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DESIGN OF ROBUST STABLE MASTER-SLAVE SYSTEMS WITH UNCERTAIN DYNAMICS AND TIME-DELAY

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Abstract: Master-slave and teleoperation systems with kinesthetic feedback enhance the dexterity of an operator performing manipulation tasks in remote, not accessible, or scaled environments. Kinesthetic feedback is achieved by closing bi-directional control loops between the operator and teleoperator site. This article proposes a method for the synthesis of robust bi-directional control loops in parameter space. Using the approach of singular frequencies a method for the synthesis of Hurwitz- and Γ-stable bi-directional controllers is developed and its application is shown by simulation. Robust performance is achieved w.r.t. parameter uncertainties of the operator and teleoperator dynamics and the bi-directional communication time delay.

Keywords: Master-slave systems, singular frequencies, parameter space transformations, parameter space control

1. INTRODUCTION

The design of telemanipulation systems for telerobotics applications calls for robust control methods to cope with critical uncertain system parameters (typically time-delay and environment stiffness). Several approaches are proposed, including the scattering matrix and "wave variable" based theory, (Anderson and Spong, 1989; Niemeyer and Slotine, 1991).

In this paper we propose a method for the synthesis of robust bilateral position controllers to achieve kinesthetic feedback with master-slave systems despite time delay and other system parameter uncertainties. In the context of robust control (Ackermann *et al.*, 1993), we use the approach of singular frequencies to solve symbolically for the Hurwitz- and Γ-stable regions taking into account the uncertainty of the parameters describing the interaction stiffness at the operator and environment port, and the signal delay time.

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The model and control structure of the master-slave system is presented in section 2. In Section 3 the method of singular frequencies is described and partial differential transformation equations are derived for the nonlinear parameter dependence. These equations are solved for the bilinear parameter dependency in Section 4. An algorithm for the detection of active regions in parameter space is presented in Section 5. In Section 6 a design of a robust controller using the proposed method is presented and verified by simulations.

2. MODEL AND CONTROL STRUCTURE OF A MASTER-SLAVE SYSTEM

Model. In this paper we will base our discussion of the parameter space on one degree-of-freedom systems in Cartesian space. The signals of the control system, such as position, velocity and force are scalar variables.

Figure 1 depicts a general structure of a master-slave system. The operator generates a command τ_o to the muscle system based on the difference between a desired anticipated position and the current position of the slave system. The arm of the operator thereby exerts an external force f_m on the master-arm. The bilateral controller couples the master and slave system by commanding the forces τ_m and τ_s to the master and slave manipulator respectively. The slave manipulator motion generates a force f_s via the slave-environment contact stiffness. In Figure 1 and in the model equations the operator and the environment are represented as impedances.

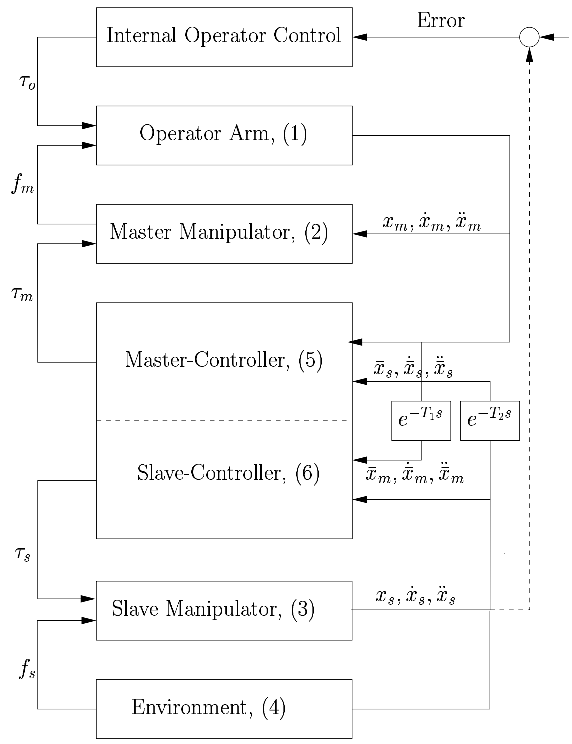

Fig. 1. General structure of a master-slave system.

