconstruct the control signal necessary to achieve speed regulation, even in the presence of unknown but constant load torque. To prove global stability and boundedness of the unforced system we exploit the by now well–known passivity property of electro-mechanical systems. We work out in detail the problem of speed regulation for a permanent magnet synchronous motor, for which normalized simulations that illustrate the properties of the design are provided.

Keywords: Nonlinear control, internal model, electrical machines, BP transformation

1. INTRODUCTION

In this paper we consider the practically important, and theoretically challenging, problem of speed regulation of rotating electrical machines without measurement of mechanical coordinates—the so–called sensorless control. Since rotational transducers and their associated digital or analogue circuits give extra costs and are often complex and rather fragile—reducing the robustness of the total system—there has been an increasing interest in industry in control schemes without rotational sensors. This has triggered an intensive research activity in this area in the last few years, both, in the industrial electronics and in the automatic control communities. While some successful practical implementations have already been reported, our theoretical understanding of this difficult robust output feedback stabilization problem is far from satisfactory, and many fundamental questions are essentially open. We refer the reader to (Feemster *et al.*, 1999; Petrovic, 2001) for an overview of the recent literature. Given that high–performance controllers are readily available when position is available for measurement, it seems reasonable to try to estimate position and, in the spirit of observer and adaptive control theories, replace in the control scheme the actual position by its estimation. Broadly speaking, there are two approaches to rotor position estimation reported in the literature. The first one concentrates on estimation of the motor *back emf* and subsequent extraction of position information from this signal. For, various observers—or even open–loop integration—of the electrical and mechanical states have been proposed. There are two fundamental drawbacks to these schemes, first, that their performance critically depends on the motor parameters, which are usually not precisely known and/or change with operating conditions. [2]

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[2] Obviousl y, the rather *naive* approach of open–loop integration furthermore requi res the "knowl edge" of the ini tial

Adding on top a parameter identification algorithm leads to a nonlinear estimation problem (involving products of unknown parameters and unknown states) for which little, if at all, theory is available. Second, since the back emf term vanishes close to standstill, the performance of these algorithms is degraded at low speeds—they are actually (theoretically) ineffectual at zero speed, where the state becomes unobservable. The second approach extracts the information about rotor position exploiting the fact that the *magnetic saliency* affects the dependence of the motor inductance on the rotor position. The methods pursuing this line of research usually involve injection of an auxiliary balanced high frequency voltage signal to probe the motor electrical subsystem. Besides the obvious undesirable feature of excitation of high frequency modes—induced by the probing signal—the quality of the estimation will depend on the effective existence of rotor saliency, which is sometimes enforced modifying the rotor slots, and the availability of good models to describe this complex electromagnetic phenomenon. On the other hand, this approach does not suffer from the aforementioned problem of singularity at low speeds. Combinations of both techniques, i.e., signal injection in start–up, and back emf based algorithms at higher speeds, have also been reported. The difficulties for position estimation mentioned above are, of course, intrinsic, and stymie the behavior of all schemes hinging upon certainty–equivalence. In this paper we abandon this perspective, and propose a radically different controller structure whose motivation stems from the following observations.

(1) As a corollary of Proposition 2.5 in Chapter 2 of (Ortega *et al.*, 1998), which establishes passivity of general Euler–Lagrange systems, we have that electromechanical systems define passive operators with conjugated port variables voltage/current and load torque/speed. Thus, passivity will be preserved for all positive real controllers relating voltages and currents.
(2) For the class of machines considered here (see below), the unique control signal that keeps the speed and current errors identically equal to zero is ω_*–periodic, where ω_* is the constant desired speed. To ensure zero tracking error it is therefore *necessary* that the controller incorporates the internal model of this signal.
(3) In (Isidori, 1995, Proposition 8.8.1) it is shown that the cascade composition of a Poisson stable exosystem and a locally exponentially stable subsystem possesses a (unique) steady–state. Under the practically reasonable assumption that the load torque is periodic—or constant—we can treat this signal, together with the ω_*–periodic current reference, as the outputs of a Poisson stable exosystem. Attractivity will then follow provided the motor/controller subsystem is locally exponentially stable.

In the light of the discussion above we propose an internal model–based linear controller, whose transfer matrix is positive real, and is driven by an ω_*–periodic reference current, whose amplitude has to be tuned to ensure the existence of a steady–state, see Figure 1. It should be underscored that, even though incorporation of the internal model of the reference current is necessary for perfect output tracking, this condition is (in general) not sufficient, as the closed loop system with zero exogeneous input should be locally exponentially stable. The resulting scheme enjoys the following features.

