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PULSE-STEP CONTROL

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Abstract The paper presents a new method for obtaining fast response to set point changes by using a nonlinear feedforward which is obtained by solving a nonlinear equation. Pulse-step control is simple and gives a settling time close to the time where the impulse response reaches its maximum. The fact that the response time is matched to the process dynamics makes the method more robust than time-optimal control. The feedforward generated by pulse-step control can conveniently be combined with feedback based on PID control.

Keywords Feedforward control, Set point response, PID control, 2DOF.

1. INTRODUCTION

Regulation is typically the main task of PID controllers, but in many applications is also important to have a fast response to set-point changes. This can be enhanced by using a controller structure having two degrees of freedom, see (Horowitz (1963)), often in the form of set point weighting, see (Araki (1984)) and (Araki (1988)). Set point weighting as well as the standard system with two degrees of freedom are linear methods. Substantial improvements can be obtained by using nonlinear methods which takes the natural limitations on the control signal into account.

It is natural to consider set-point changes as minimum-time problems. For linear systems with actuator constraints it follows from Pontryagin's maximum principle that the minimum-time solution is a bang-bang strategy, see (Pontryagin *et al.* (1962)), (Athans and Falb (1966)) and (Bryson and Ho (1969)). The complexity of the minimum-time control strategy increases with the order of the system. For systems with real poles the number of switches is less or equal to $n-1$, where n is the order of the system. For systems with complex poles there may be a very large number of switches. It is straightforward to find the switching surface for second order systems with real poles. The complexity of the switching surfaces increases rapidly with the order of the system.

A simple method to generate control signals that give fast set point changes is developed in this paper. The method is inspired by optimal control but it can also be regarded as an attempt to mimic how experienced operators make fast set point changes. The method is called *pulse-step control* because of the form of the control signal. The pulse-step strategy only has three switches. It gives surprisingly good results for the systems that are typically encountered when using simple controllers. A nice feature is that the sensitivity of the minimum-time solutions is avoided since the solution time is matched to the dynamics of the system. It is natural to combine pulse-step feedforward with feedback so that the set point change is executed under closed loop. The feedback can then compensate for modeling errors and disturbances.

2. PULSE-STEP CONTROL

The method presented in this paper is inspired by the time-optimal bang-bang control strategy. Suboptimal strategies can be obtained by restricting the control signals. The following is one possibility

$$u(t) = a\delta(t) + b\theta(t-T)$$

where $\delta(t)$ is the Dirac impulse function, $\theta(t)$ is the Heaviside step function, and a, b and T are constants. The output then becomes

$$y(t) = ah(t) + bS(t-T)$$

where h is the impulse response and S the step response. The control signal and the corresponding output are shown in Figure 1. The parameter a is chosen so that the desired steady state is reached at the time t_{max} where the impulse response has its maximum. The parameter b is chosen to give the desired steady state and the

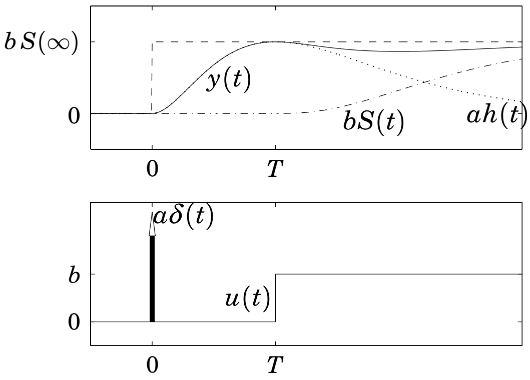

Fig. 1 Response to an impulse and a delayed step.

