15th Triennial World Congress, Barcelona, Spain

SUFFICIENT CONDITIONS TO FAULT ISOLATION IN NONLINEAR SYSTEMS: A GEOMETRIC APPROACH

C. Join, J-C. Ponsart, D. Sauter

Centre de Recherche en Automatique de Nancy (CRAN)
CNRS UMR 7039
Université Henri Poincaré, Nancy 1, BP 239,
F-54506 Vandœuvre-lès-Nancy Cedex
Fax : +33 383 912 030, Tel. : +33 383 912 069
email: cedric.join@cran.uhp-nancy.fr

Abstract: In this paper, a differential geometric approach to solve the problem of fault detection and isolation in nonlinear systems is presented. New conditions giving a solution for fault isolation problem are proposed. These conditions are less restrictive than previous work and are based on the analysis of the solution of the fundamental problem of residual generation given by De Persis and Isidori. *Copyright© 2002 IFAC*

Keywords: nonlinear system - diagnosis - detection - isolation - geometric approach.

1. INTRODUCTION

The problem of fault detection and isolation in dynamical systems is the problem of generating diagnostic signals sensitive to the occurrence of faults. Using the process signals assumed to be available for measurements (control inputs and outputs), a fault can be considered as an input acting on the system. The diagnosis must be able to detect as well as isolate this particular input from all other inputs (control, other faults and perturbations) affecting the system behavior. Massoumnia et *al.* (Massoumnia *et al.*, 1989) have shown that the problem can be addressed and successfully solved in a general setting for linear systems. Later, De Persis and Isidori ((De Persis and Isidori, 1999) and (De Persis and Isidori, 2000)) have realized the extension of this solution for nonlinear systems. This problem turns out to correspond to the solution of the dual problem of noninteracting control by means of dynamic feedback. This problem has been dealt with Isidori and *al.* in (Isidori, 1995) and (Isidori *et al.*, 1981). In addition, we consider the case in which the residual is the output signal generated by a filter which is designed as an observer. In common practice ((De Persis and Isidori, 2001), (Frank *et al.*, 2000), (Hammouri *et al.*, 1999) and (Schreier *et al.*, 2000)), a bank of observers is used. But it is not the only way of proceeding since only one filter can allow the detection and isolation of several faults (Join *et al.*, 2002). However the purpose of this contribution is not the filter design but the explanation of the sufficient conditions to fault isolation. To achieve the objective (to relax the explanation sufficient conditions), it is assumed that faults do not occur simultaneously. Moreover, if sufficient conditions are respected, it is possible to design a filter or a banc of filters to fault isolation. The resulting residual is exactly decoupled from the control and allows the distinction of the fault occurring among other eventual faults. The main innovation in the present paper is the integration of a structural analysis (based on work of Cassar et *al.* ((Cassar *et al.*, 1994)) in the linear case and recently by Staroswiecki and Comtet-Varga in (Staroswiecki and Comtet-Varga, 2001)) for nonlinear systems using a geometric approach. However, there exists other development with an analytical approach

(with Lie algebra in particular) as in (Frank and Ding, 1997) (carried out by Frank et *al.*).
The paper is organized as follows. Section 2 introduces the problem of residual generation. We recall that this paper deals with nonlinear systems and nonlinear filter (realized according to output injection). Section 3 concerns fault detectability and fault isolability conditions. These geometric conditions are based on a structural analysis. The different steps are described, in section 4, to carry out two examples, chosen to highlight the interest of this new method. Finally, some conclusions are drawn in section 5.