- Given that no position or velocity estimation is required, the parameter sensitivity and low–speed misbehavior problems mentioned above are obviated.
- Passivity is preserved in closed–loop, inheriting the well–known robust stability properties of passive systems. Unfortunately, as a transfer matrix with poles on the $j\omega$ axis—like our internal model controller—cannot be *strictly* positive real we cannot conclude $\mathcal{L}_2$–stability from here. On the other hand, under some reasonable prior knowledge assumptions on the *mechanical* dynamics, we can prove that the system is locally exponentially stable, even for *arbitrarily small* speed references.
- The stabilization mechanism is based on the internal model principle that "automatically" generates the control signal required to establish the desired periodic steady–state. To fix the latter in a practical machine, it is necessary to assume that the first harmonic in a Fourier approximation of the emf gives a sufficiently close approximation of the real emf. Hence the scheme is applicable to the well–known class of Blondel–Parks transformable machines (Liu *et al.*, 1989; Ortega *et al.*, 1998), which contains the classical induction and permanent magnet synchronous motors.
- The controller has a very simple linear time–varying structure with only three tuning knobs: the gains of the proportional and the internal model terms, and the amplitude of the desired steady state current. From our stability analysis we have that, while the gains may take any positive value, the current amplitude must satisfy an algebraic constraint to guarantee the existence of the steady–state, which requires the knowledge of bounds on the load torque.

conditions of the state that we actually want to estimate! This paradoxical situation casts serious doubts on the practicality of this "idea". (Fortunately so, otherwise the whole edifice of feedback control theory will fall to crumbles.)

2. MODEL OF THE MACHINE

2.1 *Generalized electric machine*

We present now the generalized electrical machine described in (Meisel, 1996), see also (Ortega *et al.*, 1998, Chapter 9). It consists of $n_e = n_s + n_r$ windings on stator and rotor, with possible permanent magnets or a salient rotor. Ideal symmetrical phases and sinusoidally distributed phase windings are assumed. The permeability of the fully laminated cores is assumed to be infinite, and saturation, iron losses, end winding and slot effects are neglected. Only linear magnetic materials are considered, and it is further assumed that all parameters are constant. Under these assumptions, application of Gauss's law and Ampere's law leads to the following affine relationship between the *flux linkage* vector $\lambda \in I\!R^{n_e}$ and the *current* vector $i \in I\!R^{n_e}$

$$\lambda = L(\theta)i + \mu(\theta) \tag{1}$$

with $\theta \in I\!R$ the *mechanical angular position* of the rotor, and $L(\theta) = L^\top(\theta) > 0$ the $n_e \times n_e$ multiport inductance matrix of the windings. The vector $\mu(\theta)$ represents the flux linkages due to the possible existence of *permanent magnets*. Note that both $L(\theta)$ and $\mu(\theta)$ are bounded and periodic function of θ.

With the considerations above, the voltage balance equation yields

$$\dot{\lambda} + Ri = Mu \tag{2}$$

where $u \in I\!R^{n_s}$ is the vector of voltages applied to the stator windings, $R = R^\top > 0$ is the matrix of electrical resistance of the windings, and M is a constant matrix that defines the actuated coordinates. If $n_s = n_e$ then, $M = I_{n_e}$, and we say that the motor is fully actuated. On the other hand, if $n_s < n_e$, then $M = \begin{bmatrix} I_{n_s} \\ 0 \end{bmatrix} \in I\!R^{n_e \times n_s}$, and we will call the motor underactuated.

The coupling between the electrical and the mechanical subsystems is established trough the torque of electrical origin

$$\tau = \frac{1}{2} i^\top \frac{\partial L}{\partial \theta}(\theta) i + i^\top \frac{\partial \mu}{\partial \theta}(\theta). \tag{3}$$

The model is completed replacing the latter in the mechanical dynamics

$$m\ddot{\theta} = -b\dot{\theta} + \tau - \tau_L \tag{4}$$

where $m > 0$ is the rotational inertia of the rotor, $b \geq 0$ is the viscous friction coefficient, and we have introduced a term of load torque τ_L, which we will also assume constant. As shown in (Ortega *et al.*, 1998) the model (1)–(4) contains, as particular cases, induction, permanent magnet synchronous and stepping motors. The following passivity property of the generalized machine is instrumental for our analysis and can be easily proven from direct substitution. (See also Proposition 9.11 in Chapter 9 of (Ortega *et al.*, 1998) for a stronger passive feedback decomposition property.)