The operator arm is modeled as a linear mass-damper-spring model of second order,

$$\tau_o = m_o \ddot{x}_m + b_o \dot{x}_m + c_o x_m + f_m. \qquad (1)$$

We thereby assume that the operator firmly grasps the master arm during task execution with the system. The operator arm can be seen as a passive system with spring-like behavior, although neural feedback may modify muscle impedance (Hogan, 1989). The damping b_o and stiffness c_o of the arm vary in a certain range according to the contraction of the muscle apparatus.

The dynamics of the master and the slave system is described by

$$f_m = m_m \ddot{x}_m + b_m \dot{x}_m + \tau_m, \qquad (2)$$

$$\tau_s = m_s \ddot{x}_s + b_s \dot{x}_s + f_s \qquad (3)$$

with m_m, b_m and m_s, b_s, the mass and damping of the master and slave arm, respectively. The environment contact dynamics is represented by

$$f_s = m_e \ddot{x}_s + b_e \dot{x}_s + c_e x_s, \qquad (4)$$

where m_e, b_e, and c_e are the mass, damping, and stiffness of the environment, respectively.

Control. The bilateral controller shown in Fig. 1 consists of a master and a slave part. Note that the delayed signals are marked by a bar, e.g. $\bar{x}_m(t) = x_m(t - T_1)$.

As outlined above bilateral PD position control will be considered throughout this paper

$$\begin{aligned} \tau_m &= -k_{mm} x_m - d_{mm} \dot{x}_m + k_{sm} \bar{x}_s + d_{sm} \dot{\bar{x}}_s, \\ \tau_s &= -k_{ss} x_s - d_{ss} \dot{x}_s + k_{ms} \bar{x}_m + d_{ms} \dot{\bar{x}}_m \end{aligned} \qquad (5)$$

with k_{mm}, d_{mm}, k_{sm}, d_{sm} and k_{ss}, d_{ss}, k_{ms}, d_{ms} the master and slave control parameters respectively.

3. THE METHOD OF SINGULAR FREQUENCIES

Introduction. Let $p(s, \boldsymbol{q}, \boldsymbol{k})$ be the polynomial family of a system, where $\boldsymbol{q} = [q_1, q_2, \cdots, q_p]^T$ is the vector of parameter uncertainties and $\boldsymbol{k} = [k_1, k_2, \cdots, k_c]^T$ contains the controller parameters. The vector of uncertainties $\boldsymbol{q}$ is assumed to be constant but unknown up-to a so-called uncertainty $\boldsymbol{Q}-$box which encloses all the possible operating points and is defined as $q_i^- < q_i < q_i^+, \;\; i = 1, 2, \cdots, p$.

The basic eigenvalue problem of robust control is to find the set of all controller parameters $\boldsymbol{K}_\Gamma$, s.t. the set of eigenvalues of each operating point within the $\boldsymbol{Q}-$box lie inside a prespecified $\Gamma-$region in the complex $s-$plane. It is said that the $\boldsymbol{Q}-$box is robustly $\Gamma-$stabilized,

$$\boldsymbol{K}_\Gamma = \{\boldsymbol{k} : p(s, \boldsymbol{q}, \boldsymbol{k}) \text{ is robust } \Gamma - \text{stable } \forall \boldsymbol{q} \in \boldsymbol{Q}\},$$

General items. Usually a simple controller structure is assumed in the first step. The process of controller design goes through several design-analysis steps. In a first design step, the boundary of the eigenvalue $\Gamma-$region, $\partial\Gamma$, is mapped into the controller parameter space for a multi-model (i.e. for several operating points $\{\boldsymbol{q}^{(\nu)} : \boldsymbol{q}^{(\nu)} \in \boldsymbol{Q}-box\}$, usually vertices of the $\boldsymbol{Q}-$box). The mapping equation is the characteristic equation of the system itself,

$$p(s, \boldsymbol{q}^{(\nu)}, \boldsymbol{k}) = 0 \qquad (6)$$

During the analysis step, the contrary is done, i.e. the boundary of the resulting (intersection) stable region in the controller space, $\partial \boldsymbol{K}_\Gamma$ is back-mapped to the uncertainty parameter space, $\boldsymbol{Q}_\Gamma$. This feedback design-analysis procedure, with new critical operating points included is repeated until $\boldsymbol{Q}_\Gamma \subset \boldsymbol{Q}$. Otherwise the Γ region has to be redefined. The mapping and back-mapping is usually done in the two-parameter plane, with the rest of parameters fixed, see (Ackermann *et al.*, 1993).

