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AN INTEGRATED APPROACH TO CONTROL AND DIAGNOSIS FOR THE MINIMISATION OF UNCERTAINTIES EFFECTS ON RESIDUAL GENERATION

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Abstract: The work reported in this paper focuses on the design problem of integrated control and diagnosis in the case of time-variant systems. The developed methods take the interactions between control and diagnosis into account. The main purpose is to improve fault detection performances in a closed-loop framework by minimizing the model uncertainties effect on the residual generation. The first approach consists in choosing a controller among an a-priori set. This controller is computed by means of the optimal solution of a performance criterion, which takes the model uncertainties effects into account. The second approach proposes an augmented criterion combining control and diagnosis goals. These approaches are illustrated and compared through an example of simulation.

Keywords: uncertain system, residual generation and evaluation, fault detection, closed-loop framework

1. INTRODUCTION

Most of methods for model-based fault detection are usually designed on an open-loop scheme of the system but only few studies consider diagnosis in closed-loop. Nevertheless, the existence of interactions between control and fault detection performances is described in (Niemann and Stoustrup, 1997) and (Stoustrup *et al.*, 1997). It is straightforward to note that control and diagnosis have opposite goals. Indeed, the diagnosis module aims at detecting faults whereas controller should make the measured output insensitive to both disturbances and faults.

Some methods have been developed in a closed-loop framework. Their main purpose is to design a fault detection algorithm robust to disturbances such as model uncertainties and noises. The authors of (Jacobson and Nett, 1991), (Murad *et al.*, 1996), (Nett *et al.*, 1988), (Tyler and Morari, 1994) and present an integrated approach based on a four parameters controller, proposing a joint control and detection algorithms design. Other works in this field use two or three degrees of freedom modules ((Kilsgaard *et al.*, 1996), (Stoustrup *et al.*, 1997), (Suzuki and Tomizuka, 1999)). In (Wu and Wang, 1993), the control law is synthesized from a criterion, which combines control with detection, so that the system has not only the stability robustness, but also the performance robustness in failure detection. Another approach (Ding and Guo, 1998) consists in optimizing residual generation and evaluation without taking into account how the control signal is calculated.

When uncertain plants are involved, these studies

show that the two-step method does not lead to an efficient diagnosis. In this case, it is interesting to synthesize the control and diagnosis modules simultaneously and it is essential to trade-off control performances and quality of fault detection.
However, any of these works do not consider at the beginning of their design, the interaction between fault detection and control.
The key technical contribution presented in this paper is the use of theses interactions in order to improve detection performances. Fault detection improvement is obtained by minimizing the influence of model uncertainties on residual generation.
This paper is organized as follows. In section 2, work hypotheses are presented. Then, in section 3, two structures of residual evaluation are compared when the control law is derived from a Linear Quadratic (L.Q.) criterion. In section 4, an approach improving residual generation is developed by choosing a controller among a set previously defined. Next, a quadratic criterion, combining the control and the diagnosis objectives, is proposed in section 5. Finally, these 3 approaches are illustrated and compared through an example of simulation in section 6.

2. CONTEXT

Consider time-variant systems described by

$$\underline{y}(t) = (B + \Delta B_{real}(t))\underline{u}(t) + Wf(t) + \underline{e}(t) \quad (1)$$

where $\underline{y}(t) \in \mathbb{R}^p$ is the output vector, $\underline{u}(t) \in \mathbb{R}^m$, $f(t) \in \mathbb{R}$ and $\underline{e}(t) \in \mathbb{R}^p$ are, respectively , the control input, the fault input and the measurement noises vector, which is zero mean and with standard deviation $\underline{\sigma}$. Vector $W \in \mathbb{R}^p$ are fault distribution vector and $B \in \mathbb{R}^{p*m}$ is a gain matrix. Matrix $\Delta B_{real}(t) \in \mathbb{R}^{p*m}$ represents uncertainties, which is considered as a time-variant parameter belonging to interval $[-\Delta B; \Delta B]$.
Nominal model of system (1) is defined as follows

$$\hat{\underline{y}}(t) = B\underline{u}(t). \quad (2)$$

Efficient model-based fault detection relies on the generation of a fault sensitive residual. Since the output vector is measurable and $\underline{e}(t)$ is a zero mean noise, the simplest residual is an output simulation error between real and simulated outputs

$$\underline{r}(t) = |\underline{y}(t) - \hat{\underline{y}}(t)|. \quad (3)$$

Residual evaluation in a closed loop framework is studied in the following section when the control law is deduced from L.Q. criterion.
For a sake of simplicity, only static behavior will be considered in the sequel.

