15th Triennial World Congress, Barcelona, Spain

ADAPTIVE CONTROL FOR UNCERTAIN SYSTEMS WITH SECTOR-LIKE BOUNDED NONLINEAR INPUTS

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Abstract: In this paper, a general approach to the synthesis of model reference adaptive control systems is proposed for uncertain time-delay systems with the so-called sector-like bounded nonlinear inputs via hyperstability theory. In the presence of system parameter variations and external disturbances, it can be ensured that the system output can track the reference model output asymptotically. Finally, some computer simulations are given to illustrate the performance of the proposed adaptive control scheme.

Keywords: adaptive control, uncertain system, time-delay, sector-like bounded, hyperstability.

1. INTRODUCTION

Adaptive control of linear time-invariant systems with idealized situations has been an active research area for the past three decades. From the beginning of the 1980's, as a result of the discovery that unmodeled dynamics or even small bounded disturbances can cause instability in adaptively controlled systems, robustness studies in adaptive control have been drawing much attention from researchers. Until present, several significant attempts, such as dead zone approach (Peterson and Narendra, 1982), s-modification (Hsu and Costa, 1987), persistency of excitation (Narendra and Annaswamy, 1989), parameter projection together with normalization (Praly, 1983) etc., have been proposed to construct the robust adaptive control scheme for systems subjected to system parameter uncertainties, or unmodelled dynamic errors, or external disturbances. Most of attempts to design robust adaptive control systems usually consider linear systems. In recent ten years, the adaptive control of nonlinear systems has attracted a lot of interest and has made valuable progress (Khalil, 1996; Polycarpou and Ioannou, 1996; Hotzel and Karsenti, 1998; Jiang and Praly, 1998; McLain et al., 1999). Despite these advances in adaptive control for nonlinear systems, most of the literature so far still deals with linear input problems. However, in practice, owing to physical limitation, there usually exist nonlinearities in the control input. In real control systems, actuators and sensors often have nonlinear characteristics such as dead-zone, backlash, and hysteresis. Some common examples are mechanical connections, hydraulic servo-valves and electric servomotors, magnetic suspensions and bearings, and some biomedical systems. These effects of input nonlinearities usually result in the control performance degeneration or instability in the controlled system. Hence, in system realization and control, the existence of input nonlinearities cannot be ignored. Meanwhile, time delay is also frequently encountered in various engineering systems and can be the cause of instability. Up to the present, the system with nonlinear input or time delay has still not attracted quite a lot of studies in the adaptive control (Tao and Kokotovic, 1995; Tao and Tian, 1995; Miyasato, 1991, Salem and Carroll, 1985). For the practical control reason, an uncertain time-delay system with unknown nonlinear input will be considered in this work.

In this paper, a model reference adaptive control is proposed based on hyperstability theory for a class of uncertain time-delay systems with nonlinear inputs. Meanwhile, these classes of systems are also subjected to system parameter variations and external disturbances. To cope with the problem of system uncertainties, an adaptation algorithm is developed via hyperstability theory to estimate these system uncertainties. Therefore, without the requirement of information about uncertain system parameter variations and disturbances, the proposed adaptive control scheme can be realized and the characteristics of the output asymptotical tracking can be achieved. Finally, some computer simulation results will be given to show the performance of the proposed model reference adaptive control scheme.

2. HYPERSTABILITY THEORY

In this section, we provide a brief review for the hyperstability theory (Popov, 1973). Consider the multivariable standard system shown in Figure 1 and described by the state equation of the form

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

$$y(t) = Cx(t) + Du(t) \tag{2}$$

$$u(t) = -w(t) \tag{3}$$

$$w(t) = j\,(y(t),t) \quad , \quad t \le t \tag{4}$$

where A, B, C, and D are appropriate dimensional matrices, $x(t)$, $u(t)$, and $y(t)$ are the state vector, the input vector, and the output vector, respectively. The pair (C, A) is completely observable and the pair (A, B) is completely controllable. $j(\cdot)$ denotes a nonlinear functional dependence between $w(t)$ and $y(t)$ in the interval $t \le t$. In order that the closed-loop system of equations (1)-(4) is asymptotically hyperstable, the following conditions should be satisfied:

(1) The transfer function

$$G(s) = C(sI - A)^{-1}B + D \tag{5}$$

must be strictly positive real (SPR);

(2) The so-called passivity condition (Popov integral inequality)

$$\int_0^t y^T(t)w(t)dt \ge -r_0^2 \tag{6}$$

is true for all $t \ge 0$.

where r_0 is an arbitrary finite constant.

Remark 1: The closed-loop system of equations (1)-(4) with asymptotically hyperstable implies that the result, $\lim_{t\to\infty} x(t) = 0$, will be achieved.

3. SYSTEM DESCRIPTIONS

Consider the following uncertain time-delay system with nonlinear input described by the following differential-difference equation

$$\dot{x}(t) = (A + \Delta A)x(t) + A_d x(t-h) + Bf(u(t)) + d(x,t) \tag{7}$$

$$x(t) = f(t) \quad \text{for} \quad -h < t < 0 \tag{8}$$

$$y(t) = Cx(t) \tag{9}$$

where $x(t) \in R^n$, which is assumed to be measurable at every t, is the system state vector, $u(t) \in R^m$ is the system input vector, $y(t) \in R^m$ is the system output vector, A, B, and C are constant matrices of appropriate dimension, ΔA is the uncertain matrix of matrix A, $A_d \in R^{n\times n}$ is an unknown matrix, $f(u(t)) \in R^m$ is a continuous nonlinear function and $f(0) = 0$, $d(x,t) \in R^n$ represents system disturbances, $f(t) \in R^n$ is an arbitrary known time function, and h denotes an unknown but fixed time-delay. In this paper, referring to the definition of a sector bounded function in (Slotine and Li, 1991), a so-called sector-like bounded function is defined as follows.

Definition 1: A continuous function $f_i(u(t))$ with $f_i(0) = 0$, as shown in Figure 2, is said to belong to the sector-like $[c_{i1}, \ c_{i2}]$ by u_i, if there exist two positive constants c_{i1} and c_{i2} such that

$$c_{i1} \le \frac{f_i(u)}{u_i} \le c_{i2} \quad \text{for} \quad u_i \ne 0$$

and $u = [u_1 \ \ u_2 \ \ \cdots \ \ u_m]^T$.

The following assumptions specify the class of uncertain nonlinear systems considered in this paper.

Assumption 1:

(1) Nonlinear input functions $f_i(u(t))$, $i = 1,\cdots,m$, are sector-like bounded by u_i, $i = 1,\cdots,m$, respectively. It yields that there exist positive constants c_{i1}, $i = 1,\cdots,m$, and c_{i2}, $i = 1,\cdots,m$, such that the following conditions are satisfied. $c_{i1} \le \frac{f_i(u)}{u_i} \le c_{i2}$, $i = 1,\cdots,m$.

(2) Uncertain system parameter matrices ΔA, A_d, and disturbance vector $d(x,t)$ are norm bounded. It means that there exist three positive values d_1, d_2, and d_3 such that $\|\Delta A\| \le d_1$, $\|A_d\| \le d_2$, and $\|d(x,t)\| \le d_3$.

From Assumption 1 (1), it straightly gives the following results

$$c_{i1}u_i^2 \le u_i f_i(u) \le c_{i2}u_i^2, \quad i = 1,\cdots,m.$$

Then, we have

$$\begin{aligned} & c_{11}u_1^2 + c_{21}u_2^2 + \cdots + c_{m1}u_m^2 \\ & \le u_1 f_1(u) + u_2 f_2(u) + \cdots + u_m f_m(u) \\ & \le c_{12}u_1^2 + c_{22}u_2^2 + \cdots + c_{m2}u_m^2 \end{aligned}$$

It yields that from the above inequality

$$c_1 u^T u \le u^T f(u) \le c_2 u^T u \tag{10}$$

where $c_1 = \min\{c_{i1} \mid i = 1,\cdots,m\}$ and $c_2 = \max\{c_{i2} \mid i = 1,\cdots,m\}$.