Singular frequencies. In the following we concentrate on the method of singular frequencies, which is shown to be convenient for the design of master-slave systems in parameter space. A necessary step of the method is to move to a new controller parameter space $\boldsymbol{r}$, i.e.,

$$\boldsymbol{k} = \boldsymbol{k}(\boldsymbol{r}). \tag{7}$$

For the sake of simplicity, we concentrate our further discussion on two-parameters r_1 and r_2. The mapping equations, that is, the real and imaginary part of the characteristic equation are,

$$\begin{aligned} h(s, \boldsymbol{q}^{(\nu)}, r_1, r_2) &= 0, \\ g(s, \boldsymbol{q}^{(\nu)}, r_1, r_2) &= 0 \end{aligned} \tag{8}$$

Complex frequencies on $\partial\Gamma$ usually map to *a* point (r_1, r_2). However at some certain peculiar frequencies $s^o = \sigma^o + j\omega^o$ the so called *rank-condition*,

$$rank \, \frac{\partial(h,g)}{\partial(r_1,r_2)}|_{s^o} = 1 \tag{9}$$

is met. Given that r_1 and r_2 enter *linearly* in (8), two things can happen: s^o is mapped to a *singular line*, or s^o is under no-(r_1, r_2) combination an eigenvalue of the system. In the first case, s^o is said to be a *singular frequency*, i.e.

$$\boldsymbol{p}(s^o, \boldsymbol{q}^{(\nu)}, r_1, r_2) = 0, \tag{10}$$

which is the *root-condition* for the singular frequencies. For the special class of *singular* $\Gamma-$*regions*, the rank-condition is fulfilled at each frequency $s \in \partial\Gamma$, i.e.

$$rank \, \frac{\partial(h,g)}{\partial(r_1,r_2)} = 1, \quad \forall s \in \partial\Gamma. \tag{11}$$

Geometrically, the rank-condition guarantees that for each frequency $s \in \partial\Gamma$, (8) map to two parallel lines in (r_1, r_2)-plane, while the root-condition identifies the singular frequencies at which these lines are identical. The eigenvalues of the system can enter/leave the $\Gamma-$region, at singular frequencies only. Thus only the singular frequencies have to be mapped and not the whole boundary $\partial\Gamma$.

Parameter transformation. The basic problem of the method of singular frequencies is to find the transformation equations (7) s.t. a given $\Gamma-$region in the new parameter space $\boldsymbol{r}$ turns into a singular one. We define such a parameter space as *singular parameter space* of the $\Gamma-$region. Unfortunately, not every $\Gamma-$region can turn to singular. In addition, the set of singular $\Gamma-$regions is determined by the controller structure.

Solution of (7) depends heavily on how controller parameters appear in the mapping equations (8). The affine parameter dependency is solved in (Bajcinca, 2001; Ackermann *et al.*, 2001).

We consider here the general case of nonlinear appearance of controller parameters in the mapping equations. Let

$$\boldsymbol{a} = \boldsymbol{a}(k_1, k_2, k_3, q^{(\nu)})$$

be the vector of coefficients $a_0, a_1, \cdots, a_n$ of the characteristic polynomial (6) and (k_1, k_2, k_3) three free controller parameters. The rest of controller parameters and uncertainties are included in $\boldsymbol{q}^{(\nu)}$.