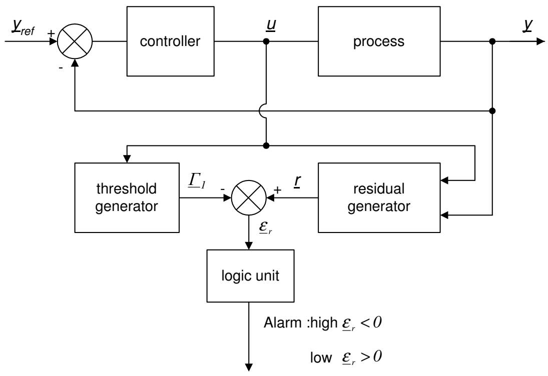


Fig. 1. principle of plant fault detection using threshold $\underline{\Gamma}_1$

3. L.Q. CRITERION

Classically, the control law structure is given by the following expression

$$\underline{u} = K(\underline{y}_{ref} - \underline{y}). \quad (4)$$

Nevertheless, the determination of the control law is usually based on the nominal model of the system. That is the reason why, the optimal control problem consists in determining the vector $\underline{u} = K(\underline{y}_{ref} - \hat{\underline{y}})$ minimizing the performance index given by

$$J = \frac{1}{2}[(\underline{y}_{ref} - \hat{\underline{y}})^T Q_1 (\underline{y}_{ref} - \hat{\underline{y}}) + \underline{u}^T Q_2 \underline{u}] \quad (5)$$

where $\underline{y}_{ref}$ is the set point value and Q_1, Q_2 are weighting matrices. The optimal controller solution associated to (5) is the constant matrix

$$K = Q_2^{-1} B^T Q_1. \quad (6)$$

The model uncertainties imply that the no faulty residual defined by

$$\underline{r} = |\Delta B_{real}\underline{u} + \underline{e}|. \quad (7)$$

is a non-zero mean signal even if no fault corrupts the system. In order to avoid false alarms caused by modelling errors and noises, the residual is compared with a detection threshold, which corresponds to the maximal value of the residual without fault.
Two threshold, which take the uncertainties effects on residual into account, can be synthesized. At first, the threshold $\underline{\Gamma}_1$ (Figure 1), deriving from an open-loop framework, is computed according to the control signal

$$\underline{\Gamma}_1 = \max \underline{r}|_{f=0} = \Delta B|\underline{u}| + \alpha\underline{\sigma}. \quad (8)$$

The second threshold $\underline{\Gamma}_2$, based on a closed-loop framework (Figure 2), is defined by

$$\underline{\Gamma}_2 = \underline{\Lambda} + \Phi\alpha\underline{\sigma}. \quad (9)$$

with $\underline{\Lambda} = \max_{\Delta B_{real}} |\Upsilon \underline{y}_{ref}|$, $\Phi = \max_{\Delta B_{real}} |I - \Upsilon|$ and $\Upsilon = \Delta B_{real}(I + K(B + \Delta B_{real}))^{-1}K$.
In these two cases, the rate of false alarms is set by the positive scalar α. For example, according

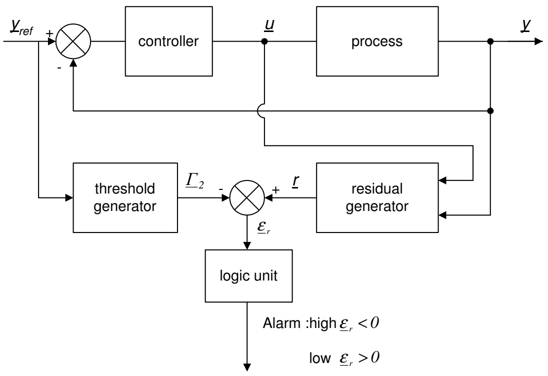


Fig. 2. principle of plant fault detection using threshold $\underline{\Gamma}_2$

to the definition of zero mean normal law distribution, the rate of false alarms is set to 1% if $\alpha = 3$. A detailed study of fault detection performances is undertaken for a S.I.S.O. system.
For this type of system, the static and nominal models are described by

