15th Triennial World Congress, Barcelona, Spain

ADAPTIVE CANONICAL FORM FOR SLIDING MODE CONTROL OF UNCERTAIN NONLINEAR SYSTEMS

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Abstract: In this article, *projection operators* are used for obtaining an adaptive canonical form for sliding mode controlled uncertain nonlinear systems. For systems already in this canonical form, a simple variable structure input coordinate transformation together with an update law for the unknown parameters are used to achieve passivity and, thus, a feedback sliding mode control can be synthesized to require that the state trajectories slide on a given submanifold of the state space.

Keywords: Nonlinear systems, Passive compensation, Sliding mode control, Adaptive control

1. INTRODUCTION

Passivity based control enjoys popularity due to several advantages related to controller simplicity, robustness and the physically appealing features of the approach. A seminal contribution is that of Willems (1972) in the context of *dissipative systems*. The works by Hill (1976, 1977) constitute also a general approach with emphasis on conditions for stability of feedback interconnected systems. A geometric approach to feedback equivalence of nonlinear passive systems was contributed by the work of Byrnes *et al* (1991). The passivity approach has found application developments in the area of robotics, electrical machines and power electronics. The reader is referred to the works by Ortega and his coworkers (1995), and Sira-Ramírez *et al* (1997). The initial developments of passivity based control were carried out within the context of Hamiltonian systems, with a view towards applications in the control of robotic manipulator systems (Takegaki and Arimoto, 1981) General developments were later advanced, within the formulation of Lagrangian systems, in the work of Ortega *et al* (1995) and Shishkin *et al* (1996), and also in the context of Hamiltonian systems (van der Schaft, 1996). Nevertheless, passivity based control should not be regarded as a control technique which is suitable only in the field of electromechanical systems (or, *lagrangian* systems in short). Applications in the context of chemical and biochemical processes are also possible, as demonstrated by the works of Sira-Ramírez and Delgado (1997), Sira-Ramírez and Angulo-Núñez (1997); and more recently, by the approach proposed by Ríos-Bolívar *et al* (2000).

On the other hand, Sliding Mode Control (SMC) is a robust control approach which employs discontinuous control to enforce the system state trajectories to slide on a prescribed manifold, (Utkin, 1978). Recently, Sira-Ramírez (1998) proposed a general canonical form for sliding mode controlled nonlinear systems which have provided further insights on the connections between passivity based control and SMC.

In this article, a further extension of this latter approach is used to achieve an adaptive canonical form for sliding mode control nonlinear systems containing constant but unknown parameters. A *natural* drift vector field decomposition is firstly obtained on the basis of a *projection operator*, induced by the given sliding surface function and the control input vector field. This decomposition is aimed at naturally exhibiting the *workless*, the *attracting* (or beneficial) and the locally *rejecting* forces acting on the uncontrolled motions of the system. The workless or conservative forces yield invariance of the switching surface coordinate, the beneficial forces make the sliding manifold attractive and try to drive the surface coordinate function to lower absolute values while, the rejecting forces drive the system to achieve higher absolute values of the switching surface coordinate. These two last forces change their nature depending on the local sign of the surface coordinate function i.e., attracting forces *above* the surface become repelling forces *below* the surface and viceversa. The adaptive canonical field is obtained by incorporating estimate of the unknown parameter and taking into account the parameter estimate error. Then, by proposing an update law for the uncertain parameters and, by suitably respecting the local beneficial nonlinearities, on each side of the sliding surface, an autonomous non-divergence from the sliding surface is guaranteed and thus, the variable structure feedback controller yielding convergence towards the surface can be easily synthesized. The controller design simply consists in injecting *damping*, or attractivity terms, which suitably complement the local beneficial nonlinearities of the system.