Lemma 1. The system (1)–(4) define a passive operator with conjugated port variables $(u, M^\top i)$ and $(-\tau_L, \dot{\theta})$ and storage function the total stored energy $H(i, \theta, \dot{\theta}) = \frac{1}{2} i^\top L(\theta) i + \frac{m}{2}\dot{\theta}^2$. More precisely, the power balance is given by the equation $\dot{H} = i^\top Mu - \dot{\theta}\tau_L - i^\top Ri - b\dot{\theta}^2$.

2.2 *Blondel–Park transformable machines*

As discussed in the introduction we propose a controller that generates in steady–state the control signal required to achieve (robust) speed regulation. To be able to explicitly characterize such control signal we must assume that, roughly speaking, the steady–state dynamics has a single harmonic behavior. To formulate this condition precisely we need to restrict the machines under consideration to the following class identified in the fundamental paper (Liu *et al.*, 1989), see also (Ortega *et al.*, 1998).

Assumption 1. There exists a *constant* matrix $U \in I\!R^{n_e \times n_e}$ such that

$$UL(\theta) - L(\theta)U = \frac{dL}{d\theta}(\theta) \tag{5}$$

$$RU = UR \tag{6}$$

$$U\frac{d\mu}{d\theta}(\theta) = \frac{d^2\mu}{d\theta^2}(\theta) \tag{7}$$

In this case, we say that the machine (1)–(4) is Blondel–Park (BP) transformable.

The main feature of BP transformable machines is that there exists a coordinates transformation for the current such that, in the new coordinates, the electrical dynamics (1)-(2) are *independent* of θ (but dependent on $\dot{\theta}$). To prove this fact we notice that (5) and (7) admit as unique solutions

$$L(\theta) = \mathrm{e}^{U\theta} L_0 \mathrm{e}^{-U\theta} \qquad \frac{d\mu}{d\theta}(\theta) = \mathrm{e}^{U\theta} m_0,$$

where we have defined $L_0 = L(0)$ and $m_0 = \frac{d\mu}{d\theta}(0)$. Using these identities it is easy to verify that in the new coordinates

$$z = \mathrm{e}^{-U\theta} i \tag{8}$$

(1)-(2) is transformed into

$$L_0\dot{z} + (UL_0\dot{\theta} + R)z + m_0\dot{\theta} = \mathrm{e}^{-U\theta} Mu. \tag{9}$$

The mechanical equations (3), (4), in turn, take the simple form

$$m\ddot{\theta} = -b\dot{\theta} + z^\top U L_0 z + m_0^\top z - \tau_L. \quad (10)$$

In the sequel we will restrict our attention to machines described by (9), (10). Obviously, this so–called dq model cannot be used for sensorless controller design, because the inverse transformation—that depends on θ—is unknown. This description will be used only to analyze the steady–state behavior of the machine.

Remark 1. In Lemma D.1 of (Ortega *et al.*, 1998) it is shown that if $\frac{dL}{d\theta}(\theta) \neq 0$ then $U = -U^\top$. Hence, the coordinate transformation is a rotation. This property will be instrumental for our further developments.

Remark 2. The underlying fundamental assumption for the machine to be BP transformable is that the windings are sinusoidally distributed (Youla and Bongiorno, 1980), giving a sinusoidal air-gap emf and sinusoidally varying elements in the inductance matrix. For a practical machine, this means that the first harmonic in a Fourier approximation of the emf must give a sufficiently close approximation of the real emf. Examples of machines in which higher order harmonics must be taken into account, are the square wave brushless DC motors, and machines with significant saliency in the air gap. The squirrel-cage induction machine is an example of a machine where the squirrel-cage rotor with non-sinusoidally distributed emf is replaced by an equivalent fictitious sinusoidally wound rotor for analytical purposes, without introducing detrimental effects to controller design. It must be noted, however, that the proposed design method can be applied also to the control of non-BP transformable machines. For simplicity, this case is treated in a separate report.