2. PROBLEM STATEMENT

In this paper, systems of the following form are considered:

$$\Sigma_{NL}: \begin{cases} \dot{x} = f_0(x) + \sum_{i=1}^{m} f_i(x)u_i + \sum_{j=1}^{q} p_j(x)w_j \\ y = h(x) \end{cases} \quad (1)$$

with $x(t) \in \mathcal{X} = \mathbb{R}^n$, $u(t) \in \mathcal{U} = \mathbb{R}^m$, $y(t) \in \mathcal{Y} = \mathbb{R}^p$ and $w(t) \in \mathcal{W} = \mathbb{R}^q$ are respectively states, inputs, outputs and faults. The functions $f_*(\bullet)$, $h(\bullet)$, $p_*(\bullet)$ are matrix-valued differentiable ($\mathcal{C}^\infty$) and all of appropriate dimensions.
We shall also find it more convenient to represent sensor failures by pseudoactuator failures (see (Massoumnia *et al.*, 1989) and (Park *et al.*, 1994)). The considered faults represent both actuator and sensor faults.
About system (1), we can refer to (Isidori, 1995), (Sontag, 1998) and (Whonam, 1985), for the definitions of the observability subspace ∇_{obs} and the unobservability subspace ∇_{inobs} (i.e. $\nabla_{inobs} = (\nabla_{obs})^\perp$).
The main problem addressed in the paper is to give sufficient conditions to fault isolation using filter (which is throughout referred to as the residual generator) modelled by equations of the form:

$$\Sigma_{FD}: \begin{cases} \dot{z} = \{f_0(z) - \Psi_1(z, y_z)\} + \sum_{i=1}^{m} \{f_i(z) - \Psi_{2,i}(z, y_z)\}u_i \\ \qquad + \{\Psi_1(z, y) + \sum_{i=1}^{m} \Psi_{2,i}(z, y)u_i\} \\ y_z = h(z) \end{cases} \quad (2)$$

where $\Psi_*(\bullet) = \Psi_1(\bullet) + \sum_{i=1}^{m} \Psi_{2,i}(\bullet)u_i$ represents the output injection.
Existence conditions of a solution of the Fundamental Problem of Residual Generation (F.P.R.G.) are shown for linear systems in (Massoumnia *et al.*, 1989) and represent one solution of this problem. In (De Persis and Isidori, 2000), existences condition are given for nonlinear systems. But these conditions are very restrictive and not necessary to realize diagnosis (fault detection and isolation). Therefore, the objective of this paper is to reduce these conditions.

3. SUFFICIENT CONDITIONS TO FAULT DETECTION/ISOLATION

To determine the sufficient conditions, a geometric approach is considered. For this aim, two non-decreasing sequences of distribution are used. The sequences are declined for each single fault ($\dim(Span\{p_j\}) = 1$), and are the base of a geometric interpretation (or structural analysis).

- The first sequence defines the maximal (in the sense of distribution inclusion) state subspace sensitive to the fault w_j:

$$\begin{cases} C_0^{p_j} = Span\{p_j\} \\ C_{i+1}^{p_j} = \overline{C}_i^{p_j} + \sum_{k=1}^{m} [f_k, \overline{C}_i^{p_j}] \end{cases} \quad (3)$$

with $\overline{C}_i^{p_j}$ denotes the *involutive closure* of the distribution $C_i^{p_j}$, i.e. if τ, $\sigma \in \overline{C}_i^{p_j}$ then $[\tau, \sigma] \in \overline{C}_i^{p_j}$ where $[\tau, \sigma]$ is the Lie bracket defined by:

$$[\tau, \sigma](x) = \frac{\partial \sigma}{\partial x}(x)\tau(x) - \frac{\partial \tau}{\partial x}(x)\sigma(x) \quad (4)$$

The stop conditions of the previous sequence are:

$$\left. \begin{array}{l} C_i^{p_j} = C_{i+1}^{p_j} \\ dim(Span\{C_i^{p_j}\}) = n \end{array} \right\} \Rightarrow C_*^{p_j} = C_i^{p_j} \quad (5)$$

$C_*^{p_j}$ expresses the fault w_j propagation within nonlinear states, i.e. the occurrence of fault w_j affects the state subset $C_*^{p_j}$.