2. A CANONICAL FORM FOR SLIDING MODE CONTROL

2.1 *Assumptions, definitions and results*

Consider the class of nonlinear single-input single-output systems described by

$$\begin{aligned} \dot{x}(t) = f(x) + g(x)u \ ; \ \ x \in \mathcal{X} \subset \mathbf{R}^n \ ; \\ y = \sigma(x) \ ; \ y \in \mathcal{Y} \subset \mathbf{R} \ \ u \in \mathcal{U} \subset \mathbf{R} \end{aligned} \qquad (1)$$

where $\mathcal{X}$ denotes the *operating region* of the system, constituted by a sufficiently large open set containing a continuum of equilibrium points, possibly parameterized by a constant control input value $u = U \in \mathcal{U}$, of the form $x = \overline{x}(U)$ and given by the solution of $f(\overline{x}) + g(\overline{x})U = 0$. In particular, for $u = 0$, we assume $f(\overline{x}) = 0$ implies $\overline{x} = 0$. However, motivated by a large class of real life systems, we are specifically interested in *nonzero* constant state equilibrium points $x = \overline{x}$, corresponding to nonzero constant control inputs $u = U$. The output function $y = \sigma(x)$ is assumed to be zero at the equilibrium point, i.e., $\sigma(\overline{x}) = 0$.

We assume that $\sigma(x)$ is a C^1 scalar function, called the *sliding surface function* $\sigma : \mathbf{R}^n \rightarrow \mathbf{R}$ such that when the state trajectories are confined to its zero level set $\mathcal{S}_0 = \{x \in \mathcal{X} \ : \ \sigma(x) = 0\}$, the behaviour of the system is asymptotically stable towards a given equilibrium.

The column vector field $\partial\sigma/\partial x$, with components $\partial\sigma/\partial x_i \ i = 1, \ldots, n$, denotes the gradient field of σ. The transpose of this gradient field is denoted by the row vector $\partial\sigma/\partial x^T$. Let $L_g\sigma(x)$ denote the directional derivative of the scalar function $\sigma(x)$ with respect to the control input vector field $g(x)$ at the point $x \in \mathcal{X}$. We assume throughout the entire article that the following assumption holds valid:

$$L_g\sigma(x) = \frac{\partial\sigma}{\partial x^T} g(x) \neq 0 \qquad \forall x \in \mathcal{X}$$

This condition is usually known as the *transversality condition* and simply establishes that the vector field $g(x)$, is not orthogonal to the gradient of $\sigma(x)$ at any point x in $\mathcal{X}$. In other words, the control vector field $g(x)$ is not tangential, at each x, to the sliding surface function level sets, defined in the state space of the system as $\mathcal{S}_k = \{x \in \mathcal{X} \ : \ \sigma(x) = k\}$. This condition is quite familiar in sliding mode control of nonlinear systems (Sira-Ramírez, 1988) and it amounts to having a sliding surface function which is locally *relative degree* one in $\mathcal{X}$. The *zero dynamics* corresponding to the ideal sliding condition $y = \sigma(x) = 0$ is assumed to be asymptotically stable towards the isolated equilibrium point $\overline{x} \in \mathcal{S}$. In other words, the system is *minimum phase* with respect to the output $y = \sigma(x)$.

For each $x \in \mathcal{X}$, we define a *projection operator, along the span of the control input vector field* $g(x)$ *onto the tangent space to the constant level sets of the sliding surface function* $\sigma(x)$, as the matrix $M(x)$ given by

$$M(x) = \left[I - \frac{1}{L_g\sigma} g(x) \frac{\partial\sigma}{\partial x^T} \right] \qquad (2)$$

The following proposition points out some properties of the matrix $M(x)$ which further justify the given name of “projection operator”

Proposition 1. The matrix $M(x)$ enjoys the following properties:

$$\begin{aligned} \frac{\partial\sigma}{\partial x} \in \mathcal{K}er \ M^T(x) \\ M(x)(I - M(x)) = 0 \\ g(x) \in \mathcal{K}er \ M(x) \end{aligned} \qquad (3)$$

The first property is equivalent to $\partial\sigma/\partial x^T M(x) = 0$ and simply states that the span of $M(x)$ lies in the tangent space to the level sets of $\sigma(x)$. The second property establishes that $M(x)$ leaves invariant any vector field, or distribution, in the tangent space to the level sets of $\sigma(x)$. Finally, the third property establishes that, locally, $M(x)g(x) = 0$. The following proposition depicts further properties of the projection matrix $M(x)$

Proposition 2. Let $f(x)$ be a smooth vector field, then the vector $M(x)f(x)$ can be written as

$$M(x)f(x) = \tilde{\mathcal{J}}(x)\frac{\partial\sigma}{\partial x}$$

where $\tilde{\mathcal{J}}(x)$ is a skew-symmetric matrix, i.e. $\tilde{\mathcal{J}}(x) + \tilde{\mathcal{J}}^T(x) = 0$.