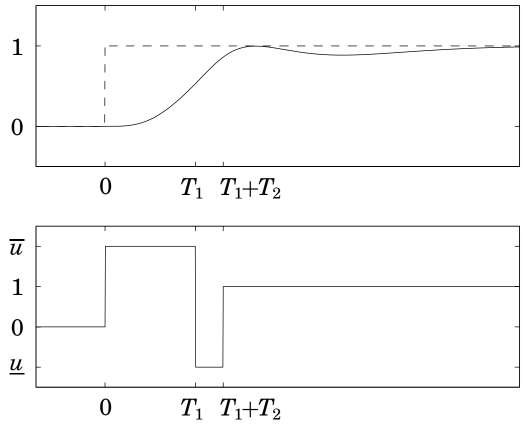

Fig. 2 Pulse-step control for the normalized problem.

parameter T is selected so that the the response is as close as possible to the desired value.

To obtain a realizable input that satisfies the constraints on the control signal the impulse is replaced by a pulse with approximately the same area as the impulse and we obtain the control signal shown in Figure 2 which we call the pulse-step control. The control signal is characterized by four parameters only: $\overline{u}$, $\underline{u}$, T_1, and T_2, where the first two parameters are typically given by the specifications. In reality the feedforward control signal is thus characterized by two parameters only. It will be shown that this simple strategy will give good results for the typical processes encountered in simple control problems. Also notice that the settling time is approximately equal to the time where the impulse response has its maximum. The response time is thus matched to the system dynamics.

3. AN OPTIMIZATION PROBLEM

Having described the method intuitively we will now state the optimization problem formally, and give an algorithm for its solution.

Consider a stable linear time-invariant system with the transfer function

$$G(s) = e^{-sT} G_0(s) \tag{1}$$

where $G_0(s)$ has the series expansion

$$G_0(s) = K(1 - T_{ar}s + \ldots) \tag{2}$$

for small s. The parameter T_{ar} is the average residence time of the system, see (Åström and Hägglund (1995)). The time delay is not important for the feedforward problem, because the feedforward strategies for $G(s)$ and $G_0(s)$ are the same.

Assume that it is desired to change the process variable from y_0 to $y_1 = y_0 + \Delta y$. The control signal changes from u_0 to $u_1 = u_0 + \Delta y/K$, where the values of the control signal should be in the interval (u_{min}, u_{max}). Inspired by the impulse-step strategy the control signal is restricted to be a combination of a pulse and a step, see Figure 2.

3.1 *Normalization*

It is useful to normalize the problem by scaling so that the desired change of the output is 1 and to normalize the control signal to

$$u(t) = \begin{cases} 0, & t < 0 \\ \overline{u}, & 0 \le t < T_1 \\ \underline{u}, & T_1 \le t < T_1 + T_2 \\ 1, & t \ge T_1 + T_2 \end{cases} \tag{3}$$

where $\overline{u}$ and $\underline{u}$ are the maximum and minimum values of the control signal in the normalized problem, see Figure 2. The problem is meaningful only if $\overline{u} \ge 1$, $\underline{u} \le 0$, $T_1 \ge 0$ and $T_2 \ge 0$. When translating the original problem to the normalized setup, the maximum and minimum levels of the control signal are transformed as

$$\overline{u} = \frac{K}{\Delta y}(u_{max} - u_0) = \frac{u_{max} - u_0}{u_1 - u_0} \tag{4}$$

$$\underline{u} = \frac{K}{\Delta y}(u_{min} - u_0) = \frac{u_{min} - u_0}{u_1 - u_0} \tag{5}$$

if $K/\Delta y > 0$. Otherwise, $\overline{u}$ and $\underline{u}$ will just exchange places in the formulas above. The normalized parameter $\overline{u}$ which is the ratio of the maximum control signal to the value required for the desired steady state is a measure of the control authority.