We are looking for a transformation into an $r-$parameter space

$$\begin{aligned} k_1 &= k_1(r_1, r_2, r_3), \\ k_2 &= k_2(r_1, r_2, r_3), \\ k_3 &= k_3(r_1, r_2, r_3), \end{aligned} \tag{12}$$

under the rank-condtion. Since the rank-condition applies over the entire $\sigma-$axis ($s = \sigma + j\omega$), it follows that[3],

$$\frac{\partial(h, \frac{1}{\omega}g)}{\partial(r_1, r_2)} = \eta(\sigma, \omega) \, \frac{\partial a}{\partial(r_1, r_2)}, \tag{13}$$

with

$$\eta(\sigma, \omega) = \begin{bmatrix} 1 & \sigma & \sigma^2 - \omega^2 & \dots & \Re s^n \\ 0 & 1 & 2\sigma & \dots & \frac{1}{\omega}\Im s^n \end{bmatrix}, \tag{14}$$

and

$$\frac{\partial a}{\partial(r_1, r_2)} = \frac{\partial a}{\partial(k_1, k_2, k_3)} \, \frac{\partial(k_1, k_2, k_3)}{\partial(r_1, r_2)}. \tag{15}$$

If the rank-condition is applied on (13), a curve family in the complex $s-$plane results, which is called *the family of singular curves*. These curves define the boundaries $\partial\Gamma$ of singular $\Gamma-$regions.

The family of singular curves is described by the equation,

$$\frac{\partial a}{\partial r_1} + \lambda \frac{\partial a}{\partial r_2} = \varphi_1\gamma_1 + \varphi_2\gamma_2 + \ldots + \varphi_{n-1}\gamma_{n-1}, \tag{16}$$

where $\lambda = \lambda(\alpha)$ and $\varphi_i = \varphi_i(\alpha)$, $i = 1, 2, \ldots, n-1$ are arbitrary functions defined on the boundary $\partial\Gamma$ and α is used as the parameter of the singular curves $\partial\Gamma$. The vectors γ_i,

$$\gamma_i^T = \begin{bmatrix} \frac{\sigma}{\omega}\Im s^{i+1} - \Re s^{i+1} \\ -\frac{1}{\omega}\Im s^{i+1} \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix}, \quad i = 1, 2, \ldots, n-1 \tag{17}$$

span the null-space of the matrix η, which we call $\gamma-$space. In order to solve for explicit equations

[3] The singular frequency at $\omega = 0$ is not considered.

of the singular curves, the arbitrary functions λ and φ_i should be eliminated in (16). The resulting singular curve (the Γ region, too) is said to get singularized by a specific set of the $\gamma-$vectors. It can be shown that a $\Gamma-$region is singularized by a unique set of γ vectors. We will show later on that our assumed controller structure for design of the master-slave system results in a $\Gamma-$space spanned just by the vector γ_1. Other $\Gamma-$spaces are analyzed in (Bajcinca, 2001). It has been shown there that the solution for the singular γ_1 family is,

$$A(\sigma^2+\omega^2)+2B\sigma+C=0 \tag{18}$$

with

$$\begin{aligned} A &= \frac{\partial a_1}{\partial r_1}\frac{\partial a_2}{\partial r_2}-\frac{\partial a_2}{\partial r_1}\frac{\partial a_1}{\partial r_2},\\ B &= \frac{\partial a_0}{\partial r_1}\frac{\partial a_2}{\partial r_2}-\frac{\partial a_2}{\partial r_1}\frac{\partial a_0}{\partial r_2},\\ C &= \frac{\partial a_0}{\partial r_1}\frac{\partial a_1}{\partial r_2}-\frac{\partial a_1}{\partial r_1}\frac{\partial a_0}{\partial r_2}. \end{aligned} \tag{19}$$

Eq. (18) suggests that the γ_1- singular $\Gamma-$region family contains the set of circles with the center on the real $\sigma-$axis and an arbitrary radius ($A \neq 0$, circle stability) and the lines parallel to imaginary axis ($A = 0$, $\sigma-$stability). Note that $A = 0$ and $C = 0$ correspond to Hurwitz-stability.