On the other hand, the vector field $(I - M(x))f(x)$ can be written as

$$(I - M(x))f(x) = -\frac{1}{2}\tilde{\mathcal{J}}(x)\frac{\partial\sigma}{\partial x} + \mathcal{S}(x)\frac{\partial\sigma}{\partial x}$$

where $\mathcal{S}(x)$ is a symmetric matrix, i.e., $\mathcal{S}(x) = \mathcal{S}^T(x)$

Proof. The first part of the proposition is straightforward from algebraic manipulations.

$$\begin{aligned}
M(x)f(x) &= \left[I - \frac{1}{L_g\sigma}g(x)\frac{\partial\sigma}{\partial x^T}\right]f(x) \\
&= \frac{1}{L_g\sigma(x)}\left[(L_g\sigma(x))f(x) - g(x)L_f\sigma(x)\right] \\
&= \frac{1}{L_g\sigma(x)}\left[\frac{\partial\sigma(x)}{\partial x^T}g(x)f(x) - g(x)\frac{\partial\sigma(x)}{\partial x^T}f(x)\right] \\
&= \frac{1}{L_g\sigma}\left[f(x)g^T(x) - g(x)f^T(x)\right]\frac{\partial\sigma}{\partial x} \\
&= \tilde{\mathcal{J}}(x)\frac{\partial\sigma}{\partial x} \qquad (4)
\end{aligned}$$

For the proof of the second part of the proposition note that,

$$\begin{aligned}
(I - M(x))f(x) &= \frac{1}{L_g\sigma}g(x)\frac{\partial\sigma}{\partial x^T}f(x) \\
&= \frac{1}{L_g\sigma}\left[g(x)f^T(x)\right]\frac{\partial\sigma}{\partial x} \qquad (5)
\end{aligned}$$

The result follows from the fact that *any* square matrix $N(x)$ and, in particular,

$$N(x) = \frac{1}{L_g\sigma}\left[g(x)f^T(x)\right]$$

can always be written as the sum of a symmetric matrix and a skew symmetric matrix, i.e.,

$$\begin{aligned}
N(x) = &1/2(N(x) - N^T(x)) \\
&+1/2(N(x) + N^T(x))
\end{aligned}$$

The first summand, which is written as,

$$\begin{aligned}
\frac{1}{2}(N(x) - N^T(x)) &= \frac{1}{2L_g\sigma}\left[g(x)f^T(x) - f(x)g^T(x)\right] \\
&= -\frac{1}{2}\tilde{\mathcal{J}}(x) = -\mathcal{J}
\end{aligned}$$

is clearly skew-symmetric, while the second summand $(1/2)(N(x)+N^T(x))$ is symmetric. For the purposes of further reference we define the matrix $\mathcal{S}(x)$ as follows

$$\begin{aligned}
\mathcal{S}(x) &= \frac{1}{2}\left[N(x) + N^T(x)\right] \\
&= \frac{1}{2L_g\sigma}\left[g(x)f^T(x) + f(x)g^T(x)\right]
\end{aligned}$$

2.2 *Vector field decompositions through projection operators*

As a consequence of the results and definitions of the previous section, we have the following proposition.

Proposition 3. An n-dimensional smooth vector field $f(x)$ can be naturally decomposed in the following sum,

$$\begin{aligned}
f(x) &= M(x)f(x) + (I - M(x))f(x) \\
&= \mathcal{J}(x)\frac{\partial\sigma}{\partial x} + \mathcal{S}(x)\frac{\partial\sigma}{\partial x}
\end{aligned}$$

where $\mathcal{J}(x)$ is skew-symmetric and $\mathcal{S}(x)$ is symmetric.