The set-point change should be done so that there is no overshoot, i.e.

$$y(t) \le 1, \qquad \forall t \tag{6}$$

and the criterion

$$IE = \min_{T1,T2} \int_0^\infty \Big(1 - y(t)\Big) dt \qquad (7)$$

is minimized. Straight forward calculations give the following simple analytical expression

$$IE = T_{ar} + \underbrace{(1-\overline{u})}_{<0} T_1 + \underbrace{(1-\underline{u})}_{>0} T_2 \qquad (8)$$

where T_{ar} is the average residence time for the system, see (Wallén (2000)). The integrated error consists of one part T_{ar} which depends on the process model, and one part $(1-\overline{u})T_1 + (1-\underline{u})T_2$ which depends on the control strategy. Since $(1-\overline{u}) < 0$ and $(1-\underline{u}) > 0$, T_1 should be large and T_2 small in order to make *IE* small. If T_1 is made too large, though, the zero overshoot constraint will clearly not be met.

3.2 *Main Result*

We have the following result:

THEOREM 1—OPTIMALITY CONDITIONS
Let the limits $\overline{u}$ and $\underline{u}$ of the control signal be given. Assume that the system has a pole excess greater than 2. The switching times that minimize the criterion *IE* are then given by

$$\begin{aligned}
&\overline{u}\, S(t^\star) + (\underline{u} - \overline{u})\, S(t^\star - T_1{}^\star) \\
&+ (1-\underline{u})\, S(t^\star - T_1{}^\star - T_2{}^\star) = 1 \\
&\overline{u}\, h(t^\star) + (1-\overline{u})\, h(t^\star - T_1{}^\star) = 0 \\
&h(t^\star - T_1{}^\star) - h(t^\star - T_1{}^\star - T_2{}^\star) = 0
\end{aligned} \qquad (9)$$

□

PROOF 1
Since the objective function is linear in the parameters T_1 and T_2, it is clear that the constraint must be active at the optimal solution $(T_1{}^\star,\, T_2{}^\star)$. Thus, there exists a time $t = t^\star$ with $y(t^\star) = 1$. Note that $t^\star$ may be infinite in some cases, for example if $\overline{u} = 1$. Furthermore, if the pole excess of $G(s)$ is greater than 2, both $y(t)$ and $\dot{y}(t)$ will be continuous, so it will also hold that $\dot{y}(t^\star) = 0$. The condition for the process variable to reach the desired value at time t is

$$\begin{aligned}
y(t, T_1, T_2) = {} & \overline{u}\, S(t) + (\underline{u} - \overline{u})\, S(t - T_1) \\
& + (1-\underline{u})\, S(t - T_1 - T_2) = 1
\end{aligned} \qquad (10)$$

and the condition that the derivative is zero at time t is

$$\begin{aligned}
\dot{y}(t, T_1, T_2) = {} & \overline{u}\, h(t) + (\underline{u} - \overline{u})\ h(t - T_1) \\
& + (1-\underline{u})\ h(t - T_1 - T_2) = 0
\end{aligned} \qquad (11)$$

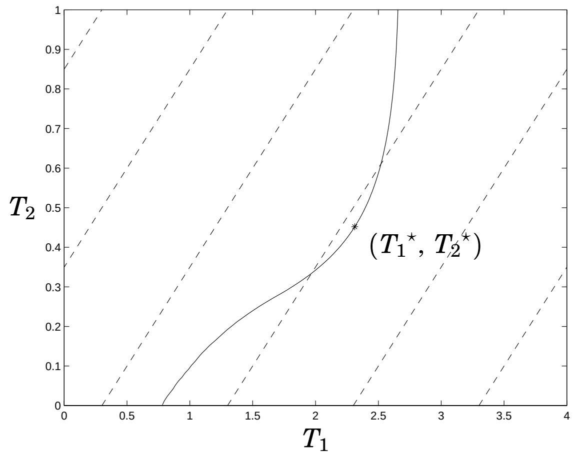

Fig. 3 Locus of $(T_1,\, T_2)$ values such that $y(t) = 1$ and $\dot{y}(t) = 0$ for some t. The dashed lines are the level curve for the criterion *IE*. The curves are constructed for $G_0(s) = 1/(s+1)^4$ with $\overline{u} = 2$ and $\underline{u} = -1$.