- The second non-decreasing sequence defines the smallest (in the sense of distribution inclusion) state subspace sensitive to the fault w_j:

$$\begin{cases} S_0^{p_j} = Span\{p_j\} \\ S_{i+1}^{p_j} = \overline{S}_i^{p_j} + \sum_{k=1}^{m} [f_k, \overline{S}_i^{p_j} \cap Ker\{dh\}] \end{cases} \quad (6)$$

with the same stop conditions as (5), the distribution solution is denoted $S_*^{p_j}$. Consequently, the new distribution $(S_*^{p_j})^\perp$ represents the maximal (in the sense of distribution inclusion) state subspace insensitive to w_j modulo an output injection. These notions have been recently introduced in (Isidori *et al.*, 1981), (De Persis and Isidori, 1999) and (De Persis and Isidori, 2000), which the reader can refer to, for more information. In (De Persis and Isidori, 2000), the dual non-decreasing sequence of codistribution is also defined.

The difference between the two sequences (3) and (6) is due to the output injection used here, that is to say:

- $C_*^{p_j}$ is the **"greatest"** state subset sensitive to the fault (output injection is not used),
- $S_*^{p_j}$ is the **"smallest"** state subset sensitive to the fault (by using an output injection).

Remark : The output injection does not modify the fault propagation, then $S_*^{p_j} \subseteq C_*^{p_j}$. We can add that, we are sure that $S_*^{p_j}$ is sensitive to the considered fault for all output injection. In the particular case of no detectability fault ($C_*^{p_j} \subset \nabla_{inobs}$), the two distributions are equal.

Before explaining if faults are isolable, a necessary condition for fault isolation is detectability of all faults. It is the interest of the next paragraph, because the considered systems (1) are not necessarily observable (i.e. $\nabla_{inobs} \neq \{0\}$).

3.1 *Sufficient conditions to fault detection*

The first stage is to know if fault w_j is detectable. For this aim, a new distribution is used:

$$\Xi^{p_j} = \nabla_{obs} - Span\{(C_*^{p_j})^{\perp}\} \qquad (7)$$

A fault w_j acts on at least one output if and only if $\Xi^{p_j} \neq 0$. **Conclusion All faults are detectable if and only if the inequality $\Xi^{p_j} \neq \{0\}$ is verified $\forall$j.**

Theorem 1. The previous condition $\Xi^{p_j} \neq \{0\}$ (with $\forall j$) is:

(1) a necessary and sufficient condition, if $w_j \in \mathbb{R}^{\mathbf{1}}$ as is defined in system (1)
(2) a necessary condition, if $w_j \in \mathbb{R}^{\mathbf{k}}$ (with $\mathbf{k} > \mathbf{1}$)

to achieve the detectability of all faults.

Proof of Thorem 1.

(1) the first point of the theorem is trivial because: if $w_j \in \mathbb{R}^{\mathbf{1}}$ then the fault w_j is scalar and $(C_*^{p_j})^{\perp}$ represents the state subspace insensitive to this fault. It is a sufficient and necessary condition to know if this scalar input acts on outputs. That is to say that the observable state subspace is not included in the insensitive part. That amounts to $\Xi^{p_j} \neq \{0\}$.
(2) the second point is more difficult because: if $w_j \in \mathbb{R}^{\mathbf{k}}$ (with $\mathbf{k} > \mathbf{1}$) and $\Xi^{p_j} \neq \{0\}$, we don't know if all components (the k components) of the vector w_j are represented in Ξ^{p_j}. It is possible that the state subspace sensitive to one (or several) part(s) of vector w_j is included in ∇_{inobs}. In this case this (or these) part(s) occurring is (are) not detectable, then the condition is not sufficient in this case. But it is necessary for the same reason as (1).

Remark : In the particular case of observable systems ($\dim(Span\{\nabla_{obs}\}) = n$, $\nabla_{inobs} = \{0\}$), all faults are detectable because $\Xi^{p_j} \neq \{0\}$ with $C_*^{p_j} \neq \{0\}\ \forall j$.

3.2 *Sufficient conditions to fault isolation*

The second stage is the determination of isolability conditions. We assume that the first step is verified and with this assumption, we are sure that each fault affects at least one output. But fault detectability is not equivalent to fault isolability. Indeed, it is conceivable that several faults act on the same outputs.
To find the sufficient conditions to fault isolation, a structural analysis is necessary. This analysis is based on (Cassar *et al.*, 1994) and (Gertler, 1998) works and concerns the structure matrix of the residuals. We recall that a residual is a nonlinear function of outputs. It is the object of this new definition.