Proof. Indeed,

$$M(x)f(x) = \tilde{\mathcal{J}}(x)\frac{\partial\sigma}{\partial x}$$

and

$$(I - M(x))f(x) = -\frac{1}{2}\tilde{\mathcal{J}}(x)\frac{\partial\sigma}{\partial x} + \mathcal{S}(x)\frac{\partial\sigma}{\partial x}$$

then,

$$\begin{aligned}
f(x) &= M(x)f(x) + (I - M(x))f(x) \\
&= \tilde{\mathcal{J}}(x)\frac{\partial\sigma}{\partial x} - \frac{1}{2}\tilde{\mathcal{J}}(x)\frac{\partial\sigma}{\partial x} + \mathcal{S}(x)\frac{\partial\sigma}{\partial x} \\
&= \frac{1}{2}\tilde{\mathcal{J}}(x)\frac{\partial\sigma}{\partial x} + \mathcal{S}(x)\frac{\partial\sigma}{\partial x} \\
&= \mathcal{J}(x)\frac{\partial\sigma}{\partial x} + \mathcal{S}(x)\frac{\partial\sigma}{\partial x} \qquad (6)
\end{aligned}$$

The following lemma is a well known decomposition result for symmetric matrices.

Lemma 4. Let $\mathcal{S}(x)$ be a symmetric matrix, then $\mathcal{S}(x)$ can be decomposed as the sum of a positive semi-definite matrix $\mathcal{S}_p(x)$ and a negative semi-definite matrix $\mathcal{S}_n(x)$. If the above decomposition is not possible, then either $\mathcal{S}(x)$ is positive definite or, at least, positive semi-definite or, else, it is negative definite or, at least, negative semi-definite.

2.3 *A canonical form for sliding mode controlled nonlinear systems*

As a corollary to the above results, a nonlinear system of the form (1), with a positive definite storage function $\sigma(x)$, satisfying the transversality condition $L_g\sigma(x) \neq 0$, can always be rewritten as

$$\dot{x} = \mathcal{J}(x)\frac{\partial\sigma}{\partial x} + \mathcal{S}_p(x)\frac{\partial\sigma}{\partial x} + \mathcal{S}_n(x)\frac{\partial\sigma}{\partial x} + g(x)u$$

with $\mathcal{J}(x)$ being skew-symmetric, and $\mathcal{S}_p(x)$ being positive semi-definite and $\mathcal{S}_n(x)$ negative semi-definite. However, if $\mathcal{S}_p(x)$ is positive definite, then $\mathcal{S}_n(x)$ is zero and conversely if $\mathcal{S}_n(x)$ is negative definite then $\mathcal{S}_p(x)$ is zero.

3. AN ADAPTIVE CANONICAL FORM FOR SLIDING MODE CONTROLLED UNCERTAIN NONLINEAR SYSTEMS

Consider a SISO nonlinear system with linearly parameterized uncertainty in the form

$$\begin{aligned} \dot{x} &= f(x) + \sum_{i=1}^{p} f_i(x)\theta_i + g(x)u \\ y &= \sigma(x) \end{aligned} \tag{1}$$

where $x \in \chi \subset \Re^n$ is the state; u, y the scalar input and output respectively; and θ_i, $i = 1, \ldots, p$ is a set of constant unknown parameters. The drift vector fields $f(x)$, $f_1(x), \ldots, f_p(x)$, and the "input" vector field $g(x)$ are smooth n-dimensional vector fields.

System (1) may always be rewritten as follows

$$\begin{aligned} \dot{x} &= f(x) + \Phi(x)\theta + g(x)u \\ y &= \sigma(x) \end{aligned} \tag{2}$$

where the matrix $\Phi(x) \in \Re^{n\times p}$ and the vector θ of unknown parameters are defined as follows

$$\Phi(x) = [f_1(x), \ldots, f_p(x)]\ ; \theta = [\theta_1, \ldots, \theta_p]^T$$

By incorporating a parameter estimate vector $\hat{\theta}$ of the constant unknown parameters, (2) may be rewritten as follows

$$\begin{aligned} \dot{x} &= f(x) + \Phi(x)\hat{\theta} + g(x)u + \Phi(x)(\theta - \hat{\theta}) \\ y &= \sigma(x) \end{aligned}$$