The pairs $(T_1,\, T_2)$ which satisfy the Equations (10) and (11) implicitly define a curve C in the $(T_1,\, T_2)$-plane as is shown in Figure 3. The level curves of the loss function *IE* given by (8) are straight lines in the $(T_1,\, T_2)$-plane. The value of *IE* decreases from the top left to the bottom right corner in the figure. Provided that the curve C is smooth at the optimum, it is clear that the tangent to C at the optimum will be parallel to the level curves of *IE*. This will give a third condition for the optimal solution. By setting *IE* constant in Equation (8) we obtain the following condition for optimality

$$\frac{dT_2}{dT_1} = \frac{\overline{u} - 1}{1 - \underline{u}}$$

for the level curves. Using the implicit function theorem, the slope of C is given by

$$\begin{aligned}
\frac{dT_2}{dT_1} &= -\frac{\partial y(\hat{t}, T_1, T_2)}{\partial T_1} \Big/ \frac{\partial \dot{y}(\hat{t}, T_1, T_2)}{\partial T_2} = \\
&= -\frac{(\underline{u} - \overline{u})\ h\,(\hat{t} - T_1)}{(1-\underline{u})\ h\,(\hat{t} - T_1 - T_2)} - 1
\end{aligned}$$

Equating the expressions for the slope at the optimal solution we get

$$\frac{\overline{u} - 1}{1 - \underline{u}} = -\frac{(\underline{u} - \overline{u})\ h\,(t^\star - T_1{}^\star)}{(1-\underline{u})\ h\,(t^\star - T_1{}^\star - T_2{}^\star)} - 1$$

which gives the following condition for optimality

$$h\,(t^\star - T_1{}^\star) = h\,(t^\star - T_1{}^\star - T_2{}^\star) \qquad (12)$$

Evaluating Equation (11) for $t = t^\star$ and combining with Equation (12) it follows that

$$\overline{u}\, h\,(t^\star) + (1-\overline{u})\, h\,(t^\star - T_1{}^\star) = 0 \qquad (13)$$

The conditions for optimality (10), (11) and (12) can be written as (9). □

Remark 1: A sufficient condition for the existence of an optimal solution is that $S(t) \leq 1$, $\forall t$. $(T_1, T_2) = (0, 0)$ will then be a feasible solution with $IE = T_{ar}$. IE may be smaller than this by selecting $T_1 > 0$. However, if $\overline{u} > 1$, there exists a time T_{1max} such that $\overline{u}S(t)$ becomes greater than 1 at $t = T_{1max}$. This gives a lower (mostly unachievable) bound on $IE = T_{ar} + T_{1max}(1 - \overline{u})$.

Remark 2: If the impulse response is unimodal it follows from Equation (12) that

$$t_{max} + T_1 < t^\star < t_{max} + T_1 + T_2 \qquad (14)$$

Remark 3: The zero-overshoot constraint in the calculations above may easily be relaxed to $y(t^\star) = y_{max}$ for some constant $y_{max} \geq 1$.

3.3 *Numerical solution*

The optimality conditions given by Equation (9) is a nonlinear equation for the switching times T_1 and T_2 and the time $t^\star$ which represents the settling time for the system. The conditions are expressed in terms of the step response S and the impulse response of the system. If the transfer function $G_0(s)$ is known analytically it is possible to derive analytical conditions. The equations can be solved using a Newton-Raphson iteration. By using optimization methods that only use function evaluation the switching times can also be computed from measured step responses. Good initial conditions can be found from the impulse-step approximation. Details are given in (Wallén (2000)).

4. EXAMPLES

The properties of the pulse-step method will be illustrated by two examples.