$$y = (b + \Delta b_{real})u + f + e \tag{10}$$

$$\hat{y} = bu \tag{11}$$

with $\Delta b_{real} \in [-\Delta b; \Delta b]$. The controller determined from the minimization of the criterion (5), is given by

$$k = \frac{q_1 b}{q_2}. \tag{12}$$

According to (8) and (9), Γ_1 and Γ_2 are defined by

$$\Gamma_1 = \Delta b|u| + \alpha\sigma \tag{13}$$

$$\Gamma_2 = \lambda|y_{ref}| + \phi\alpha\sigma \tag{14}$$

with $\lambda = \max\limits_{\Delta b_{real}} |\upsilon|$, $\phi = \max\limits_{\Delta b_{real}} |1 - \upsilon|$ and $\upsilon = \Delta b_{real}(1 + k(b + \Delta b_{real}))^{-1}k$.
Functions λ and ϕ are monotone functions of Δb_{real}. Thus, the values of λ and ϕ are obtained for $\Delta b_{real} = -sign(b)\Delta b$. If it is assumed that $0 < \Delta b < |b|$, kb and $k(b \pm \Delta b)$ are positive and it can be shown that

$$\lambda = \frac{|\Delta bk|}{1 + k(b - sign(b)\Delta b)} \tag{15}$$

$$\phi = \frac{1 + bk}{1 + k(b - sign(b)\Delta b)}. \tag{16}$$

The conservatism performances of each threshold in residual evaluation, is studied by calculating the minimal magnitude of all the positive and negative detectable faults for all the uncertainties belonging to $[-\Delta b; \Delta b]$. According to the table (1) presented in appendix and since $0 < \Delta b < |b|$, it can be noted that the threshold Γ_1 allows us to detect similar or lower magnitude of fault than Γ_2.
The minimal amplitude of all the positive detectable faults (resp. negative for $by_{ref} < 0$) is given by

$$|f^+_{min}| = |\frac{2\Delta bk}{1 + k(b + \Delta b)} y_{ref} + \frac{2(1 + bk)}{1 + k(b + \Delta b)} \alpha\sigma| \tag{17}$$

and the minimal amplitude of all the negative faults (resp. positive for $by_{ref} < 0$) is expressed by

$$|f^-_{min}| = |\frac{2\Delta bk}{1 + k(b - \Delta b)} y_{ref} + \frac{2(1 + bk)}{1 + k(b - \Delta b)} \alpha\sigma|. \tag{18}$$

4. MINIMISATION OF UNCERTAINTIES EFFECTS ON RESIDUAL

In order to detect the smallest magnitude fault, a method is developed in this section for minimizing uncertainties effects on residual. According to the definition of the no-faulty residual in a M.I.M.O case

$$\underline{r}|_{f=0} = |\Upsilon \underline{y}_{ref} + (I - \Upsilon)\underline{e}|$$

where Υ is defined in (9), it appears that an optimal choice for the controller is the only way to reduce the magnitude of $\underline{r}|_{f=0}$.
Since the uncertainty ΔB_{real} belongs to $[-\Delta B; \Delta B]$, a set of models can be defined according to all the particular values of ΔB_{real}. A set K_T of controllers is then described by

$$K_T = K + \Delta K \tag{19}$$

with $K = Q_2^{-1} B^T Q_1$, $\Delta K = Q_2^{-1} \Delta B_c^T Q_1$ and $\Delta B_c \in [-\Delta B; \Delta B]$.
The optimal controller is chosen in order to minimize the maximal value of J with respect to ΔB_{real}

$$J = \frac{1}{2}[(\underline{y}_{ref} - \underline{y})^T Q_1 (\underline{y}_{ref} - \underline{y}) + \underline{u}^T Q_2 \underline{u}] \tag{20}$$

where $\underline{u} = K_c(\underline{y}_{ref} - \underline{y})$ is the practical control law.
The optimal choice of the controller, $K_c^* \in K_T$, is then defined by

$$K_c^* = \arg\min_{\underline{u}} \max_{\Delta B_{real}} (J) \tag{21}$$

and implies the computation of ΔB_c^*

$$\Delta B_c^* = \arg\min_{\Delta B_c} \max_{\Delta B_{real}} (J). \tag{22}$$

The residual evaluation is based on the new controller K_c^* and on the threshold $\underline{\Gamma}_1$

$$\underline{\Gamma}_1 = \Delta B|\underline{u}| + \alpha\underline{\sigma}. \tag{23}$$