Substitution of (15) in(19) (after some obvious steps) yields the following PDE system,

$$\frac{\partial k_1}{\partial r_1}\frac{\partial k_2}{\partial r_2}-\frac{\partial k_2}{\partial r_1}\frac{\partial k_1}{\partial r_2}=F, \tag{20}$$

$$H\frac{\partial k_1}{\partial r_1}-G\frac{\partial k_2}{\partial r_1}+F\frac{\partial k_3}{\partial r_1}=0, \tag{21}$$

$$H\frac{\partial k_1}{\partial r_2}-G\frac{\partial k_2}{\partial r_2}+F\frac{\partial k_3}{\partial r_2}=0. \tag{22}$$

Its solution depends heavily on the functions F, G and H. However we do not have to solve for the general solution, since we aim to find just a singular parameter space, i.e. a particular solution.

Robustness. Assume the characteristic polynomial of the system in the form,

$$\boldsymbol{p}(s,\boldsymbol{q},\boldsymbol{k})=A(s,\boldsymbol{q})\ a(s,\boldsymbol{k})+B(s,\boldsymbol{q}). \tag{23}$$

The plant parameters $\boldsymbol{q}$ are free to appear arbitrarily in the polynomials $A(s,\boldsymbol{q})$ and $B(s,\boldsymbol{q})$. $A(s,\boldsymbol{q})$ and $B(s,\boldsymbol{q})$ can also contain exponential terms with time-delays, that is they can be quasipolynomials. This matches exactly with our master-slave system.

The Sylvester inequality about the rank of the product of two matrices applied on (23) yields,

$$rank\ \frac{\partial \boldsymbol{p}}{\partial \boldsymbol{r}}=rank\ \eta\frac{\partial \boldsymbol{a}}{\partial \boldsymbol{k}}\frac{\partial \boldsymbol{k}}{\partial \boldsymbol{r}}. \tag{24}$$

Note that the plant uncertainties $\boldsymbol{q}$ in (23) have no effect on the rank-condition (24), i.e. they do not disturb the singularity of a $\Gamma-$region. However, uncertainties effect the singular frequencies on the boundary $\partial\Gamma$. An important consequence is that the gridding orientation planes, $r_3 = const$, in the transformed parameter space does not depend on the plant parameters.

4. BILINEAR PARAMETER DEPENDENCE

The characteristic polynomial of a linear master-slave system with the controller structure (5) is,

$$p(s,\boldsymbol{q},\boldsymbol{k})=a(s,\boldsymbol{k'})\ e^{-(T_1+T_2)s}\ +\ b(s,\boldsymbol{q},\boldsymbol{k''}), \tag{25}$$

with

$$a(s,\boldsymbol{k'})=a_0+a_1s+a_2s^2, \tag{26}$$

$$b(s,\boldsymbol{q},\boldsymbol{k''})=b_0+b_1s+b_2s^2+b_3s^3+b_4s^4. \tag{27}$$

$\boldsymbol{k'}$ contains the coupled controller parameters k_{sm}, k_{ms}, d_{sm}, $\boldsymbol{k''}$ the local ones k_{mm}, k_{ss}, d_{mm} and d_{ss} and $\boldsymbol{q}$ the plant parameters. Given that $\boldsymbol{k''}$ parameters are designed, i.e. fixed, (25) gets the typical form (23) for the method of singular frequencies. Note that the $\gamma-$space of this polynomial is spanned just by the vector γ_1, i.e. circle-, $\sigma-$ and Hurwitz stability design requirements are realizable.

We introduce the notation $k_1 = k_{sm}$, $k_2 = d_{sm}$, $k_3 = d_{sm}$ and fix $d_{ms} = d^*_{ms}$, so,

$$\begin{aligned} a_0 &= k_1k_3,\\ a_1 &= k_1d^*_{ms}+k_2k_3,\\ a_2 &= k_2d^*_{ms}. \end{aligned} \tag{28}$$

Now we will try to solve the problem of nonlinear parameter transformation (7) for this special case.