EXAMPLE 1—COMPARISON WITH PID CONTROL
Consider a system with the transfer function

$$G(s) = \frac{1}{(s+1)^4} \qquad (15)$$

Figure 4 shows a comparison between the fast set point response method and two different PID settings. The PID controllers have been designed using the method in (Panagopoulos (2000)) with maximum sensitivity $M_s = 1.4$ and $M_s = 2.0$, respectively. The controller parameters are $K = 1.14$, $T_i = 2.23$, $T_d = 1.0$ and $b = 0$ for $M_s = 1.4$, and $K = 2.29$, $T_i = 1.92$, $T_d = 0.98$ and $b = 0$ for $M_s = 2.0$. The PID controllers have a settling time of about 12. The

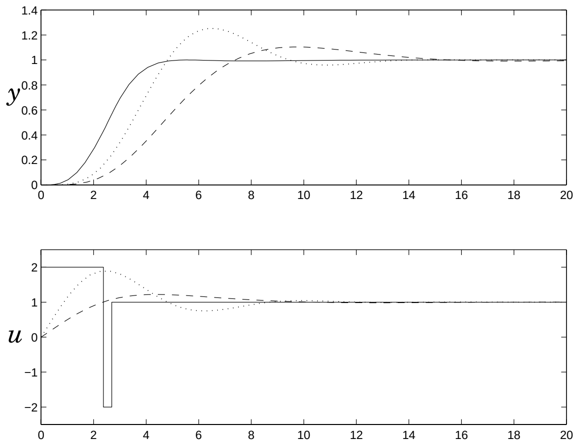

Fig. 4 Comparison between the fast set point response strategy (full) and PID control with $M_s = 1.4$ (dashed) and $M_s = 2.0$ (dotted) for $G(s) = 1/(s+1)^4$. The control signal is limited to $\overline{u} = 2$ and $\underline{u} = -2$ and the feedforward signal is characterized by $T_1 = 2.37$, $T_2 = 0.32$ and $t^\star = 5.54$.

pulse-step response in Figure 4 were computed with $\overline{u} = 2$ and $\underline{u} = -2$, and the resulting rise time and settling time are approximately 4 time units. This is a little larger than the time it takes the impulse response of the open loop system to reach its maximum, which is 3. With pulse-step control the settling time can be reduced towards 3 by increasing the span of the control signal. There is a significant improvement over PID control even if the largest values of the control signal for PID control with $M_s = 2$ (the dashed curve) is almost the same as $\overline{u}$. The figure clearly illustrates the power of pulse-step control. □

Other examples in (Wallén (2000)) are similar to the example given above. Pulse-step control typically gives settling times that are 2 to 3 times faster than well tuned PID controllers.

Pulse-step control is equivalent to minimum-time control for second order systems with real poles and no zeros. The strategies are different for system with zeros because the output y and its derivative $\dot{y}$ are not states for such systems. For systems of higher order the pulse-step control and minimum-time control may differ significantly because minimum-time control may have many more switches. We illustrate this by an example.

EXAMPLE 2—COMPARISON WITH MINIMUM-TIME CONTROL
Consider the same system as in Example 1 with the transfer function (15). Since the system has real poles and is of fourth order, the time-optimal strategy may have two more switches than the pulse-step strategy.