Definition 1. A co-distribution (Δ) is said "*reconstructible*" if, and only if, it exists $\Gamma(y)$ such that:

$$\frac{\partial \Gamma \circ h(x)}{\partial x} = \Delta$$

The set of reconstructible co-distribution is denoted by γ and we can add that $\gamma \subseteq Span\{dh\}$.
Remark : In the linear case, the equality $\gamma = Span\{dh\}$ is always verified.

Then, the existence conditions of an isolation filter are bounded by the output injection and output function (composing the residual vector) choices. These choices are based on a structural study in which each distribution sensitive to one fault ($S_*^{p_j}$) is examined. ($S_*^{p_j}$) is used instead of ($C_*^{p_j}$), because ($C_*^{p_j}$) can to be reduce (in sense of dimension) by output injection. That is to say that this previous distribution must be insensitive to the greatest number of other faults. Indeed, the more significant this number is, the bigger the number of simultaneous faults which can be isolate is. This criterion is transformed in a mathematical problem ((8) and (10)) with the following binary matrix:

$$\mathcal{A}^{p_j} = \begin{bmatrix} (\mathcal{A}^{p_j})_1^1 & (\mathcal{A}^{p_j})_1^2 & \cdots & (\mathcal{A}^{p_j})_1^{q-1} \\ (\mathcal{A}^{p_j})_2^1 & (\mathcal{A}^{p_j})_2^2 & \cdots & (\mathcal{A}^{p_j})_2^{q-1} \\ \vdots & \vdots & \vdots & \vdots \\ (\mathcal{A}^{p_j})_{2^{q-1}-1}^1 & (\mathcal{A}^{p_j})_{2^{q-1}-1}^2 & \cdots & (\mathcal{A}^{p_j})_{2^{q-1}-1}^{q-1} \end{bmatrix} \qquad (8)$$

In this paper, the following notation is used: $(\square)_i^j$ denotes the element at the i^{th} row and the j^{th}

column of the matrix □. In the particular case of a distribution (resp. co-distribution), the index j (resp. i) is not necessary.
The value of $(\mathcal{A}^{p_j})_i^k$ is given according to two choices:

- $(\mathcal{A}^{p_j})_i^k = 1$, if and only if $(S_*^{p_k})^\perp \subseteq (S_*^{p_j})^\perp$ and if the inclusion is not tested,
- $(\mathcal{A}^{p_j})_i^k = 0$, if and only if $(S_*^{p_k})^\perp \nsubseteq (S_*^{p_j})^\perp$, it exists at least one distribution in $(S_*^{p_k})^\perp$ not included in $(S_*^{p_j})^\perp$.

therefore $(\mathcal{A}^{p_j})_i^k \in \{0,1\}$ and $k \in \{1,\cdots,q-1\}$. **We can add that $\mathbf{i \in \{1,\cdots,2^{q-1}-1\}}$, because all combinations of intersections between the different distributions are tested**.

Example
A four faults case (q=4) is considered and the fault p_2 (j=2) is studied. In this particular case the matrix (8) becomes:

$$\mathcal{A}^{p_2} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ & \vdots & \\ 1 & 1 & 1 \end{bmatrix} \begin{array}{l} \Leftarrow (S_*^{p_1})^\perp \nsubseteq (S_*^{p_2})^\perp \\ \Leftarrow (S_*^{p_3})^\perp \nsubseteq (S_*^{p_2})^\perp \\ \Leftarrow (S_*^{p_4})^\perp \subseteq (S_*^{p_2})^\perp \\ \Leftarrow (S_*^{p_1})^\perp \cap (S_*^{p_3})^\perp \subseteq (S_*^{p_2})^\perp \\ \vdots \\ \Leftarrow (S_*^{p_1})^\perp \cap (S_*^{p_3})^\perp \cap (S_*^{p_4})^\perp \subseteq (S_*^{p_2})^\perp \end{array} \quad (9)$$

The matrix $\mathcal{A}^{p_2}$ is composed of 3 columns and $2^3-1=7$ rows, moreover it is easy to deduce the last three rows from the 3^{rd} row.