Solving for F, G, H in (25) and substituting them in (21) and (22) results with the solution,

$$Bk_2k_3-Ak_1k_3+Bd^*_{ms}k_1-Cd^*_{ms}k_2=U(r_3) \tag{29}$$

with $U(r_3)$ being an arbitrary function of r_3. Since we are looking just for a particular solution, we assume $U(r_3) = r_3$. In addition, we assume the simple condition $k_2 = r_2/d^*_{ms}$. Now (29) is solved for k_3, and subsequently substituted in (20). A PDE solvable in closed form results,

$$\frac{\partial k_1}{\partial r_1}=-\frac{(Br_2-Ad^*_{ms}k_1)^2}{d^*_{ms}(-r_2r_3+2Bd^*_{ms}r_2k_1-Cr_2^2-Ad^{*2}_{ms}k_1^2)}. \tag{30}$$

The parameters A, B, C define the singular $\Gamma-$region.

Hurwitz region ($\sigma < 0$) turns singular for $A = 0, C = 0, B = 1$. A Hurwitz singular parameter space is then defined by,

$$\begin{aligned} k_1 &= (r_3+\sqrt{f})/2d^*_{ms},\\ k_2 &= r_2/d^*_{ms},\\ k_3 &= d^*_{ms}(r_3-\sqrt{f})/2r_2,\\ \\ f &= r_3^2-4r_1r_2. \end{aligned} \tag{31}$$

$\sigma-$ Singular regions ($\sigma < -\sigma_o < 0$) result if $A = 0, B = 1/2, C = \sigma_0, \sigma_0 > 0$. The transformation equations are,

$$\begin{aligned} k_1 &= (r_3 + \sigma_0 r_2 - \sqrt{f})/d^*_{ms}, \\ k_2 &= r_2/d^*_{ms}, \\ k_3 &= (r_3 + \sigma_0 r_2 + \sqrt{f})d^*_{ms}/r_2, \\ f &= r_3^2 + 2\sigma_0 r_2 r_3 + \sigma_0^2 r_2^2 - r_1 r_2/2. \end{aligned} \quad (32)$$

Note that, the condition $f \geq 0$ restricts the space of controllers in $\boldsymbol{r}-$parameter space.

5. DETECTION OF ACTIVE REGIONS IN PARAMETER SPACE

In this section we state a theorem which we use for automatic detection of the $\Gamma-$stable regions in parameter space. Consider a singular $\boldsymbol{r}-$parameter [4] space and a singular curve

$$\partial\Gamma: \ F(\sigma, \omega) = 0. \quad (33)$$

It can be shown that the condition $r_3 = const$ generates a set $\mathcal{S} = \{s_1^o, s_2^o, \ldots\}$ of singular frequencies. Each singular frequency generates a singular line in the $(r_1, r_2)-$plane and consequently a set of convex polygons results. If r_3 is gridded then the whole $(r_1, r_2, r_3)-$space is divided in 3D regions. The boundary surface between two regions, $S(r_1, r_2, r_3) = 0$, is called a *singular surface.* We search for the active ones, i.e. those that bound the parameter space region(s) with a maximal number of system eigenvalues inside the $\Gamma-$region.

Theorem 1. Let $S(r_1, r_2, r_3) = 0$ be a singular surface and $\boldsymbol{r}^o = (r_1^o, r_2^o, r_3^o)^T$ a point on it. Let s^o be the corresponding singular frequency on the singular curve (33). If the singular surface $S(r_1, r_2, r_3) = 0$ is crossed at $\boldsymbol{r}^o$ with,

$$d\boldsymbol{r} = \left(\frac{\partial r_1}{\partial \rho}, \frac{\partial r_2}{\partial \rho}, \frac{\partial r_3}{\partial \rho}\right)^T_{r^o} d\rho$$

($d\rho > 0$), a pair of eigenvalues crosses the singular curve (33) at s^o with,

$$\mu = \left(\frac{d\sigma}{d\rho}, \frac{d\omega}{d\rho}\right)^T_{s^o, r^o} \quad (34)$$

with

$$\begin{aligned} \left.\frac{d\sigma}{d\rho}\right|_{s^o, r^o} &= \sum_{i=1}^{3} \left|\frac{\partial(h,g)}{\partial(\omega, r_i)}\right|_{s^o} \left.\frac{\partial r_i}{\partial \rho}\right|_{r^o}, \\ \left.\frac{d\omega}{d\rho}\right|_{s^o, r^o} &= \sum_{i=1}^{3} \left|\frac{\partial(h,g)}{\partial(r_i, \sigma)}\right|_{s^o} \left.\frac{\partial r_i}{\partial \rho}\right|_{r^o}. \end{aligned} \quad (35)$$

[4] It is assumed that parameters r_1, r_2, r_3 enter linearly in the transformed characteristic polynomial.

