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DYNAMIC MODELLING AND IDENTIFICATION OF A CAR

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Abstract : The aim of this paper is to give a general and unifying presentation of the dynamic modelling and identification issues of a car. The modelling is based on the modified Denavit-Hartenberg geometric description, which is commonly used in robotics. The kinematics and dynamic models are automatically calculated using the software package SYMORO+. The dynamic model is used to simulate the behaviour of the car and to identify the dynamic parameters. Primarily experimental results on the dynamic identification of a real car are given.

Key words: parameter identification, automobiles, dynamic modelling, least squares.

1. INTRODUCTION

The aim of this paper is to present a general and unifying way of modelling a car, in order to simulate its behaviour and to identify the dynamic parameters. For this purpose we make use of the modified Denavit Hartenberg notations which are widely used in robotics fields.
Several techniques of derivation of kinematics and dynamic models for mobile robots are available in the literature (Tilbury *et al.*, 1994, Zodiac, 1996), but the usual approach considers systems made up of a rigid cart and rigid wheels, moving in a horizontal plane, with the constraint of rolling without slipping. Meanwhile real working conditions of motion do not satisfy such hypothesis. Consequently, it is necessary that the dynamic model takes into account the 3D motion and the forces between the wheels and the soil.

With such a complexity, a systematic method of geometrical description, based on the modified Denavit Hartenberg parameterisation (Khalil and Kleinfinger, 1986) facilitates the derivation of the dynamic and identification models. The car is considered as a tree structure multi body system, where the four wheels are the terminal links. This description allows to automatically calculate the symbolic expression of the geometric, kinematics and dynamic models by using robotics techniques or even by a symbolic software package like SYMORO+ (Symbolic Modelling of Robots)(Khalil and Creusot, 1997). The car suspensions are modelled with a lumped elasticity. Such a model allows us to calculate the inverse dynamic model which is linear with respect to the dynamic parameters, (Khosla, 1986, Khalil and Dombre, 2002, Guillo and Gautier, 2001).

2. GEOMETRICAL DESCRIPTION OF A CAR WITH ROBOTICS FORMALISM

2.1. Robotic formulation of the description of a multi-bodies system

The car is considered as a system Σ, composed of $n+1$ bodies (links) connected together by L joints, that can be prismatic or revolute, rigid or elastic.

$$\Sigma = \cup_{j=0}^{n} C_j$$

The Modified Denavit Hartenberg (MDH) notations can be applied to obtain the geometric parameters.

A frame R_j is defined attached to each body C_j. The axis z_j is defined along the joint axis j and x_j is defined as the common perpendicular with z_j and one of the following z axis.
Let us call C_i the body which is antecedent to C_j ($i = a(j)$). Denoting x_i' as the common perpendicular of z_i and z_j. The homogeneous transformation ${}^{i}T_j$ of the frame R_j with respect to R_i is expressed as a function of the following six parameters (Khalil & Dombre 2002):

- γ_j : angle between x_i and x_i' around the axis z_i,
- b_j : distance between x_i and x_i' along z_i.
- α_j : angle between z_i and z_j around the axis x_i',
- d_j : distance from z_i to z_j along x_i',
- θ_j : angle between x_i' and x_j around the axis z_j,
- r_j : distance from x_i' to x_j along z_j,

Each joint is described with two parameters σ_j and μ_j:

- σ_j describes the type of joint:
  $\sigma_j = 0$: revolute joint, the joint variable q_j is θ_j
  $\sigma_j = 1$: prismatic joint, the joint variable q_j is r_j
  $\sigma_j = 2$: locked joint
- μ_j describes the type of actuation:
  $\mu_j = 1$: actuated joint
  $\mu_j = 0$: passive joint.