The resolution of (22) and the performances of fault detection are now studied for a S.I.S.O. system (10). For all uncertainties belonging to

$[-\Delta b; \Delta b]$ and with $0 < \Delta b < |b|$, the control criterion is given by

$$\Delta b_c^* = \arg \min_{\Delta b_c} \max_{\Delta b_{real}} (J) \quad (24)$$

where $J = \frac{1}{2}[(y_{ref} - y)^2 q_1 + u^2 q_2]$. It can be proved that the solution of (24) results in a diagnosis improvement.

The performance index J is monotone with respect to (w.r.t.) Δb_{real} for a fixed Δb_c. So, the maximal value of J w.r.t. Δb_{real} is obtained for $\Delta b_{real} = -sign(b)\Delta b$. In the same way, the minimal value of this maximal value of J w.r.t. Δb_c is given for $\Delta b_c = -sign(b)\Delta b$.

Since, the no-faulty residual amplitude increases w.r.t. k_c

$$r = \left| \frac{\Delta b_{real} k_c}{1 + k_c(b + \Delta b_{real})} y_{ref} + (1 - \frac{\Delta b_{real} k_c}{1 + k_c(b + \Delta b_{real})}) e \right|.$$

It can be noticed that the residual generation is actually optimal for $\Delta b_c = -sign(b)\Delta b$ ($|k_c^*| < |k_c|$).

The optimal controller k_c^* is then defined by the following expression

$$k_c^* = \frac{q_1(b - sign(b)\Delta b)}{q_2} \quad (25)$$

Referring to the previous section, the minimal amplitude of positive detectable faults (resp. negative for $by_{ref} < 0$) is given by

$$|f_{min}^+| = \left| \frac{2\Delta b k_c^*}{1 + k_c^*(b + \Delta b)} y_{ref} + \frac{2(1 + bk_c^*)}{1 + k_c^*(b + \Delta b)} \alpha\sigma \right|. \quad (26)$$

and the minimal amplitude of negative detectable faults (resp. positive for $by_{ref} < 0$) is expressed by

$$|f_{min}^-| = \left| \frac{2\Delta b k_c^*}{1 + k_c^*(b - \Delta b)} y_{ref} + \frac{2(1 + bk_c^*)}{1 + k_c^*(b - \Delta b)} \alpha\sigma \right|. \quad (27)$$

These amplitudes are lower than (17) and (18) since $|k_c^*| < |k|$ and $\Delta b < |b|$.

5. AUGMENTED CRITERION

In the previous section, the diagnosis performances have been improved by a judicious choice of a controller among a set. This controller represents the optimal solution for the control criterion (21). However, in order to reflect the interactions between the control and the diagnosis, the fault detection goals can be integrated into the control law synthesis. The following augmented criterion is proposed

$$\widetilde{J} = J + \Phi_d \quad (28)$$

where the criterion J is defined by (5), $\Phi_d = (\underline{y} - \hat{\underline{y}} - \Delta B_c \underline{u})^T Q_3 (\underline{y} - \hat{\underline{y}} - \Delta B_c \underline{u})$ and Q_3 is a weighting matrix. Since the simulation error without noise is given by $\underline{y} - \hat{\underline{y}} = \Delta B_{real} \underline{u}$, ΔB_c is now exclusively dedicated to minimize no-faulty residual magnitude. Then, the problem consists in finding the vector

$$\underline{u} = K_1(\underline{y}_{ref} - \hat{\underline{y}}) + K_2(\underline{y} - \hat{\underline{y}}) \quad (29)$$

minimizing the performance index given by

$$\widetilde{J} = \frac{1}{2}[(\underline{y}_{ref} - \hat{\underline{y}})^T Q_1 (\underline{y}_{ref} - \hat{\underline{y}}) + \underline{u}^T Q_2 \underline{u} + (\underline{y} - \hat{\underline{y}} - \Delta B_c \underline{u})^T Q_3 (\underline{y} - \hat{\underline{y}} - \Delta B_c \underline{u})]. \quad (30)$$

The trade-off between the control and the diagnosis is achieved by an appropriate choice of the weighting matrices.