2.3 *An example*

The dynamics of a 3–phase permanent magnet synchronous motor may be approximated by (1)–(4), with $n_e = 2$, $n_s = 2, n_r = 0$, $R = R_s I_2$, with $R_s > 0$ the stator resistance, $M = I_2$, and

$$L(\theta) = \begin{bmatrix} L_2 + L_1 \cos(2n_P\theta) & L_1 \sin(2n_P\theta) \\ L_1 \sin(2n_P\theta) & L_2 - L_1 \cos(2n_P\theta) \end{bmatrix}$$

$$\mu(\theta) = \Phi \begin{bmatrix} \cos(n_P\theta) \\ \sin(n_P\theta) \end{bmatrix}$$

where Φ is the DC component of the flux harmonic expansion, and n_P denotes the number of pole pairs. The machine is BP transformable with $U = n_P J$, where $J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, yielding the rotation matrix

$$\mathrm{e}^{n_P\theta J} = \begin{bmatrix} \cos(n_P\theta) & -\sin(n_P\theta) \\ \sin(n_P\theta) & \cos(n_P\theta) \end{bmatrix} \quad (11)$$

The rotated model (9) becomes then

$$\begin{bmatrix} L_d & 0 \\ 0 & L_q \end{bmatrix} \dot{z} + \begin{bmatrix} R_s & -L_q n_P \dot{\theta} \\ L_d n_P \dot{\theta} & R_s \end{bmatrix} z + \begin{bmatrix} 0 \\ \Phi n_P \end{bmatrix} \dot{\theta} = \mathrm{e}^{-n_P\theta J} u$$

where we defined $L_d = L_1 + L_2$ and $L_q = L_2 - L_1$. The torque equation becomes $\tau = n_P[(L_d - L_q)z_2 z_1 + \Phi z_2]$. Now, if we denote $[i_d, i_q]^\top = z$, $[v_d, v_q]^\top = \mathrm{e}^{-n_P\theta J}u$, $\omega = n_P\dot{\theta}$ we, of course, recover the classical dq model of the synchronous motor (Krause, 1986).

3. CONTROLLER STRUCTURE

3.1 *Passivity*

Motivated by the passivity properties discussed in Lemma 1 we consider the sensorless controller structure depicted in Figure 1, where the transfer matrix $W(s) \in \mathbb{R}(s)^{(n_s+1)\times(n_s+1)}$ is of the form

$$W(s) = \begin{bmatrix} W_1(s) & 0 \\ 0 & 0 \end{bmatrix}$$

with $W_1(s) \in \mathbb{R}(s)^{n_s \times n_s}$ positive real, and $M^\top i_*(t)$ is the current reference that we will define below. That is, we propose a controller

$$\dot{\chi} = A\chi + BM^\top(i_* - i)$$
$$u = C\chi + DM^\top(i_* - i) \quad (12)$$

where $\chi \in \mathbb{R}^{n_c}$ and $W_1(s) = C(sI - A)^{-1}B + D$. From Lemma 1 and positive realness of $W(s)$ we

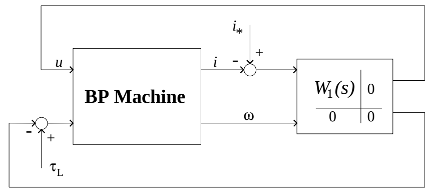

Fig. 1. Block diagram of the sensorless control.

can easily establish passivity of the closed–loop. Indeed, consider the storage function

$$V(x) = H(i, \theta, \dot{\theta}) + \frac{1}{2}\chi^\top P \chi$$

where the state vector is $x = [i^\top, \theta, \dot{\theta}, \chi^\top]^\top$, and $P = P^\top > 0$ is the solution of the Kalman–Yakubovich–Popov algebraic equations $PA + A^\top P = -L^\top L$, $PB = C^\top - L^\top S$ and $S^\top S = D^\top + D$, for some matrices L, S. After some standard calculations we obtain the dissipation inequality

$$\int_0^t [i_*^\top(t')Mu(t') - \tau_L\omega(t')]dt' \geq$$
$$\int_0^t [i^\top(t')Ri(t') + b\omega^2(t')]dt' + V[x(t)] - V[x(0)],$$

that establishes the claim. From this inequality, and the fact that $V(x)$ is radially unbounded, we also conclude that all trajectories of the *unforced* system, i.e., with $i_* = \tau_L = 0$, are bounded. Unfortunately, our passivity–based analysis cannot proceed any further because, as we will explain below, to achieve the control objective the transfer matrix $W(s)$ must have poles in the $j\omega$ axis, hence it cannot be *strictly* positive real.