2.2. Application for a car

Let R_0 be a fixed reference frame attached to the ground and C_r the reference body of the studied structure be the chassis. It corresponds to the body whose location ζ (i.e. position & orientation) gives the system posture in the frame R_0:
$\xi = [q_1\ q_2\ q_3\ q_4\ q_5\ q_6]^T$ for a movement in the three dimension space. The six degrees of freedom (d.o.f.) are described with 5 virtual bodies and one real body, with one d.o.f. for each body (Guillo and Gautier, 2000).
In that case the model of a car system can be composed of 20 bodies C_j such that (Fig.1):

- C_0 is the base attached to the ground,
- C_1, C_2,..., C_5 are virtual bodies used to define the car posture. Their variables are $x\ y\ z\ \theta\ \phi$.
- C_6 is the chassis, its variable is $q6 = \psi$
- C_7, C_{10}, C_{13} and C_{17} are the dampers, represented by linear springs. They are taken as virtual bodies, with no mass and no inertia, to simplify the system.
- C_9 and C_{12} are the rear wheels, such that $q_9 = \omega_1$ and $q_{12} = \omega_2$
- C_{16} and C_{20} are the front wheels, such that $q_{16} = \omega_3$ and $q_{20} = \omega_4$
- C_8, C_{11}, C_{15}, C_{19} are virtual bodies used to define a second frame attached to the wheel axes. They represent the frames of the measured contact forces. (see section 3.2)

Table 1: geometric parameters of the model of a car

J	a(j)	σ_j	b_j	α_j	d_j	θ_j	r_j
1	0	1	0	$\pi/2$	0	$\pi/2$	-y
2	1	1	0	$\pi/2$	0	$\pi/2$	x
3	2	1	0	$\pi/2$	0	$\pi/2$	z
4	3	0	0	0	0	ψ	0
5	4	0	0	$\pi/2$	0	$-\phi + \pi/2$	0
6	5	0	0	$\pi/2$	0	$\theta + \pi/2$	0
7	6	1	0	$\pi/2$	$-D_3$	$\pi/2$	q_7
8	7	2	0	$-\pi/2$	0	0	0
9	8	0	0	0	0	q_9	0
10	6	1	0	$\pi/2$	D_4	$\pi/2$	q_{10}
11	10	2	0	$-\pi/2$	0	0	0
12	11	0	0	0	0	q_{12}	0
13	6	1	B_{13}	$\pi/2$	$-D_1$	$\pi/2$	q_{13}
14	13	0	0	0	0	β	0
15	14	2	0	$-\pi/2$	0	0	0
16	15	0	0	0	0	q_{16}	0
17	6	1	B_{17}	$\pi/2$	D_2	$\pi/2$	q_{17}
18	17	0	0	0	0	β	0
19	18	2	0	$-\pi/2$	0	0	0
20	19	0	0	0	0	q_{20}	0

According to this description, the vehicle motion is completely described by the vector q of the 16 generalised co-ordinates:

$$q = [\xi^T\ q_7\ q_9\ q_{10}\ q_{12}\ q_{13}\ \beta\ q_{16}\ q_{17}\ \beta\ q_{20}]^T$$

- q_7, q_{10}, q_{13} and q_{17} are the dampers clearance,
- q_9, q_{12}, q_{16} and q_{20} are the angular positions of the four wheels with respect to their axis,
- $q_{14}=q_{18}=\beta$, is the steering angle.

This description allows to calculate the geometric, kinematic and dynamic models automatically with the help of SYMORO+, a software of symbolic calculations developed by the robotics team of the IRCCyN. (Khalil and Creusot, 1997)

3. DYNAMIC MODEL

The inverse dynamic model gives the joint torques as a function of the joint co-ordinates, speeds and accelerations.