Then, using the following structures of K_1,K_2

$$\begin{cases} K_1 = D^{-1} N_1 \\ K_2 = D^{-1} N_2 \end{cases} \quad (31)$$

the solution (29) of the criterion (30) is given by

$$K_1 = (Q_2 + (B^T + \Delta B_c^T) Q_3 \Delta B_c)^{-1} B^T Q_1 \quad (32)$$

$$K_2 = (Q_2 + (B^T + \Delta B_c^T) Q_3 \Delta B_c)^{-1} (B^T + \Delta B_c^T) Q_3. \quad (33)$$

Referring to the previous section, the optimal parameter ΔB_c^* is determined by solving

$$\Delta B_c^* = \arg \min_{\Delta B_c} \max_{\Delta B_{real}} (J) \quad (34)$$

where

$$J = \frac{1}{2}[(\underline{y}_{ref} - \underline{y})^T Q_1 (\underline{y}_{ref} - \underline{y}) + \underline{u}^T Q_2 \underline{u} + (\underline{y} - \hat{\underline{y}})^T Q_3 (\underline{y} - \hat{\underline{y}})] \quad (35)$$

and $\underline{u}$ represents the practical control law

$$\underline{u} = K_1(\underline{y}_{ref} - \underline{y}) + K_2(\underline{y} - \hat{\underline{y}}). \quad (36)$$

The threshold Γ_1 can be computed as follows

$$\underline{\Gamma}_1 = \Delta B |\underline{u}| + \alpha \underline{\sigma}. \quad (37)$$

The resolution of (34) is studied for a S.I.S.O. system (10). In this case, the problem (34) becomes

$$\Delta b_c^* = \arg \min_{\Delta b_c} \max_{\Delta b_{real}} (J) \quad (38)$$

with

$$J = \frac{1}{2}[(y_{ref} - y)^2 q_1 + u^2 q_2 + (y - \hat{y})^2 q_3]. \quad (39)$$

Since the criterion (39) is not monotone w.r.t. Δb_c and Δb_{real}, the solution to (38) is computed by means of the following algorithm approach. It consists for each $\Delta b_c \in [-\Delta b; \Delta b]$ in searching the maximum of J w.r.t. Δb_{real}. Then, the solution Δb_c^* (38) is the optimal value that generates the minimal value of these maxima.

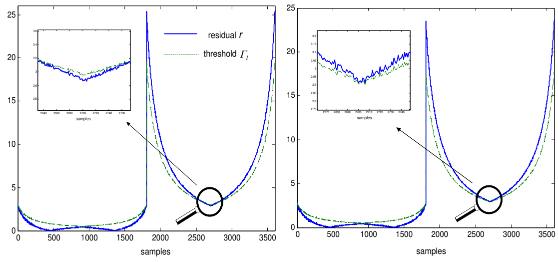
Control and fault detection performances are examined in the following section through an example of a S.I.S.O. system.

6. SIMULATION EXAMPLE

The purpose of this section is to compare, through the simulation of a S.I.S.O. system, control and fault detection performances obtained by the methods developed in this article.
For the sake of simplicity, the methods developed in sections 3, 4 and 5 are called method 1, 2 and 3 respectively. In this example, the values of the system parameters (10) are $b = 6$, $\Delta b = 4.5$ and $y_{ref} = 1$. The particular choice of weighting coefficients, $q_1 = 6$ and $q_2 = 2$, ensures a small static error. The standard deviation of the measurement noise is equal to 0.025. A fault defined by (27) and equal to -5.7677, occurs at sample 1803. The time evolution of residual, threshold and output is depicted on figures 3 to 5. For this simulation, the value of the time-variant parameter Δb_{real} decreases from Δb to $-\Delta b$ in time intervals $[0; 901]$ and $[1803; 2703]$. On the contrary, its value increases from $-\Delta b$ to Δb in time intervals $[902; 1802]$ and $[2704; 3604]$. Referring to figures 3, it can be noted that method 2 improves the detection performances since the residual is always higher than the threshold Γ_1 after the fault occurrence for all the particular values of Δb_{real}. Figures 4 show the capacity of the augmented criterion to take the fault detection performances into account according to an appropriate choice of q_3. As it can be noticed in figures 5, the improvement of fault detection performances implies an increasing of static errors. Moreover, the control and fault detection performances obtained by method 3 and by method 1 are similar when $q_3 = 0$, since in this case $\widetilde{J} = J$.