Figure 5 shows comparisons with time-optimal

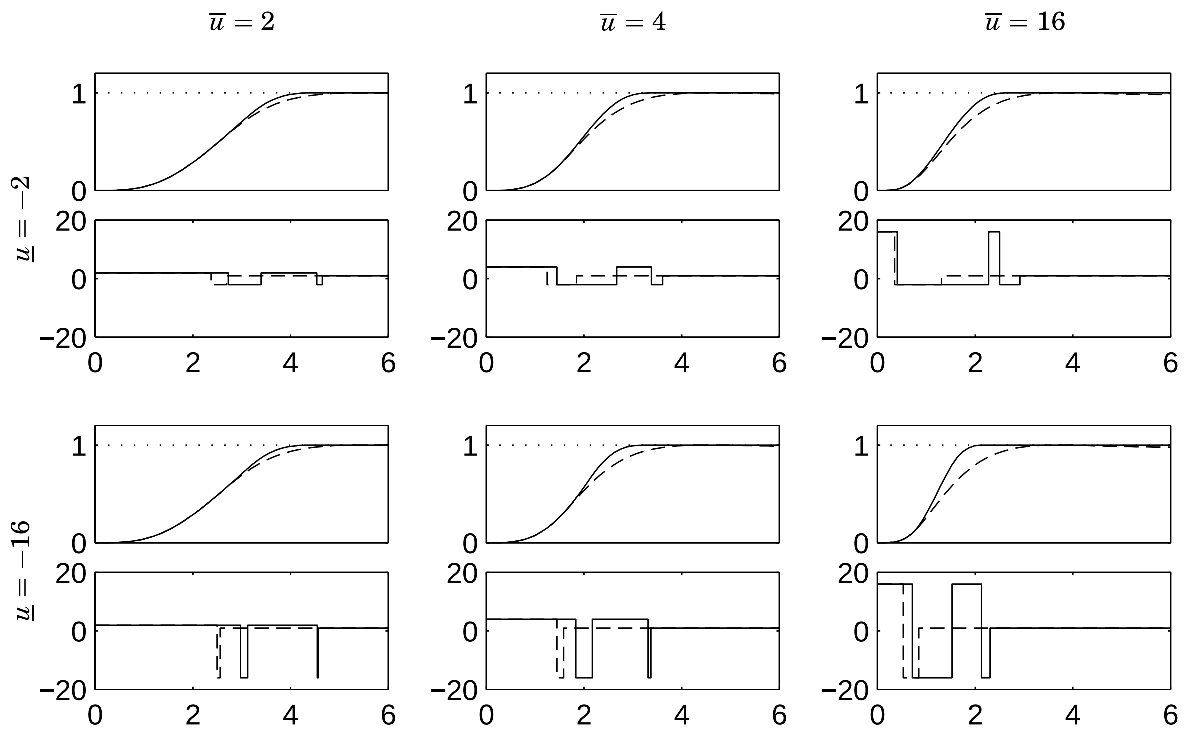

Fig. 5 Comparison between the time-optimal controller (solid lines) and pulse-step strategy (dashed lines) for $G(s) = 1/(s+1)^4$. The benefit of large negative control signals is marginal, when combined with small positive control signals.

control for different control authorities. The largest control signal varies between 2 and 16. Each group of two plots shows the behavior for one pair $(\underline{u}, \overline{u})$. The upper plot in each group shows the output and the lower plot shows the control signal, with full lines for the time-optimal strategy and dashed lines for the pulse-step method. The figure indicates that the relative merit of the time-optimal strategy increases as the size of $\underline{u}$ and $\overline{u}$ increase. The reason for this is that the control signal drives the system very hard in one direction and then attempts to stop the motion of the system, see for example the case with $\overline{u} = -\underline{u} = 16$. As one might expect, Figure 5 also shows that $\overline{u}$ is the more important parameter. The gain by having a large negative $\underline{u}$ compared to $\underline{u} = 0$ is very small, particularly when $\overline{u}$ also is small.

The settling time, defined as the first time when $y(t) = 1$ and $\dot{y}(t) = 0$, as a function of $\overline{u}$ is shown in Figure 6. When the input range is $[-\overline{u}, \overline{u}]$, the settling time for minimum-time control goes to zero when the control authority $\overline{u}$ goes to infinity. For pulse-step the settling time approaches a constant value equal to the time for the open loop impulse response to reach its maximum. If the input range is instead $[0, \overline{u}]$, the settling time converges to a positive constant for both strategies. For the pulse-step method the value is a little larger than the time where the impulse response has its maximum.