From Assumption 1 (1), it also yields that

$$c_1^2 u^T u \le f^T(u)f(u) \le c_2^2 u^T u$$

$$\text{or} \quad c_1\|u\| \le \|f(u)\| \le c_2\|u\| \tag{11}$$

The control objective in this work is to find an appropriate control $u(t)$ such that the system output vector $y(t)$ can asymptotically follow the desired output vector $y_m(t)$. Consider the reference model in the following form.

$$\dot{x}_m(t) = A_m x_m(t) + B_m r(t) \tag{12}$$

$$y_m(t) = Cx_m(t) \tag{13}$$

where $x_m(t) \in R^n$ is the reference model state vector, $r(t) \in R^m$ is the reference model input vector, which is piecewise continuous and bounded,

$y_m(t) \in R^m$ is the reference model output vector, A_m and B_m are matrices of appropriate dimension and A_m is a Hurwitz matrix.

4. MODEL REFERENCE ADAPTIVE CONTROL SCHEME

In this section, an adaptive control input will be derived for the uncertain time-delay system (7)-(9) such that the control objective can be achieved via hyperstability theory. Define the output tracking error vector

$$e(t) = y(t) - y_m(t) \tag{14}$$

Then, it yields that the dynamics of the system output tracking error vector can be written as

$$\begin{aligned}\dot{e} &= C[(A+\Delta A)x + A_d x_d + Bf(u) + d - A_m x_m - B_m r] \\ &= \bar{A}e + (CA - \bar{A}C)x + C\Delta A x + CA_d x_d + CBf(u) \\ &\quad + Cd + (\bar{A}C - CA_m)x_m - CB_m r \\ &= \bar{A}e - \bar{B}w\end{aligned}$$

where $\bar{A} \in R^{m\times m}$ is a designed Hurwitz matrix, $\bar{B} \in R^{m\times m}$ is a designed nonsingular matrix,

$$w = -\bar{B}^{-1}[(CA - \bar{A}C)x + C\Delta A x + CA_d x_d + CBf(u) + Cd + (\bar{A}C - CA_m)x_m - CB_m r],$$

and the delayed state is represented as $x_d = x(t-h)$ for simplification. Define a linear processing block

$$E = He$$

where $H \in R^{m\times m}$ is a constant matrix.
Hence, it follows that a linear time invariant system with the output E is

$$\dot{e} = \bar{A}e - \bar{B}w \tag{15}$$

$$E = He \tag{16}$$

As the previous descriptions in section 2, the linear time invariant system is asymptotically hyperstable under that matrix H is chosen from the following equations

$$\bar{A}^T P + P\bar{A} = -Q \tag{17}$$

$$P\bar{B} = H^T \tag{18}$$

where matrices Q and P are symmetric, positive definite matrices. To design a model reference adaptive control system such that the system output can track the reference model output asymptotically, the control law is designed in the following form.

$$u = -\frac{(E^T\bar{B}^{-1}CB)^T}{\|E^T\|} f(x,t) \tag{19}$$

$$\begin{aligned}f(x,t) = \frac{1}{c_1 l_{\min}[\bar{B}^{-1}CB(\bar{B}^{-1}CB)^T]}\Big[\, &\|\bar{B}^{-1}(CA - \bar{A}C)x\| \\ &+ \hat{k}_1(t)\|x\| + \hat{k}_2(t)\max\|x(t_i)\| + \hat{k}_3(t) \\ &+ \|\bar{B}^{-1}(\bar{A}C - CA_m)x_m\| + \|\bar{B}^{-1}CB_m r\|\,\Big]\end{aligned} \tag{20}$$

where $t_i \in [-h, t]$, feedback gains $\hat{k}_1(t)$, $\hat{k}_2(t)$, and auxiliary signal $\hat{k}_3(t)$ are three time-varying constants which are adapted by the following adaptation laws.