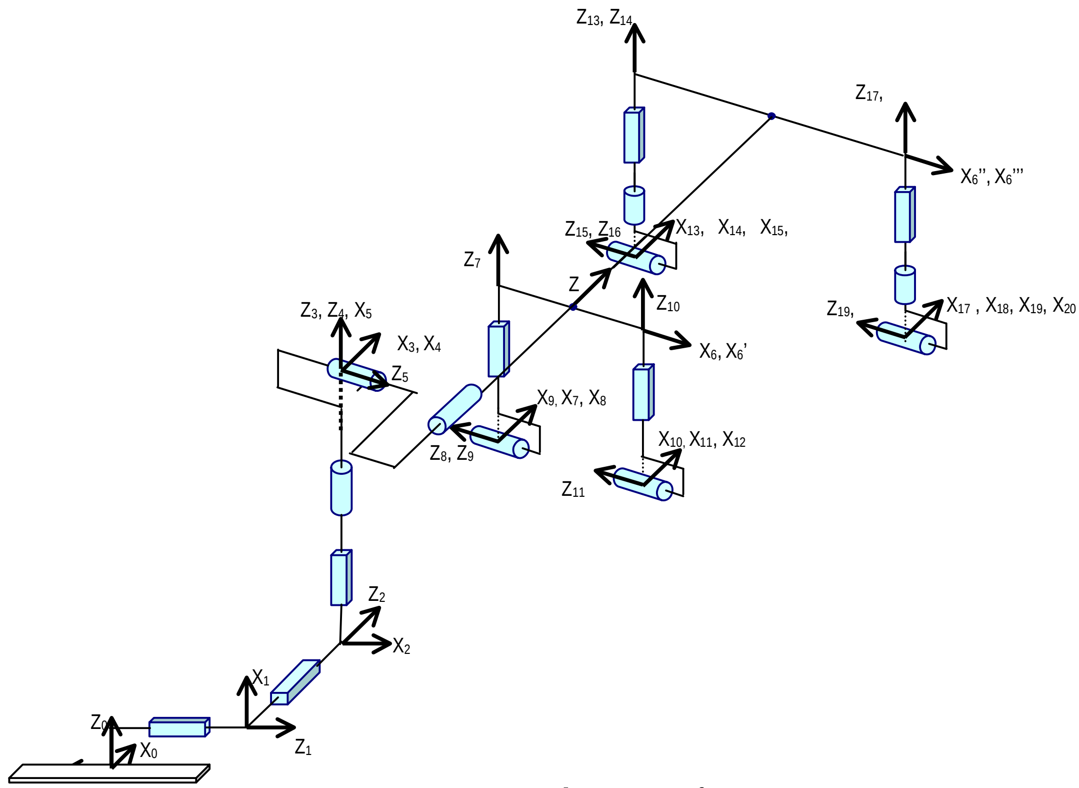

Fig. 1: geometric description of a car

Let the Inverse Dynamic Model (IDM) be written as:

$$M(q)\ddot{q} + H(q,\dot{q}) = \boldsymbol{L} + \boldsymbol{Q^c} \qquad (1)$$

- $M(q)$ is the mass matrix of the system Σ
- $H(q,\dot{q})$ is the vector of centrifugal, Coriolis and gravity terms.
- $\boldsymbol{L}$ is the vector of the internal forces between the vehicle bodies : motor torque, friction, lumped elasticity...
- $\boldsymbol{Q^c}$ is the vector of the contact forces between the ground and the wheels.

3.1. Internal forces

The internal forces vector is composed of:
- the elastic forces

The j-component of $\boldsymbol{L^e_j}$, the elastic forces vector can be written, with k_j the stiffness of the j-joint, such as:

$$\boldsymbol{L^e_j} = -k_j q_j \qquad \text{if } j \text{ is an elastic joint,}$$

where q_j is the joint displacement with respect to the initial position.
- the actuation vector component.

It depends on the value of the Boolean parameter μ_j and on the motor torque of the j-joint.

$$\boldsymbol{L^a_j} = u_j \qquad \text{if } \mu_j = 1$$

$$\boldsymbol{L^a_j} = 0 \qquad \text{if } \mu_j = 0$$

- the friction vector component, is simplified as:

$$\boldsymbol{L^f_j} = -F_{vj}\dot{q}_j - F_{sj} sign(\dot{q}_j)$$

Where:
- F_{vj} is the viscous friction coefficient.
- F_{sj} is the Coulomb friction force.

3.2. Contact forces between the wheels and the ground

In our experiments, the contact forces are measured by four dynamometric wheels. They give the 6 directional forces and torques expressed in the frames R_8, R_{11}, R_{15}, R_{19}, attached to the wheel axes.

4. IDENTIFICATION

4.1. Standard inertial parameters

For each link there are 13 standard dynamic parameters (Gautier and Khalil, 1990), composed of 10 standard inertial parameters:
- [XX_j XY_j XZ_j YY_j YZ_j ZZ_j]: the 6 elements of the inertia matrix of link j with respect to frame j,
- [MX_j MY_j MZ_j]: the 3 first moments of link j around the origin of frame j,
- M_j, the mass of link j.