After proceeding with the natural decomposition of the vector field $f(x)$, according to the procedure explained above, an *adaptive canonical form for sliding mode controlled uncertain nonlinear systems* is obtained as follows

$$\begin{aligned} \dot{x} &= \mathcal{J}(x)\frac{\partial\sigma}{\partial x} + \mathcal{S}_p(x)\frac{\partial\sigma}{\partial x} + \mathcal{S}_n(x)\frac{\partial\sigma}{\partial x} \\ &\quad + \Phi(x)\hat{\theta} + g(x)u + \Phi(x)(\theta - \hat{\theta}) \\ y &= \sigma(x) \end{aligned} \tag{3}$$

3.1 *Adaptive feedback sliding mode control for uncertain systems in canonical form*

Consider an uncertain nonlinear system in the canonical form (3). Along the solutions of the system, the time derivative of the "energy function"

$$V(x, \hat{\theta}) = \frac{1}{2}\sigma^2(x) + \frac{1}{2}(\theta - \hat{\theta})^T\Gamma^{-1}(\theta - \hat{\theta}) \tag{4}$$

is evaluated as

$$\begin{aligned} \dot{V} &= \sigma(x)\dot{\sigma}(x) + (\theta - \hat{\theta})^T\Gamma^{-1}(-\dot{\hat{\theta}}) \\ &= \sigma\left[\frac{\partial\sigma}{\partial x^T}\mathcal{J}(x)\frac{\partial\sigma}{\partial x} + \frac{\partial\sigma}{\partial x^T}(\mathcal{S}_n(x) + \mathcal{S}_p(x))\frac{\partial\sigma}{\partial x}\right. \\ &\qquad \left. + \frac{\partial\sigma}{\partial x^T}\Phi(x)\hat{\theta} + L_g\sigma(x)u\right] \\ &\qquad + (\theta - \hat{\theta})\Gamma^{-1}\left[-\dot{\hat{\theta}} + \Gamma\Phi^T(x)\frac{\partial\sigma}{\partial x}\sigma(x)\right] \end{aligned} \tag{5}$$

In order to eliminate the destabilizing estimate error term in $\dot{V}(x, \hat{\theta})$, we choose the *update law*

$$\dot{\hat{\theta}} = \Gamma\Phi^T(x)\frac{\partial\sigma}{\partial x}\sigma(x) \tag{6}$$

to obtain

$$\begin{aligned} \dot{V} = \sigma &\left[\frac{\partial\sigma}{\partial x^T}(\mathcal{S}_n(x) + \mathcal{S}_p(x))\frac{\partial\sigma}{\partial x}\right. \\ &\left. + \frac{\partial\sigma}{\partial x^T}\Phi(x)\hat{\theta} + L_g\sigma(x)u\right] \end{aligned} \tag{7}$$

For $\sigma(x) > 0$ we have,

$$\begin{aligned} \dot{V}(x) \leq \sigma &\left[\frac{\partial\sigma}{\partial x^T}\mathcal{S}_p(x)\frac{\partial\sigma}{\partial x}\right. \\ &\left. + \frac{\partial\sigma}{\partial x^T}\Phi(x)\hat{\theta} + L_g\sigma(x)u\right] \end{aligned} \tag{8}$$

while for $\sigma(x) < 0$ we obtain

$$\begin{aligned} \dot{V}(x) \leq \sigma &\left[\frac{\partial\sigma}{\partial x^T}\mathcal{S}_n(x)\frac{\partial\sigma}{\partial x}\right. \\ &\left. + \frac{\partial\sigma}{\partial x^T}\Phi(x)\hat{\theta} + L_g\sigma(x)u\right] \end{aligned} \tag{9}$$

Consider now the following variable structure input coordinate transformation, with υ denoting a new external independent control input.