Note that $A_{\beta j}^{p_j}$ is a solution of the following criterion:

$$\beta j = \min_i \left(\sum_{k=1}^{q-1} (\mathcal{A}^{p_j})_i^k \right) \quad (10)$$

then the set of solutions $(\mathcal{A}_{\beta j}^{p_j})$ can be written as a set of row vectors according to the solution number:

$$\mathcal{A}_{\beta j}^{p_j} = \begin{bmatrix} (A_{\beta j}^{p_j})_1 \\ (A_{\beta j}^{p_j})_2 \\ \vdots \\ (A_{\beta j}^{p_j})_{\epsilon_{p_j}} \end{bmatrix} \quad (11)$$

where ϵ_{p_j} is the number of solutions of the equation criterion (10).
A solution (a subspace of co-distributions) can be adopted only if it belongs to the subset γ.

Matrix (12) synthesizes the solution(s) for each fault, and allows to state isolation condition.

$$\mathcal{A} = \begin{bmatrix} [1] & (\mathcal{A}_{\beta 1}^{p_1})^1 & \cdots & \cdots & (\mathcal{A}_{\beta 1}^{p_1})^{q-1} \\ (\mathcal{A}_{\beta 2}^{p_2})^1 & \ddots & (\mathcal{A}_{\beta 2}^{p_2})^2 & \cdots & (\mathcal{A}_{\beta 2}^{p_2})^{q-1} \\ \vdots & \ddots & [1] & \ddots & \vdots \\ (\mathcal{A}_{\beta q-1}^{p_{q-1}})^1 & \cdots & (\mathcal{A}_{\beta q-1}^{p_{q-1}})^{q-2} & \ddots & (\mathcal{A}_{\beta q-1}^{p_{q-1}})^{q-1} \\ (\mathcal{A}_{\beta q}^{p_q})^1 & \cdots & \cdots & (\mathcal{A}_{\beta q}^{p_q})^{q-1} & [1] \end{bmatrix} \quad (12)$$

where [1] is a column vector of ones.

Conclusion
if $\mathbf{dim(Span\{\mathcal{A}\}) = q}$ then it is possible to isolate all faults.

Remark : If $\beta j = 0\ \forall j$ then it is possible to build an isolation filter (as previously) with a **diagonal residual structure. It is equivalent with saying that, in this case, the filter is solution of the F.P.R.G. (De Persis and Isidori, 1999).** It is also the most constraining structure but the most interesting, because all faults can be detected and isolated at the same time.

Thus a residual vector can be generated (according to the γ inclusion) by $R = \Pi(y) - \Pi(y_z) = \Pi \circ h(x) - \Pi \circ h(z)$ and to conclude matrix $\mathcal{A}$ can be interpreted by the following signature table (Table 1.).

4. EXAMPLES

In this section, we present two examples inspired from (Fossard and Normand-Cyrot, 1995) to highlight the interest of these new existence conditions. In the first example, conditions developed in (De Persis and Isidori, 2000) are satisfied, i.e. it is possible to synthesize a filter with a diagonal residual structure and it is shown that these results are similar to those of the method presented in this article. In the second example, the method interest is shown, because the diagonal residual structure conditions are not respected but the diagnosis : detection and isolation are possible.

4.1 *Example 1*

The studied system is modelled by the following equations:

$$\Sigma_{NL1} : \begin{cases} \dot{x} = \begin{pmatrix} x_1 x_4 \\ x_3(1-x_4) \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & x_1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} u_1 + w_1 \\ u_2 + w_2 \end{pmatrix} \\ y = \begin{pmatrix} x_1 \\ x_3 \end{pmatrix} \end{cases} \quad (13)$$

with, $x(t) \in \mathcal{X} = \mathbb{R}^4$, $u(t) \in \mathcal{U} = \mathbb{R}^2$, $y(t) \in \mathcal{Y} = \mathbb{R}^2$ and $w(t) \in \mathcal{W} = \mathbb{R}^2$ are respectively states, inputs, outputs and the unknown disturbances. This system is not observable and the inobservable subspace is $\nabla_{inobs} = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \neq \{0\}$. Several distribution calculations are executed according to non-decreasing sequences (3) and (6).