$$\hat{k}_1(t) = g_{11}\|E^T\|\|x\| + g_{12}\int_0^t \|E^T\|\|x\| dt + k_1(t) \tag{21}$$

$$\begin{aligned}\hat{k}_2(t) &= g_{21}\|E^T\|\max\|x(t_i)\| \\ &\quad + g_{22}\int_0^t \|E^T\|\max\|x(t_i)\| dt + k_2(t)\end{aligned} \tag{22}$$

$$\hat{k}_3(t) = g_{31}\|E^T\| + g_{32}\int_0^t \|E^T\| dt + k_3(t) \tag{23}$$

where positive constants g_{11}, g_{12}, g_{21}, g_{22}, g_{31}, and g_{32} are adaptation gains to determine adaptation rates.

Remark 2: If the adaptation process is fast, in equations (21)-(23), the time functions $k_1(t) = \|\bar{B}^{-1}C\| d_1$, $k_2(t) = \|\bar{B}^{-1}C\| d_2$, and $k_3(t) = \|\bar{B}^{-1}C\| d_3$ can be treated as unknown and slowly time varying. In practice, these are set to arbitrary constant values (often zero).

The performance of the proposed model reference adaptive control system can be described by the following theorem.

Theorem 1: Consider the uncertain system (7)-(9) subjected to Assumption 1 and the reference model (12), (13). If the adaptive control input $u(t)$ in (19), (20) and the adaptation laws in (21)-(23), then the linear time invariant system in (15), (16) is asymptotically hyperstable, namely the system output $y(t)$ can asymptotically follow the desired output $y_m(t)$, i.e. $\lim_{t\to\infty}\|e(t)\| = 0$.

Proof: The transfer function of the linear time invariant system (15), (16), $H(sI - \bar{A})^{-1}\bar{B}$, is strictly positive real under the appropriate set for matrix H, which is obtained from equations (17) and (18). Then, we calculate

$$\begin{aligned}&-E^T w \\ &= E^T[\,\bar{B}^{-1}(CA - \bar{A}C)x + \bar{B}^{-1}C\Delta A x + \bar{B}^{-1}CA_d x_d \\ &\quad + \bar{B}^{-1}Cd + \bar{B}^{-1}(\bar{A}C - CA_m)x_m - \bar{B}^{-1}CB_m r\,] \\ &\quad + E^T\bar{B}^{-1}CBf(u) \\ &\le \|E^T\|\,[\,\|\bar{B}^{-1}(CA - \bar{A}C)x\| + \|\bar{B}^{-1}C\|\|\Delta A\|\|x\| \\ &\quad + \|\bar{B}^{-1}C\|\|A_d\|\max\|x(t_i)\| + \|\bar{B}^{-1}C\|\|d\| \\ &\quad + \|\bar{B}^{-1}(\bar{A}C - CA_m)x_m\| \\ &\quad + \|\bar{B}^{-1}CB_m r\|\,] + E^T\bar{B}^{-1}CBf(u) \\ &\le \|E^T\|\,[\,\|\bar{B}^{-1}(CA - \bar{A}C)x\| + bd_1\|x\| + bd_2\max\|x(t_i)\| \\ &\quad + bd_3 + \|\bar{B}^{-1}(\bar{A}C - CA_m)x_m\| + \|\bar{B}^{-1}CB_m r\|\,] \\ &\quad + E^T\bar{B}^{-1}CBf(u) \\ &= \|E^T\|\,[\,\|\bar{B}^{-1}(CA - \bar{A}C)x\| + k_1\|x\| + k_2\max\|x(t_i)\| \\ &\quad + k_3 + \|\bar{B}^{-1}(\bar{A}C - CA_m)x_m\| + \|\bar{B}^{-1}CB_m r\|\,] \\ &\quad + E^T\bar{B}^{-1}CBf(u)\end{aligned} \tag{24}$$

where $\|\overline{B}^{-1}C\| = b$, $bd_1 = k_1$, $bd_2 = k_2$, and $bd_3 = k_3$. From equation (19) and Assumption 1 (1), it is obtained that

$$u^T f(u) = -\frac{E^T \overline{B}^{-1} CB}{\|E^T\|} f(x,t) f(u) \ge c_1 u^T u$$