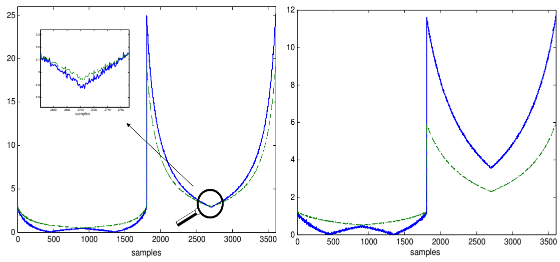
7. CONCLUSION

In this paper, a joint control and diagnosis algorithm design is proposed. It takes the interactions between the control and the diagnosis into account. The first approach consists in choosing a controller among an a-priori set. This controller is computed by means of the optimal solution of a performance criterion, which takes the model uncertainties into account. The smallest amplitude of all the detectable fault is lower with this method than when the control law derives from a traditional L.Q. criterion. In the same time, the control becomes less efficient.
The second approach proposes an augmented criterion combining control and diagnosis objectives. The various compromises between the control and the diagnosis are then carried out by a suitable choice of the weighting matrices.



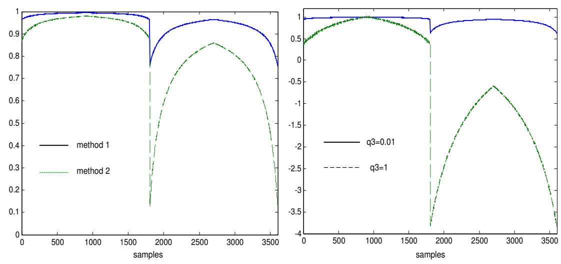
(a) method 1 (b) method 2

Fig. 3. magnitudes of residual and threshold Γ_1



(a) $q_3 = 0.01$ (b) $q_3 = 1$

Fig. 4. magnitudes of residual and threshold Γ_1 (method 3)



(a) methods 1 and 2 (b) method 3

Fig. 5. output y

8. APPENDIX

Table 1 presents, in a no-noisy context, the minimal amplitude of positive and negative detectable faults obtained according to the detection thresholds Γ_1 and Γ_2. These amplitudes are given by $\max f|_{r=\Gamma_1}$ and $\max f|_{r=\Gamma_2}$ for all $\Delta b_{real} \in [-\Delta b; \Delta b]$. The residual in the presence of a fault is defined by

$$r = \left| \frac{\Delta b_{real} k}{1 + k(b + \Delta b_{real})} y_{ref} + \frac{(1 + bk)}{1 + k(b + \Delta b_{real})} f \right|.$$

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			positive fault	negative fault
Γ_1		$by_{ref} > 0$	$\frac{2\Delta bk}{1+k(b+\Delta b)}y_{ref}$	$\frac{-2\Delta bk}{1+k(b-\Delta b)}y_{ref}$
		$by_{ref} < 0$	$\frac{-2\Delta bk}{1+k(b-\Delta b)}y_{ref}$	$\frac{2\Delta bk}{1+k(b+\Delta b)}y_{ref}$
Γ_2	$\Delta b < \frac{1+2kb}{2k}$	$by_{ref} > 0$	$\frac{2\Delta bk}{1+kb}y_{ref}$	$\frac{-2\Delta bk}{1+k(b-\Delta b)}y_{ref}$
		$by_{ref} < 0$	$\frac{-2\Delta bk}{1+k(b-\Delta b)}y_{ref}$	$\frac{2\Delta bk}{1+kb}y_{ref}$
	$\Delta b > \frac{1+2kb}{2k}$	$by_{ref} > 0$	$\frac{2\Delta b^2k^2}{(1+kb)(1+k(b-\Delta b))}y_{ref}$	$\frac{-2\Delta bk}{1+k(b-\Delta b)}y_{ref}$
		$by_{ref} < 0$	$\frac{-2\Delta bk}{1+k(b-\Delta b)}y_{ref}$	$\frac{2\Delta b^2k^2}{(1+kb)(1+k(b-\Delta b))}y_{ref}$

Table 1. magnitudes of the detectable faults by Γ_1 and Γ_2 according to the sign of by_{ref}, the value of Δb.

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