3.2 *Zero dynamics*

As discussed in the introduction, we adopt in this paper an internal model based approach, that "automatically" generates the control signal that drives the trajectories towards the desired steady–state. In this case, we are interested in steady-states corresponding to speed regulation and stator current tracking. Instrumental for the characterization of the control signal that achieves this objective is the study of the zero dynamics of the machine with respect to the *outputs*

$$y = \begin{bmatrix} \omega - \omega_* \\ z - z_* \end{bmatrix} \qquad (13)$$

where ω_*, z_* are the speed and current references, respectively. It is clear from (9), and the fact that $\mathrm{e}^{-U\theta}$ is full rank, that this study is relatively simple for fully actuated motors, as we only have to consider the solutions, $z_* \in I\!R^{n_e}$, of the quadratic algebraic equation

$$z_*{}^\top U L_0 z_* + m_0^\top z_* - \tau_L - b\omega_* = 0 \qquad (14)$$

for a given speed reference ω_*. The result can be easily summarized as follows.

Lemma 2. Consider the *fully actuated* BP transformable machine (1)–(4). Assume that, for the given ω_*, there exists a constant reference current $z_* \in I\!R^{n_e}$ solution of (14). Then, the zero dynamics of the machine is simply $\dot{\theta} = \omega_*$. Furthermore, the *only* control that makes the set $\{y = 0\}$ invariant is $u_* = \mathrm{e}^{U(\omega_* t + \theta(0))}[(UL_0\omega_* + R)z_* + m_0\omega_*]$.

In words, Lemma 2 states that, if the machine (1)–(4) admits an equilibrium at the given desired speed, then a *necessary* condition for perfect regulation is that the control signal converges to an ω_*–periodic signal. This observation, together with the discussion on passivity of the previous subsection, motivates the choice $A = \omega_* U$ in the controller (12). As discussed in (Ortega *et al.*, 1998; Marino *et al.*, 1993; Ortega *et al.*, 2001) the zero dynamics of underactuated machines is far more complicated, hence sensorless control of this class of machines—in particular, for induction motors—will be discussed in a separate report.

4. MAIN RESULT

In this section we show that the controller (12) solves (locally) the sensorless speed regulation problem. For, note that the controller (12), with $i_* = \mathrm{e}^{-U\omega_* t} i_0$, for some constant vector i_0, can be written, in the new variable $\eta = \mathrm{e}^{-U\theta}\chi$ as

$$\begin{aligned} \dot{\eta} &= -\dot{\theta}U\eta + \mathrm{e}^{-U\theta}(A\mathrm{e}^{U\theta}\eta + BM^\top \mathrm{e}^{U\theta}(z_* - z)) \\ u &= C\mathrm{e}^{U\theta}\eta + DM^\top \mathrm{e}^{U\theta}(z_* - z) \end{aligned} \qquad (15)$$

where z is as defined in (8), $z_* = \mathrm{e}^{-U\theta} z_0$ and z_0 is a constant vector.

Proposition 1. Consider the system (9)-(10) and the controller (15). Suppose A, B, C and D are such that

- the transfer matrix $W_1(s) = C(sI - A)^{-1}B + D$ is positive real;
- A, $BM^\top$, MC and $MDM^\top$ commute with $\mathrm{e}^{-U\theta}$;
- the spectrum of A has eigenvalues at $\pm j\omega_*$ and $\omega_* \neq 0$ and sufficiently small;
- the following matrix is Hurwitz:

$$\begin{bmatrix} -L_0^{-1}(R + MDM^\top) & -L_0^{-1}m_0 & L_0^{-1}MC \\ m_0^\top/m & -b/m & 0 \\ -BM^\top & 0 & A \end{bmatrix}.$$

Then the closed loop system (9)-(10)-(15) with $z_* = 0$ and $\tau_L = 0$ is globally stable. Moreover, for any sufficiently small constants z_* and τ_L the state of the closed loop system (9)-(10)-(15) converge to a well defined steady state $(z_{ss}, \dot{\theta}_{ss}, \eta_{ss})$ with $z_{ss} = z_*$. Moreover, if z_* and τ_L are such that

$$z_*^\top U L_0 z_* + m_0^\top z_* - \tau_L - b\omega_* = 0, \qquad (16)$$

then $\dot{\theta}_{ss} = \omega_*$.