$$C_*^{P_1} = Span \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -x_1 \\ x_3 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ x_1 \\ x_1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ x_1 + x_2 \\ x_1 \\ 0 \end{bmatrix} \right\}$$

<table>
<tr><td>Faults
Residuals</td><td>$\mathbf{w_1}$</td><td>$\mathbf{w_2}$</td><td>$\cdots$</td><td>$\mathbf{w_q}$</td></tr>
<tr><td rowspan="3">R_1</td><td rowspan="3">sensitive to $\mathbf{w_1}$</td><td>insensitive to $\mathbf{w_2}$
if $(A^{p_1}_{\beta 1})^1_1 = 0$ else sensitive</td><td rowspan="3">$\cdots$</td><td>insensitive to $\mathbf{w_q}$
if $(A^{p_1}_{\beta 1})^q_1 = 0$ else sensitive</td></tr>
<tr><td>$\vdots$</td><td>$\vdots$</td></tr>
<tr><td>insensitive to $\mathbf{w_2}$
if $(A^{p_1}_{\beta 1})^1_{\epsilon_{p_1}} = 0$ else sensitive</td><td>insensitive to $\mathbf{w_q}$
if $(A^{p_1}_{\beta 1})^q_{\epsilon_{p_1}} = 0$ else sensitive</td></tr>
<tr><td rowspan="3">R_2</td><td>insensitive to $\mathbf{w_1}$
if $(A^{p_2}_{\beta 2})^1_1 = 0$ else sensitive</td><td rowspan="3">sensitive to $\mathbf{w_2}$</td><td rowspan="3">$\cdots$</td><td>insensitive to $\mathbf{w_q}$
if $(A^{p_2}_{\beta 2})^q_1 = 0$ else sensitive</td></tr>
<tr><td>$\vdots$</td><td>$\vdots$</td></tr>
<tr><td>insensitive to $\mathbf{w_1}$
if $(A^{p_2}_{\beta 2})^1_{\epsilon_{p_2}} = 0$ else sensitive</td><td>insensitive to $\mathbf{w_q}$
if $(A^{p_2}_{\beta 2})^q_{\epsilon_{p_2}} = 0$ else sensitive</td></tr>
<tr><td>$\vdots$</td><td>$\vdots$</td><td>$\vdots$</td><td>$\ddots$</td><td>$\vdots$</td></tr>
<tr><td rowspan="3">R_q</td><td>insensitive to $\mathbf{w_1}$
if $(A^{p_q}_{\beta q})^1_1 = 0$ else sensitive</td><td>insensitive to $\mathbf{w_2}$
if $(A^{p_q}_{\beta q})^2_1 = 0$ else sensitive</td><td rowspan="3">$\cdots$</td><td rowspan="3">sensitive to $\mathbf{w_q}$</td></tr>
<tr><td>$\vdots$</td><td>$\vdots$</td></tr>
<tr><td>insensitive to $\mathbf{w_1}$
if $(A^{p_q}_{\beta q})^1_{\epsilon_{p_q}} = 0$ else sensitive</td><td>insensitive to $\mathbf{w_2}$
if $(A^{p_q}_{\beta q})^2_{\epsilon_{p_q}} = 0$ else sensitive</td></tr>
</table>

T able 1.

$$C_*^{P_2} = Span\left\{\begin{bmatrix}0\\0\\x_1\\0\end{bmatrix}, \begin{bmatrix}0\\-x_1(1-x_4)\\x_1x_4\\0\end{bmatrix}\right\}$$

$$(S_*^{P_1})^\perp = Span\left\{\begin{bmatrix}0\\0\\1\\0\end{bmatrix}, \begin{bmatrix}x_3\\x_1\\0\\0\end{bmatrix}\right\}$$

$$(S_*^{P_2})^\perp = Span\left\{\begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\0\\1\end{bmatrix}\right\}$$