And 3 drive chain parameters:
- Ia_j, the inertia of the motor for an actuated joint
- Fv_j and Fs_j, the viscous and Coulomb friction parameters.

The standard inertial parameters of the real bodies are shown in Table 2, where we did not include the parameters of a virtual body whose all parameters are zero. We take into account the symmetry of the wheels that leads to have:

$XX_i = YY_i$ for i = 9, 12, 15, 19.

4.2. Base inertial parameters

The base inertial parameters are defined as the minimum inertial parameters that can be used to obtain the dynamic model. They represent the set of inertial parameters which can be identified using the dynamic model, thus its determination is essential for the identification of the inertial parameters of the system.
They are obtained from the standard inertial parameters by eliminating those which have no effect on the dynamic model and by grouping some others. There are two techniques to obtain those parameters: a symbolic one (Gautier and Khalil, 2002), or a numerical one (Gautier, 1991). The symbolic method gives the 23 base inertial parameters given in Table 3, where we did not include the parameters of a body whose all parameters are zero. The grouping relations are:

$$XX_{5R} = XX_{12} + YY_6 + XX_9$$
$$ZZ_{5R} = XX_{12}+XX_9 + YY_6$$
$$XX_{6R} = XX_{12}+XX_6- XX_9 - YY_6$$
$$ZZ_{6R} = XX_{12}+XX_9 + ZZ_6$$
$$ZZ_{14R} = YY_{16} + ZZ_{14}$$
$$ZZ_{18R} = YY_{20} + ZZ_{18}$$

Table 2: standard inertial parameters

j	XX	XY	XZ	YY	YZ	ZZ	MX	MY	MZ	M
6	XX_6	XY_6	XZ_6	YY_6	YZ_6	ZZ_6	MX_6	MY_6	MZ_6	M_6
9	XX_9	0	0	XX_9	0	ZZ_9	0	0	0	M_9
12	XX_{12}	0	0	XX_{12}	0	ZZ_{12}	0	0	0	M_{12}
16	XX_{16}	0	0	XX_{16}	0	ZZ_{16}	0	0	0	M_{16}
20	XX_{20}	0	0	XX_{20}	0	ZZ_{20}	0	0	0	M_{20}

The base inertial parameters are computed with the symbolic method. They are given in Table 3.

Table 3: base inertial parameters

j	XX	XY	XZ	YY	YZ	ZZ	MX	MY	MZ	M
5	XX_{5R}	0	0	0	0	ZZ_{5R}	0	0	0	0
6	XX_{6R}	XY_6	XZ_6	0	YZ_6	ZZ_6	MX_6	MY_6	MZ_6	M_6
7	0	0	0	0	0	0	0	0	0	M_9
9	0	0	0	0	0	ZZ9	0	0	0	0
10	0	0	0	0	0	0	0	0	0	M_{12}
12	0	0	0	0	0	ZZ_{12}	0	0	0	0
13	0	0	0	0	0	0	0	0	0	M_{16}
14	XX_{16}	0	0	0	0	ZZ_{14R}	0	0	0	0
16	0	0	0	0	0	ZZ_{16}	0	0	0	0
17	0	0	0	0	0	0	0	0	0	M_{20}
18	XX_{20}	0	0	0	0	ZZ_{18R}	0	0	0	0
20	0	0	0	0	0	ZZ_{20}	0	0	0	0

4.3. Base dynamic parameters

The base dynamic parameters, defining the vector X to be identified, consists of the base inertial parameters, the friction parameters, the stiffness parameters of the dampers **ks7, ks10 ks13 ks17**, and their offsets off7, off10, off13, and off17.

4.4. Identification model

The dynamic model (1) can be expressed as a linear relation with respect to the identifiable parameters. It can be written as:

$$y=D(q,\dot{q},\ddot{q}).X \qquad (2)$$

4.5. Identification method

The dynamic parameters are estimated by solving an over-determined linear system with the weighted least squares techniques (W.L.S.). The system is obtained by sampling the identification dynamic model (3) along a trajectory.
The whole system is written as follow:

$$Y = W.X + \rho \qquad (3)$$

- W is the observation matrix (n_{exp} x n_b)
- Y the (n_{exp} x 1) vector of joint and reaction forces
- ρ the (n_{exp} x 1) vector of model errors
- X the vector of the dynamic parameters to identify (n_b x 1)

Some practical features about the weighting procedure and data filtering are given in (Gautier, 1997), in order to obtain the .L.S. estimation of parameters with minimum distortion and bias.