Proof. Global stability has been proved in Subsection 3.1. By Hypothesis the closed loop system (9)-(10)-(15) is locally exponentially stable. Hence, for any sufficiently small input signal there is a unique well defined steady state (Isidori, 1995). Moreover, by the structure of the controller, it is obvious that $z_{ss} = z_*$. Finally, by the results in Section 3, it is obvious that $\dot{\theta}_{ss} = \omega_*$, provided condition (16) holds. ◁

Remark 3. Note that conditions (16) implies that the amplitude of the reference signal i_*, or equivalently $||z_*||$ should be sufficiently large. In fact, a simple but tedious computation shows that equation (16) can always be satisfied (by some z_*) provided that $||UL_0 z_*|| \geq \tau_L$.

Remark 4. The conditions imposed by Proposition 1 on the matrices A, B, C and D of the controller are not very strong. Note, in particular, that if $n_c = n_s = n_e$ the selection $A = \omega_* U$, $B = bI$, $C = cI$ and $d = dI$, with $b > 0$, $c > 0$ and $d > 0$ is such that the first three conditions of Proposition 1 hold.

5. SIMULATIONS

In this section we discuss the application of the general theory developed so far to the sensorless control problem for the permanent magnet synchronous motor described in Section 2.3. Assume (for simplicity) $n_P = 2$ and consider the control law (12) with $A = \mathrm{diag}(\omega_* J, \omega_* J)$, $B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}^T$, $C = B^\top$ and $D = 0$, and $i_* = 10[\cos(\omega_* t) \ \sin(\omega_+ t)]^\top$. The amplitude of i_* has been selected assuming that the bound $\|\tau_L\| \leq 10$ is known. Simulations (shown in Figure 2) on a normalized[3] machine have been run. In the interval $[0, 50)$ we have selected $\omega_* = 1$ and $\tau_L = 0$. In the interval $[50, 100)$ we have set $\omega_* = 1$ and $\tau_L = 10$, whereas in the interval $[100, 150)$ we have set $\omega_* = -1$ and $\tau_L = -10$. Finally, in the interval $[150, 200]$ we have set $\omega_* = 1/4$ and $\tau_L = 0$. Note that the speed achieves the desired value, the constant torque disturbances are completely rejected, and, even for low speeds, no loss of performance is observed.

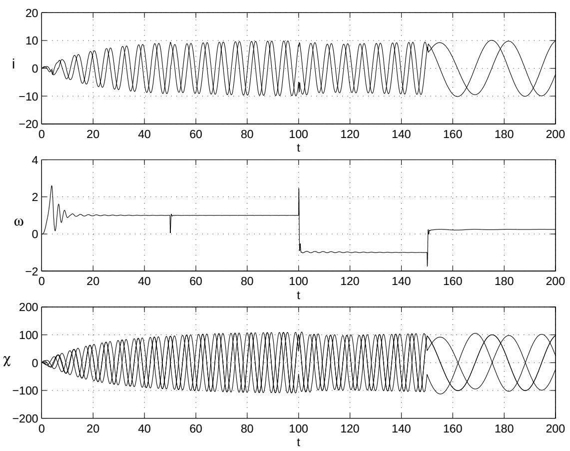

Fig. 2. Simulations results. Time history of the currents (top), of the speed (center) and of the controller states (bottom).

6. CONCLUSIONS

The problem of sensorless speed regulation for a class of electrical machines has been addressed. It is shown that asymptotic speed regulation and exact rejection of constant torque disturbances can be (locally) obtained by means of a linear control law, which exploits the passivity properties of the machine and the internal model principle. The use of these properties implies that the performance of the closed loop system does not degrade at slow speed. In fact, no attempt is done to reconstruct unmeasured states, which become difficult to observe at slow speed, and the control mechanism does not rely on the classical certainity equivalence approach.

The general theory has been developed for the class of machines having magneto motive force that can be approximated by a first harmonic Fourier expansion. However, this property is not necessary for the derivation of the main results, and more general machines, such as square wave brushless DC motors, can be studied (and controlled) using similar considerations.

Applications of the theory to the problem of sensorless speed regulation, in the presence of constant torque disturbances, for a permanent magnet synchronous motor have been detailed, and some preliminary simulations have been displayed. It is worth mentioning that the control law possesses only a few tuning parameters, whose influence on the behavior of the closed loop system can be easily assessed, via experiments or simulations.

Further studies on the problem of sensorless control of induction motors, and in general underactuated machines, are in progress, and will be reported in a separate paper.

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[3] The parameters have been selected as $L_2 = 1$, $L_1 = 1/5$, $R_s = 10$, and $\Phi = 1$.