To conclude whether the eigenvalues leave or enter the $\Gamma-$ region, one should compute,

$$e = sign\ (\boldsymbol{\mu}^T \cdot \boldsymbol{N}) \quad (36)$$

where $\boldsymbol{N} = \left(\frac{\partial F}{\partial \sigma}, \frac{\partial F}{\partial \omega}\right)_{s^o}$ is the normal vector on (33) at s^o. If $e = +1$, the eigenvalues leave the $\Gamma-$region, while for $e = -1$ they enter it.

6. DESIGN OF THE ROBUST CONTROLLER

Our aim is to find the set of all robust stabilizers for the master-slave system in 3D $r-$parameter space. The typical and most critical uncertainties in a teleoperating system are the operator stiffness, c_o, environment stiffness, c_e and bi-lateral communication time delay $T = T_1 + T_2$. The data for the master and slave robot used in this article correspond to those of the force controlled DLR light-weight-robot.

Operator:	$m_o = 0,\ d_o = 0,\ c_o =$ uncertain
Master robot:	$m_M = 1.5$ kg, $b_M = 16.5$ N/m/s
Slave robot:	$m_S = 1.5$ kg, $b_S = 16.5$ N/m/s
Environment:	$m_e = 0,\ d_e = 5$N/m/s, $c_e =$ uncertain,

We assume the following uncertainty $\boldsymbol{Q}$-box:

Time delay	$T = 0.1$ s $\ldots 0.5$ s
Operator stiffness:	$c_o = 50$ N/m$\ldots 500$ N/m
Environment stiffness:	$c_e = 1000$ N/m$\ldots 10000$ N/m.

The local controller parameters are assumed to be $K_m = K_s = 20$ N/m and $D_m = D_s = 5$ N/m/s.

Hurwitz-stability. If (31) is substituted in (25) the characteristic polynomial transforms to

$$p(s, r) = (r_2 s^2 + r_3 s + r_1) + b(s)e^{Ts}, \quad (37)$$

where $b(s)$ is a polynomial of order $= 4$. Note that the necessary principle term condition for the stability of the quasipolynomial is fulfilled, (Pontryagin, 1955).

The analysis of (37) is especially elegant since the imaginary part, $r_3 = -A(\omega)/\omega \sin(\omega T + \phi(\omega))$ generates the singular frequencies, while the real part, $r_1 - \omega^2 r_2 = -A(\omega)\cos(\omega T + \phi(\omega))$ generates the corresponding singular lines, where $A(\omega) = |b(j\omega)|$ and $\phi(\omega) = \arg\ b(j\omega)$.

As discussed in section 3, a practical procedure for the robust control of the $\boldsymbol{Q}-$box is to find simultaneous stabilizers for a finite number of critical operating points. Usually the vertices of the $\boldsymbol{Q}-$box are chosen as the critical operating points.

The execution of the algorithm in the interval $-20000 < r_3 < +20000$ and for $\omega < 100$, yields the region of candidate stabilizers shown in Fig. 2. The controllers in $-11492.48239 < r_3 < 11480.43529$ are equistable, i.e. the stability of the whole region can be concluded based on the stability of any controller in that region. For the

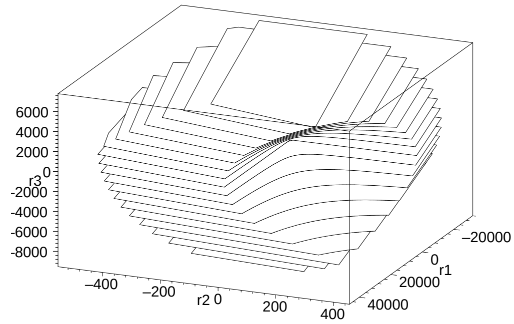

Fig. 2. Robust Hurwitz-stable region.