5. EXPERIMENTAL SET UP

Primarily experimental results are given. For the data acquisition, a real car is equipped with many sensors which allow to estimate the variables needed for the identification model.
Those sensors are: four dynamometric wheels, an inertial unit, a laser sensor for vertical position, video camera “Zimmer”, and a speed sensor “Correvit”.

5.1. Measurement and filtering

The joints are provided with the following sensors:
- Joints 1 and 2: the velocity sensor used is a “Correvit”.
- Joint 3: the sensors used are the four lasers which measure the altitude of the four corner of the car. The vertical position is then deduced geometrically, and the speed and acceleration are obtained by derivation.
- Joints 4 and 5: the sensor used is a “Zimmer” camera, it gives the joint speed. The joint accelerations and positions are obtained by a derivation and an integration respectively.
- Joint 6: the sensor used are the inertial unit.
- Joints 7, 10, 13 and 17: the sensor used is a specific sensor for clearance.
- Joints 9, 12, 16 and 20: the sensors used are the dynamometric wheels.

- Joints 14 and 18: the sensor used is a "Zimmer" autocollimator.

The sensor measurements (position, velocity or acceleration), are filtered in order to estimate the position, the velocity and the acceleration for each joint. The filter chosen is a pass band filtering. This operation is carried out using the function *filtfilt* of Matlab with an order between 5 to 8 and a cut-off frequency chosen according to the dynamic of the system and the sensor (Gautier, 1997).

The derivatives are estimated without phase shift using a central difference algorithm. And the integration is estimated with a trapeze method.

5.2. Computation of the vector of joint and reaction forces: Y

In robotics applications it is possible to directly measure *Y*, the right terms of equation (3). But in the case of the car this measure is not possible and we have to take into account the external contact forces. Thus the dynamic model can be written as:

$$\Gamma = DX + \sum J_i^T F_j = DX + \Gamma_{ext} \qquad (4)$$

Though , the general expression used for the identification is the following:

$$\Gamma - \Gamma_{ext} = D.X \qquad (5)$$

Where Γ_{ext} is computed using SYMORO$^+$ and the determination of Γ depends on the type of joint considered:

- For a virtual joint $\Gamma = 0$,
- For an actuated joint $\Gamma = \Gamma_m$,
- For an elastic joint of known stiffness k_i: $\Gamma = - ks_i.q_i$,
- For an elastic joint of unknown stiffness: $\Gamma = 0$, and the stiffness has to be identified.

6. RESULTS AND INTERPRETATION

The following results are obtained with two different tests: a ramp steer at 90km/h on the left, and a ramp steer at 110km/h on the right. Trajectories can be seen on Figure 4. The sample time is 0.016 second, and the number of samples is about 3000. Parameters are given in SI units. For this special trajectory, not all the parameters are identified because of a lack of excitation.

6.1. Results

The estimated values and the relative standard deviation are shown in table 4.

The estimation concerning the dampers even with simple trajectories are good, also good results concerning the mass and the first moments have been obtained.

6.2. Interpretation and validation

The identification of the dynamic parameters gives us an estimation of the mass of the vehicle (without wheels) and of the position of the centre of gravity with respect to the frame R_6 attached to the vehicle.

The estimated error for the mass is very low which allow to say that the identification gives a good estimation. For the position of the centre of gravity, the tests give a good estimation of its firs moments (MX_6, MY_6, MZ_6).

The results of the estimation of the mass M_6 can be checked by calculating the mass from the measurement of contact forces at time t = 0 (the car is stopped).