Since

$$c_1 u^T u = c_1 \frac{E^T \overline{B}^{-1} CB}{\|E^T\|} f(x,t) \frac{(\overline{B}^{-1} CB)^T E}{\|E^T\|} f(x,t)$$

$$\ge \frac{c_1 l_{\min}[\overline{B}^{-1} CB (\overline{B}^{-1} CB)^T] \|E^T\|^2}{\|E^T\|^2} f^2(x,t)$$

$$= c_1 l_{\min}[\overline{B}^{-1} CB(\overline{B}^{-1} CB)^T] f^2(x,t)$$

we have

$$-\frac{E^T \overline{B}^{-1} CB}{\|E^T\|} f(x,t) f(u) \ge c_1 l_{\min}[\overline{B}^{-1} CB(\overline{B}^{-1} CB)^T] f^2(x,t)$$

It follows that

$$E^T \overline{B}^{-1} CBf(u) \le -c_1 l_{\min}[\overline{B}^{-1} CB(\overline{B}^{-1} CB)^T] \|E^T\| f(x,t) \tag{25}$$

Substituting inequality (25) into inequality (24), it yields that

$$\begin{aligned} &-E^T w \\ &\le \|E^T\| \ \{ \|\overline{B}^{-1}(CA - \overline{A}C)x\| + k_1 \|x\| \\ &\quad + k_2 \max\|x(t_i)\| + k_3 + \|\overline{B}^{-1}(\overline{A}C - CA_m)x_m\| \\ &\quad + \|\overline{B}^{-1} CB_m r\| \\ &\quad - c_1 l_{\min}[\overline{B}^{-1} CB(\overline{B}^{-1} CB)^T] f(x,t) \ \} \end{aligned}$$

From equation (20), it yields that

$$\begin{aligned} &-E^T w \\ &\le \|E^T\| [\ k_1(t)\|x\| + k_2(t) \max\|x(t_i)\| + k_3(t) \\ &\quad - \hat{k}_1(t)\|x\| - \hat{k}_2(t) \max\|x(t_i)\| - \hat{k}_3(t)\ ] \\ &= -\|E^T\| \{ [\ g_{11}\|E^T\| \|x\| + \int_0^t g_{12}\|E^T\| \|x\| dt\ ] \|x\| \\ &\quad + [\ g_{21}\|E^T\| \max\|x(t_i)\| + \int_0^t g_{22}\|E^T\| \max\|x(t_i)\| dt\ ] \cdot \\ &\quad \max\|x(t_i)\| + g_{31}\|E^T\| + \int_0^t g_{32}\|E^T\| dt\ \} \end{aligned}$$

It follows that from equations (21)-(23)

$$\begin{aligned} &\int_0^t E^T(t) w(t) dt \\ &\ge \int_0^t \|E^T\| \{ [\ g_{11}\|E^T\| \|x\| + \int_0^t g_{12}\|E^T\| \|x\| dt\ ] \|x\| \\ &\quad + [\ g_{21}\|E^T\| \max\|x(t_i)\| + \int_0^t g_{22}\|E^T\| \max\|x(t_i)\| dt\ ] \cdot \\ &\quad \max\|x(t_i)\| + g_{31}\|E^T\| + \int_0^t g_{32}\|E^T\| dt\ \} dt \\ &\ge -r_0^2 \end{aligned}$$

Hence, the Popov integral inequality

$$\int_0^t E^T(t) w(t) dt \ge -r_0^2$$

can be ensured for all $t \ge 0$. Since that the transfer function matrix $H(sI - \overline{A})^{-1}\overline{B}$ is strictly positive real and the Popov integral inequality is achieved, it follows that the proposed model reference adaptive control system is asymptotically hyperstable. Namely, the result of $\lim_{t\to\infty}\|e(t)\| = 0$ is proved.