Before the fault treatment, the detectability of all faults must be established.
However $\nabla_{obs} - Span\{(C_*^{p_1})^\perp\} \neq 0$ and $\nabla_{obs} - Span\{(C_*^{p_2})^\perp\} \neq 0$ because $C_*^{p_1}$ and $C_*^{p_2}$ are different from $\begin{bmatrix}0 & 1 & 0 & 0\end{bmatrix}^T$.
In this case the structural analysis requires tw o intersection tests.
$(S_*^{p_2})^\perp \not\subseteq (S_*^{p_1})^\perp$ and one co-distribution $(\begin{bmatrix}1 & 0 & 0 & 0\end{bmatrix} \subseteq (S_*^{p_2})^\perp)$ is only included in γ thus the binary vector is $\begin{bmatrix}0\end{bmatrix}$ and $\beta_1 = 0$.
$(S_*^{p_1})^\perp \not\subseteq (S_*^{p_2})^\perp$ and the co-distribution $(\begin{bmatrix}0 & 0 & 1 & 0\end{bmatrix} \subseteq (S_*^{p_1})^\perp)$ is in γ th us the binary v ector is $\begin{bmatrix}0\end{bmatrix}$ and $\beta_2 = 0$. With the previous study, matrix $\mathcal{A}$ (equation (12)) can be defined as follows:

$$\mathcal{A} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} \quad (14)$$

$\dim(Span(\mathcal{A})) = 2 = \dim(\mathcal{W})$, so the localization is possible.
The residual signature table is:

Faults / Residuals	$\mathbf{w_1}$	$\mathbf{w_2}$
R_1	sensitiv e	insensitiv e
R_2	insensitiv e	sensitiv e

If the faults w_1 **and/or** w_2 occur, then the residual has different signatures. We add that this filter is solution of the F.P.R.G. since the signature table is diagonal.
The second example has a similar state equation. The output equation is different. Thus the observability characteristics are changed. Then, the study has to be done again.

4.2 *Example 2*

Consider the nonlinear system:

$$\Sigma_{NL2} : \begin{cases} \dot{x} = \begin{pmatrix} x_1x_4 \\ x_3(1-x_4) \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & x_1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} u_1 + w_1 \\ u_2 + w_2 \end{pmatrix} \\ y = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \end{cases} \quad (15)$$

here, $x(t) \in \mathcal{X} = \mathbb{R}^4$, $u(t) \in \mathcal{U} = \mathbb{R}^2$, $y(t) \in \mathcal{Y} = \mathbb{R}^2$ and $w(t) \in \mathcal{W} = \mathbb{R}^2$ are respectively states, inputs, outputs and the unknown disturbances. This system is completely observable ($\nabla_{inobs} = \{0\}$). Lik e previously, follo wing distributions can be found (with distributions $C_*^{P_1}$ and $(S_*^{P_1})^\perp$ as previously).

$$C_*^{P_2} = Span\left\{\begin{bmatrix}0\\0\\x_1\\0\end{bmatrix}, \begin{bmatrix}0\\-x_1(1-x_4)\\x_1x_4\\0\end{bmatrix}\right\}$$

$$S_*^{P_2} = Span\left\{\begin{bmatrix}0\\0\\x_1\\0\end{bmatrix}, \begin{bmatrix}0\\-x_1(1-x_4)\\x_1x_4\\0\end{bmatrix}\right\}$$

$$(S_*^{P_2})^\perp = Span\left\{\begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\0\\1\end{bmatrix}\right\}$$

Of course faults are detectable because $\nabla_{inobs} = 0$ and the conditions $\nabla_{obs} - (C_*^{p_j})^\perp \neq \{0\} \forall j \in 1, 2$ are always verified. So, the structural study can be used.