sake of simplicity we chose the controller $r_1 = 0, r_2 = 0, r_3 = 1$. According to the theorem of Pontryagin, (Pontryagin, 1955) a quasipolynomial $p(j\omega) = h(\omega) + jg(\omega)$ with principle term is Hurwitz stable if all roots $\{\omega_o\}$ of $h(\omega)$ are real and at each ω_o, $g(\omega)\frac{dh}{d\omega}\big|_{\omega_o} < 0$ applies. Substitution of our controller in (37) yields,

$$\begin{aligned} h(\omega) &= A(\omega)\cos(\omega T + \phi(\omega)) \\ g(\omega) &= \omega + A(\omega)\sin(\omega T + \phi(\omega)). \end{aligned}$$

Note that $A(\omega)$ has no real roots. Also $d\phi/d\omega > 0, \forall\omega > 0$. Thus the roots of $h(\omega)$ are real and they are defined by the condition $\phi(\omega) = (k + 1/2)\pi,\ k = 0, 1, 2, \ldots$. At each root ω_o,

$$g(\omega)\frac{dh}{d\omega}\bigg|_{\omega_o} = -\alpha_k(\omega_o)\left(T + \frac{d\phi}{d\omega}\right)(\omega_o + \alpha_k(\omega_o)),$$

where $\alpha_k(\omega_o) = \alpha(\omega_o)\sin(k + 1/2)\pi$. Since for all operating points of the system, $\alpha(\omega) > \omega,\ \forall\omega > 0$, the condition $g(\omega)\frac{dh}{d\omega}\big|_{\omega_o} < 0$ is valid, i.e. the controller $r_1 = r_2 = 0,\ r_3 = 1$ is stable. This shows that the region in Fig. 2 stabilizes simultaneously all vertices of the $\boldsymbol{Q}-$box of the master-slave system.

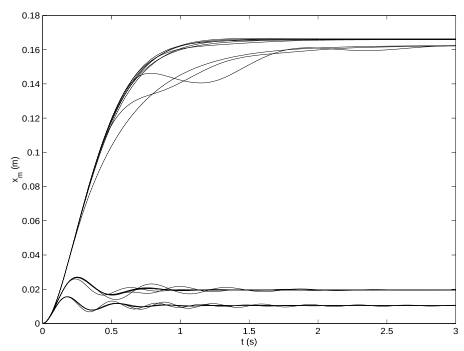

Fig. 3. Step response of x_m for 24 operating points of the $\boldsymbol{Q}-$Box, $\tau_o = 5$ (N).

The rigorous robustness analysis of the whole $\boldsymbol{Q}-$box of our system is not done in this article. Instead we analyze the stability of a finite set of operating points. Consider a controller inside the region in Fig. 2, e.g. $r_3 = 3751.86992, r_1 = 785.13565, r_2 = 38.71231$. In Fig. 3 we show the step response of x_m for 24 operating points (8 vertices + 8 centers of the edges + 8 centers of the surfaces of the $\boldsymbol{Q}-$box).

σ-Stability. An analogous design in parameter space can be done also for the $\sigma-$stability specifications, performing faster decaying system responses. In Fig. 4 the resulting region of robust stabilizers is shown for $\sigma_o = 2$.

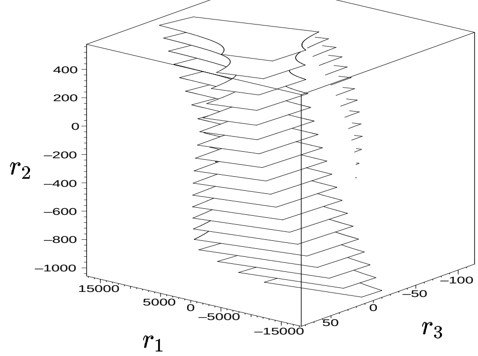

Fig. 4. Robust $\sigma-$stable in parameter space.

7. CONCLUSION

In this paper a method for the synthesis of bilateral master-slave controllers in parameter space has been proposed. The design takes into account the parameter uncertainty in the signal delay, the operator and the environment contact stiffness. Using the approach of singular frequencies, the synthesis of robust Hurwitz- and $\Gamma-$stable controllers has been shown and validated by simulations.

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