5. NUMERICAL SIMULATIONS

Consider an unstable, time-delay uncertain dynamical system represented in the following form

$$\dot{x}(t) = (A + \Delta A)x(t) + A_d x(t-h) + Bf(u(t)) + d(x,t)$$
$$x(t) = f(t) \ \text{ for } \ -h < t < 0$$
$$y(t) = Cx(t)$$

The corresponding parameters of the illustrated system are given as follows:

$$A = \begin{bmatrix} 0 & 1 \\ 2 & 2 \end{bmatrix},\ \Delta A = \begin{bmatrix} \Delta a_1 & 0 \\ \Delta a_2 & \Delta a_3 \end{bmatrix},$$

$$\Delta a_1 = 0.3\sin(0.05t),\ \Delta a_2 = SW(0.5,10),$$
$$\Delta a_3 = 0.5\sin(0.1t),$$

$$A_d = \begin{bmatrix} 0 & 0 \\ 0.5 + 0.2\sin(t) & 0.5 + 0.2\sin(t) \end{bmatrix},$$

$$x(t-h) = \begin{bmatrix} x_1(t-0.2) \\ x_2(t-0.2) \end{bmatrix},\ B = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

$$f(u) = [2 - 0.3\sin(u)]u,$$

$$d(x,t) = \begin{bmatrix} 0 \\ 0.4\sin(t) + 0.3\sin(x_1) \end{bmatrix},$$

and $C = [1 \quad 1]$

where $SW(a,p)$ represents a square waveform with amplitude a and period p (sec). The reference model is chosen as

$$\dot{x}_m(t) = A_m x_m(t) + B_m r(t)$$
$$y_m(t) = Cx_m(t)$$

where $A_m = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$, and $B_m = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. For numerical simulation, adaptation gains are chosen as $g_{11} = g_{21} = g_{31} = 10$, $g_{12} = g_{22} = g_{32} = 1$, $c_1 = 1.5$, $\overline{A} = -1$, $\overline{B} = 1$, and $H = 0.5$. Initial gain values and initial state values are set as $[\hat{k}_1(0) \quad \hat{k}_2(0) \quad \hat{k}_3(0)] = [0 \quad 0.5 \quad 0.4]$, $x^T(0) = [2 \quad 1]$, and $x_m^T(0) = [0 \quad 0]$, respectively. Simulation results are shown in figures 3 and 4 as reference inputs are set to be a sine waveform, $r(t) = \sin(t)$ and a square waveform $r(t) = SW(1,6)$, respectively. According to the proposed control scheme, for this single-input single-output illustrated system, the control input can be written as

$$u = -\frac{2E}{|E|} f = -2\,\text{sgn}(E) f$$

Since there is a term $\text{sgn}(E)$ in the control input, it will result in control chattering. To reduce the chattering amplitude, the control input can be modified as follows

$$u = -\frac{2E}{|E| + l} f$$

where l is a sufficiently small positive constant. A smaller l value will give a better output tracking performance but result in a larger control chattering. In the contrary, a larger l value will give a worse output tracking performance but result in a smaller control chattering. Hence, the setting of l value is a compromise between output tracking performance and control chattering.

6. CONCLUSIONS

In this paper, a model reference adaptive control scheme for time-delay uncertain systems with sector-like bounded nonlinear inputs is developed via hyperstability theory. Based on hyperstability theory, general adaptation algorithms are derived to update the feedback control gain and an auxiliary signal in the control input for coping with the system uncertainties. Hence, the proposed model reference adaptive controller can be implemented without the requirement about the knowledge of system parameter uncertainties and disturbances. Nevertheless, it only shows that the asymptotical output tracking performance can be achieved in this work but the internal stability of the proposed system is still an open question in the future.