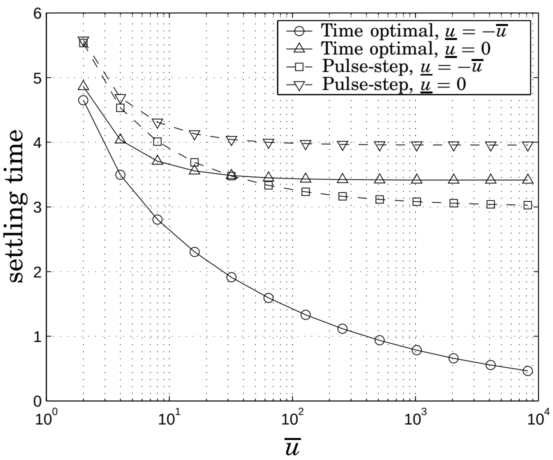

Fig. 6 Settling time as a function of the magnitude of the control signal with time-optimal controller and the pulse-step strategy for $G(s) = 1/(s+1)^4$.

The reason for the qualitative difference between the cases is that when $\underline{u} = -\overline{u}$ the process can be driven very rapidly in one direction and then stopped by applying a large negative control signal. This is not possible when $\underline{u} = 0$. In this case the settling time is essentially given by the time where the impulse response has its maximum. It is intuitively clear that control strategies that drive the system very hard in one direction and later stops the motion by applying very large negative control signals are sensitive, see (Åström and Furuta (2000)). The strategies where $\underline{u}$ is zero or has a small negative value are therefore more robust. □

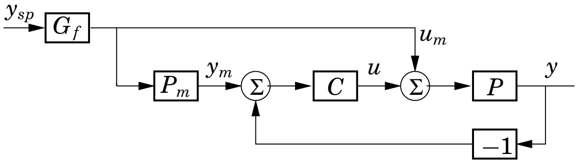

Fig. 7 Block diagram of a system having two degrees of freedom where the feedforward is generated by pulse-step control.

5. IMPLEMENTATION

Pulse-step control is an open loop strategy. It is natural to combine it with PID control using a controller strategy with two degrees of freedom. A block diagram for such a system is shown in Figure 7. The block G_f is a nonlinear block which generates the pulse-step feedforward signal u_m based on the current set point, the desired set point and the limitations on the control signal. The signal u_m is characterized by two parameters times T_1 and T_2, which are obtained by solving Equation (9). This calculation also gives an estimate of the achievable settling time $t^\star$. The signal y_m which is the ideal output is generated by feeding the signal u_m through a model P_m of the process.

The feedforward generator G_f can also be implemented as a table. This is particularly simple when $\underline{u} = 0$. In this case the table entries are the desired set point change, T_1 and T_2, see Equation (4). The table can be precomputed based on a step response that is typically obtained when tuning the PID loop.

The feedback C is typically a PI or a PID controller. The feedforward signal will drive the system towards the desired response. In the ideal case the signals y and y_m will match and the feedback error will be zero. Deviations due to modeling errors or disturbances will be taken care of by the feedback. Thus, the robustness of the system is more related to the feedback design than to the pulse-step control itself. See (Wallén (2000)) for further discussion and results.

6. CONCLUSIONS

A new method which gives fast set point changes that is suitable for simple controllers has been developed. The basic idea is to use a feedforward signal of bang-bang type with only three switches. Such a signal is characterized by two parameters which can be determined by solving an optimization problem. Optimality conditions are given in terms of a nonlinear equation which can be solved iteratively. Simple estimates of good initial conditions for the iteration have been provided.

The settling time obtained is close to the time where the impulse response of the system has its maximum. It is thus matched to the dynamics of the system. Because of this it is much more robust than minimum-time control which often drives the system very hard in one direction and then attempts to stop it by using several switches.

The strategy has been tested in a number of cases. It typically gives a significant improvement over what is achieved by PID control. Since it is not overly complex it seems very worth while to use it.

The method can also be extended to signals with rate limitations as is discussed in (Wallén (2000)).

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