$(S_*^{p_2})^\perp \not\subseteq (S_*^{p_1})^\perp$ and one co-distribution ($[1\ 0\ 0\ 0] \subseteq (S_*^{p_2})^\perp$) is included in γ thus the binary vector is $[0]$ and $\beta_1 = 0$.
$(S_*^{p_1})^\perp \not\subseteq (S_*^{p_2})^\perp$ but any co-distribution is in γ thus the binary vector is $[1]$ and $\beta_2 = 1$.
With the previous study, matrix $\mathcal{A}$ (equation (12)) can be defined as follows:

$$\mathcal{A} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \tag{16}$$

with $\dim(Span(\mathcal{A})) = 2$, so fault isolation is possible.
The signature table is:

Residuals \ Faults	w_1	w_2
R_1	sensitive	insensitive
R_2	sensitive	sensitive

Unlike the first example, we can isolate only one fault at the same time, because if faults w_1 and w_2 appear simultaneously, fault signature is identical to the case where only fault w_1 appears.
This example highlights the interest of the proposed method since although the signature table is not diagonal, we show that faults are isolable. In this case, it is assumed that faults do not occur simultaneously, which in practice is relatively current.

5. CONCLUSION

Sufficient conditions for solving the fault detection and isolation problem for a class of nonlinear systems are given. This approach is an extension of the work about the complete decoupling of the residual from the faults. The less constraining conditions proposed, compared to Fundamental Problem of Residual Generation, require a structural analysis. Based on geometric results, this analysis assumes that no simultaneous faults occur. Thus sufficient conditions to faults isolation are given.

6. REFERENCES

Cassar, J.P., R.G. Litwak, V. Cocquempot and M. Staroswiecki (1994). Approche structurelle de la conception de systèmes de surveillance pour les procédés industriels complexes. *Diagnostic et sûreté de fonctionnement* **4**(2), 179–202.

De Persis, C. and A. Isidori (1999). On the problem of residual generation for fault detection in nonlinear systems and some related facts. In: *European Control Conference*. Karlsruhe - Germany.

De Persis, C. and A. Isidori (2000). A geometric approach to nonlinear fault detection and isolation. In: *Safeprocess*. Budapest - Germany.

De Persis, C. and A. Isidori (2001). A geometric approach to nonlinear fault detection and isolation. *IEEE Transaction of Automatic Control* **46**(6), 853–865.

Fossard, A.J. and D. Normand-Cyrot (1995). *Nonlinear systems* Vol. 1. Chapman & Hall.

Frank, P.M. and X. Ding (1997). Survey of robust residual generation and evaluation methods in observer-based fault detection systems. *Journal of Process Control* **7**(6), 403–424.

Frank, P.M., S.X. Ding and B. Koppen-seliger (2000). Observer-based approach to fault detection and isolation for nonlinear systems. In: *Safeprocess*. Budapest - Germany. pp. 16–27.

Gertler, J.J. (1998). *Fault detection and diagnosis in engineering systems*. Marcel Dekker.

Hammouri, H., M. Kinnaert and E.H. El Yaagoubi (1999). Observer based approach to fault detection and isolation for non linear systems. *IEEE Transaction of Automatic Control* **44**(10), 1879–1884.

Isidori, A. (1995). *Nonlinear control systems*. Springer Verlag.

Isidori, A., A.J. Krener, C. Gori-Giorgi and S. Monaco (1981). A geometric approach to synthesis of failure detection filters. *IEEE Transaction of Automatic Control* **26**(2), 331–345.

Join, C., J-C. Ponsart and D. Sauter (2002). Fault detection and isolation via nonlinear filters. In: *15th IFAC World Congress on Automatic Control*. Barcelona - Spain.

Massoumnia, M.-A, G.C. Verghese and A.S. Willsky (1989). Failure detection and identification. *IEEE Transaction of Automatic Control* **34**(3), 316–321.

Park, J., G. Rizzoni and W. B. Ribbens (1994). On the representation of sensor faults in fault detection filters. *Automatica* **30**(11), 1793–1795.

Schreier, G., E.A. Garcia and P.M. Frank (2000). Observer-based residual generation for a class of nonlinear systems. In: *Safeprocess*. Budapest - Germany. pp. 727–732.

Sontag, E. D. (1998). *Mathematical control theory*. Springer.

Staroswiecki, M. and G. Comtet-Varga (2001). Analytical redundancy relations for fault detection and isolation in algebraic systems. *Automatica* **37**, 687–699.

Whonam, W.M. (1985). *Linear multivariable control : a geometric approach*. Springer Verlag.