ACKNOWLEDGEMENT

This work supported by the National Science Council, Taiwan, Republic of China, under Grant NSC 90-2212-E-151-001.

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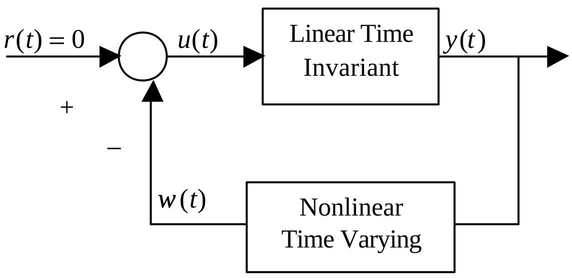


Figure 1. Standard nonlinear time varying feedback system.

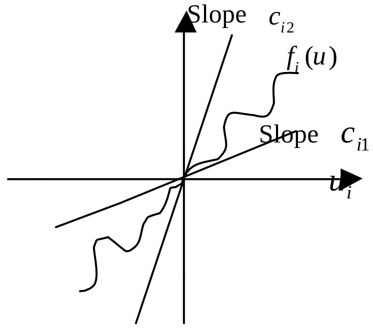
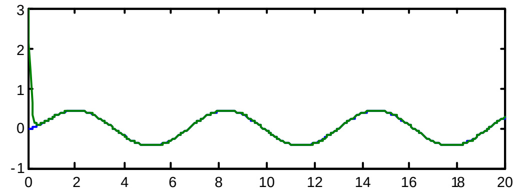
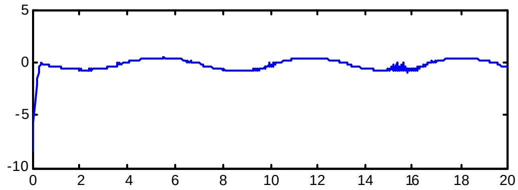


Figure 2. The sector-like bounded function.



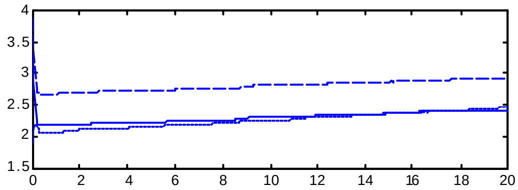
Time (s)

(a) $y(t)$ and $y_m(t)$



Time (s)

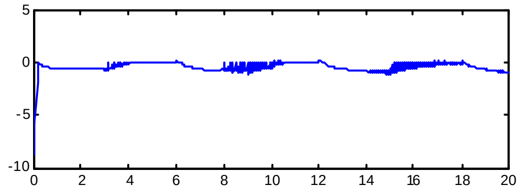
(b) $u(t)$



Time (s)

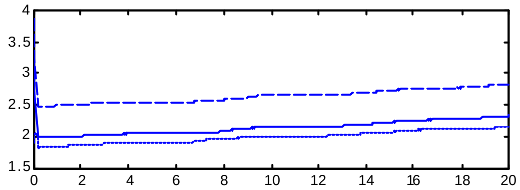
(c) $\hat{k}_1(t)$: ——— , $\hat{k}_2(t)$: – – – , and $\hat{k}_3(t)$: ··········

Figure 3. Simulation results: $r(t) = \sin(t)$.



Time (s)

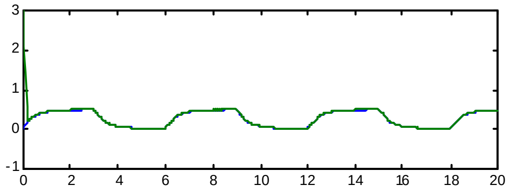
(b) $u(t)$



Time (s)

(c) $\hat{k}_1(t)$: ——— , $\hat{k}_2(t)$: – – – –, and $\hat{k}_3(t)$: ··········

Figure 4. Simulation results: $r(t) = \sin(t)$.



Time (s)

(a) $y(t)$ and $y_m(t)$