For $\sigma(x) > 0$,

$$u = \frac{1}{L_g\sigma}\left[\upsilon - \frac{\partial\sigma}{\partial x^T}\mathcal{S}_p(x)\frac{\partial\sigma}{\partial x} - \frac{\partial\sigma}{\partial x^T}\Phi(x)\hat{\theta}\right]$$

For $\sigma(x) < 0$,

$$u = \frac{1}{L_g\sigma}\left[\upsilon - \frac{\partial\sigma}{\partial x^T}\mathcal{S}_n(x)\frac{\partial\sigma}{\partial x} - \frac{\partial\sigma}{\partial x^T}\Phi(x)\hat{\theta}\right]$$

It is clear that the transformed system is given by the following variable structure system:

For $\sigma(x) > 0$

$$\begin{aligned}\dot{x} &= \mathcal{J}(x)\frac{\partial \sigma}{\partial x} + \mathcal{S}_n(x)\frac{\partial \sigma}{\partial x} \\ &\quad + \Phi(x)(\theta - \hat{\theta}) + \frac{1}{L_g\sigma}g(x)\upsilon \\ &\quad \left(I - \frac{1}{L_g\sigma}g(x)\frac{\partial \sigma}{\partial x^T}\right)\left(\mathcal{S}_p(x) + \Phi(x)\hat{\theta}\right)\frac{\partial \sigma}{\partial x} \\ \dot{\hat{\theta}} &= \Gamma\Phi^T(x)\frac{\partial \sigma}{\partial x}\sigma(x) \\ y &= \sigma(x)\end{aligned} \tag{10}$$

while, for $\sigma(x) < 0$

$$\begin{aligned}\dot{x} &= \mathcal{J}(x)\frac{\partial \sigma}{\partial x} + \mathcal{S}_p(x)\frac{\partial \sigma}{\partial x} \\ &\quad + \Phi(x)(\theta - \hat{\theta}) + \frac{1}{L_g\sigma}g(x)\upsilon \\ &\quad \left(I - \frac{1}{L_g\sigma}g(x)\frac{\partial \sigma}{\partial x^T}\right)\left(\mathcal{S}_n(x) + \Phi(x)\hat{\theta}\right)\frac{\partial \sigma}{\partial x} \\ \dot{\hat{\theta}} &= \Gamma\Phi^T(x)\frac{\partial \sigma}{\partial x}\sigma(x) \\ y &= \sigma(x)\end{aligned} \tag{11}$$

Notice that the projected vector field given by either

$$\left(I - \frac{1}{L_g\sigma(x)}g(x)\frac{\partial \sigma}{\partial x^T}\right)\left(\mathcal{S}_p(x) + \Phi(x)\hat{\theta}\right)\frac{\partial \sigma}{\partial x}$$

or

$$\left(I - \frac{1}{L_g\sigma(x)}g(x)\frac{\partial \sigma}{\partial x^T}\right)\left(\mathcal{S}_n(x) + \Phi(x)\hat{\theta}\right)\frac{\partial \sigma}{\partial x}$$

can be rewritten, respectively, as

$$\mathcal{K}_p(x,\hat{\theta})\frac{\partial \sigma}{\partial x} \quad \text{and} \quad \mathcal{K}_n(x,\hat{\theta})\frac{\partial \sigma}{\partial x}$$

with $\mathcal{K}_p(x,\hat{\theta})$ and $\mathcal{K}_n(x,\hat{\theta})$ being skew-symmetric matrices. In other words, the transformed system is of the form,

For $\sigma(x) > 0$,

$$\begin{aligned}\dot{x} &= \mathcal{I}_p(x,\hat{\theta})\frac{\partial \sigma}{\partial x} + \mathcal{S}_n(x)\frac{\partial \sigma}{\partial x} \\ &\quad + \Phi(x)(\theta - \hat{\theta}) + \frac{1}{L_g\sigma(x)}g(x)\upsilon \\ \dot{\hat{\theta}} &= \Gamma\Phi^T(x)\frac{\partial \sigma}{\partial x}\sigma(x) \\ y &= \sigma(x)\end{aligned} \tag{12}$$

with $\mathcal{I}_p(x,\hat{\theta}) = \mathcal{J}(x) + \mathcal{K}_p(x,\hat{\theta})$ being skew-symmetric and,