Table 4: results by concatenating both ramps steer

Parameters	Value	Relative standard deviation
mx6	-7.70	14.11
my6	-726.52	1.55
mz6	**2611.82**	**0.42**
m6	**1566.20**	**0.78**
off7	814.11	2.53
ks7	**70699.05**	**1.01**
h7	4986.81	14.40
off10	223.04	5.39
ks10	**65954.44**	**0.70**
h10	5483.00	9.87
off13	**719.89**	**1.63**
ks13	**60327.89**	**0.69**
h13	5201.78	9.12
off17	**1548.17**	**1.02**
ks17	**66724.52**	**0.89**
h17	4251.09	15.14

We can consider at t = 0 that the following relation is true:

$$F_{z1}+F_{z2}+F_{z3}+F_{z4} = M_{tot}.g$$

$$M_{tot} = (3100 + 3190 + 4720 + 4710)/9.8 = 1604 \text{ kg}$$

Where M_{tot} is the total mass of the vehicle, plus all the sensors and the driver. Thus we have to subtract the unsprung mass, that is composed of the four wheels and the two rear axles and the two front axles. Which gives us:

$$M_{6cal} = 1604 – (4*18.8 + 16*2 + 12.8*2) = 1471 \text{ kg}$$

Which is, compared to the identified mass:

$$(1566.20 – 1471)/1471 = 6.47 \%$$

Another validation method consists of comparing *Y* to the reconstructed vector *W.X*. (Figure 2,3,4)

Figures 2 and 3 give the results of a simple validation: *W.X* is compared to *Y* on one of the trajectory that has served to estimate *X*.

Figure 7 gives the results of a cross validation: the trajectory is not the one that has been used for the estimation: the test used for the validation is a braking in straight line.

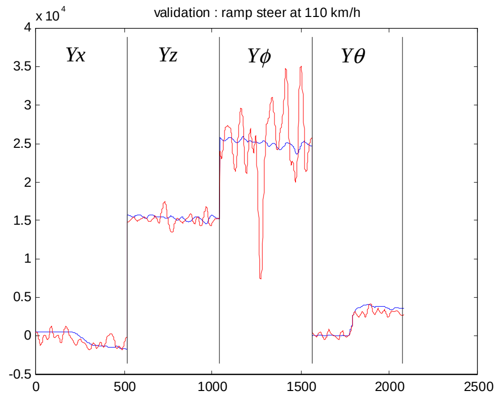

Figure 2: validation by comparing Y to $W.X$

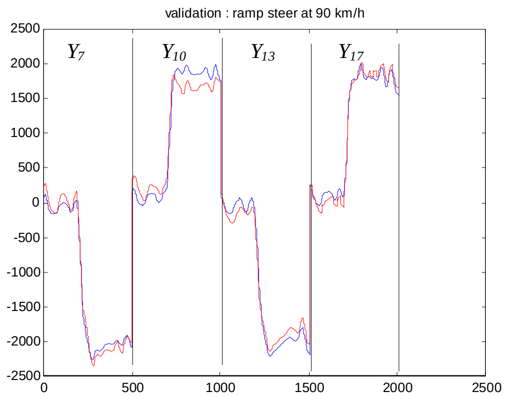

Figure 3: simple validation with the ramp steers

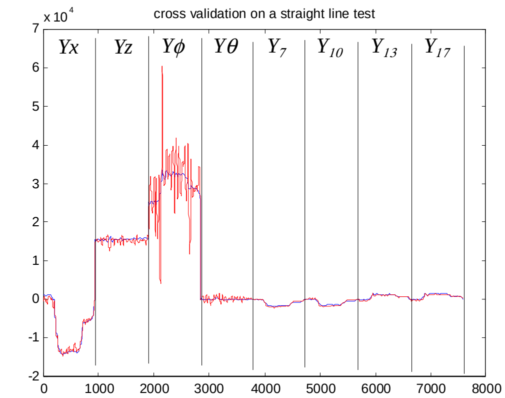

Figure 4: cross validation on a braking straight line

7. CONCLUSION

Despite the encouraging results for the estimation of some parameters, it is important to notice that the used trajectories do not excite all the parameters. Besides, the model used is a simplified one, which does not take into account all the behaviour of a car such as the camber angle. These points have to be considered in order to improve the estimation quality, and to be able to estimate all the parameters. Although, the contact between the ground and the wheels has an important effect on the behaviour of a car, an identification of the specific parameters of the tire: Pacejka parameters, then will be added as to have a better idea of this behaviour.

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