For $\sigma(x) < 0$,

$$\begin{aligned}\dot{x} &= \mathcal{I}_n(x,\hat{\theta})\frac{\partial \sigma}{\partial x} + \mathcal{S}_p(x)\frac{\partial \sigma}{\partial x} \\ &\quad + \Phi(x)(\theta - \hat{\theta}) + \frac{1}{L_g\sigma(x)}g(x)\upsilon \\ \dot{\hat{\theta}} &= \Gamma\Phi^T(x)\frac{\partial \sigma}{\partial x}\sigma(x) \\ y &= \sigma(x)\end{aligned} \tag{13}$$

with $\mathcal{I}_n(x,\hat{\theta}) = \mathcal{J}(x) + \mathcal{K}_n(x,\hat{\theta})$ being skew-symmetric.

The input coordinate transformation, viewed as a partial variable structure feedback, has achieved *compensation* of the destabilizing nonlinear terms in the system. The partial variable structure feedback has also achieved passivity for the variable structure system with respect to the sliding surface function viewed as a storage function. This latter point is established in the following proposition.

Proposition 5. The system (3) is *passive* with respect to the storage function (4), viewed as a positive semidefinite storage function, whenever $\mathcal{S}_n(x)$, (respectively $\mathcal{S}_p(x)$) is negative semidefinite (resp. positive semidefinite) and it is strictly passive if $\mathcal{S}_n(x)$ is strictly negative definite (resp. strictly positive definite).

Proof. Taking the time derivative of $V(x,\hat{\theta})$, along the solutions of the transformed system, and using the update law (6) one obtains:

For $\sigma(x) > 0$

$$\begin{aligned}\dot{V}(x,\hat{\theta}) &= \sigma(x)\left[\frac{\partial \sigma}{\partial x^T}\mathcal{I}_p(x,\hat{\theta})\frac{\partial \sigma}{\partial x} + \frac{\partial \sigma}{\partial x^T}\mathcal{S}_n(x)\frac{\partial \sigma}{\partial x} \right. \\ &\quad \left. + \frac{\partial \sigma}{\partial x^T}\frac{1}{L_g\sigma(x)}g(x)\upsilon\right] \\ &= \sigma(x)\left[\frac{\partial \sigma}{\partial x^T}\mathcal{S}_n(x)\frac{\partial \sigma}{\partial x}\right] + \sigma(x)\upsilon \\ &\leq \sigma(x)\upsilon = y\upsilon\end{aligned} \tag{14}$$

The calculation is similar for $\sigma(x) < 0$.

3.2 *A Sliding Mode Controller*

The sliding mode controller for the input υ may now be obtained by simply injecting complementary *damping* to the natural beneficial nonlinearities acting at each side of the sliding manifold. Let $\mathcal{S}_{nI}(x)$ be a symmetric negative semidefinite matrix such that $\mathcal{S}_n(x) + \mathcal{S}_{nI}(x)$ is negative definite. Similarly, let $\mathcal{S}_{pI}(x)$ be a symmetric positive semidefinite matrix such that $\mathcal{S}_p(x) + \mathcal{S}_{pI}(x)$ is positive definite. The following variable structure controller achieves the reaching of the sliding surface and the creation of a local sliding regime on such a surface

$$\upsilon = \begin{cases} \frac{\partial \sigma}{\partial x^T} \mathcal{S}_{nI}(x) \frac{\partial \sigma}{\partial x} & \text{for } \sigma(x) > 0 \\ \frac{\partial \sigma}{\partial x^T} \mathcal{S}_{pI}(x) \frac{\partial \sigma}{\partial x} & \text{for } \sigma(x) < 0 \end{cases} \tag{15}$$

4. CONCLUSIONS

We have proposed a new adaptive canonical form for sliding mode controlled systems with constant but unknown parameters. The canonical form is motivated from passivity based considerations for the same class of nonlinear systems, on which a natural decompositions of the drift vector field is achieved to help in designing a more efficient feedback variable structure controller. Furthermore, an update law is synthesized for the estimate of the unknown parameters. This update law together with a variable structure input coordinate transformation render the system passive with respect the given sliding surface function. The sliding mode control is simply the injection of complementary damping terms to the natural beneficial nonlinearities acting on each side of the sliding manifold.

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