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FLOQUET THEORY FOR MIMO SAMPLED-DATA SYSTEMS

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Abstract. The paper transfers some classical results by Floquet from the theory of linear differential equations with periodically varying coefficients to MIMO sampled-data systems. The problem of modal control is formulated for sampled-data systems, and the general solution is given in a polynomial form.

Keywords: Sampled-data control, Time-varying systems, Periodic motion, MIMO, Modal control

1. INTRODUCTION

Digital controllers and filters for continuous-time processes are widely used in contemporary applications. For analysis and (optimal) design of such systems the theory of sampled-data systems was developed over the last decade. Different approaches in state-space or frequency domain are known (Hara *et al.* 1994, Yamamoto 1994, Chen and Francis 1995, Hagiwara and Araki 1995, Rosenwasser and Lampe 2000*b*).

Sampled-data systems are a subclass in the class of periodically varying systems. Therefore, the ideas and results of the theory of periodically varying systems can be incorporated into methods for the solution of sampled-data problems. The paper shows that applying the fundamental ideas of Floquet (Floquet 1883) leads to solutions for general problems in sampled-data systems.

Especially, the modal control problem for MIMO sampled-data systems is formulated, and its general solution is presented. The polynomial form of the solution allows the application of the polynomial calculus that is a powerful tool in theory (Kučera 1979, Grimble and Kučera 1996) and numerical realization (PolyX 1999, EUROPOLY *1998).

After the statement of the problem the main results are formulated in form of four theorems. For better reading, the proofs, auxiliary concepts and lemmata are arranged behind the results.

2. PROBLEM

1. Consider the standard sampled-data system (Chen and Francis 1995, Rosenwasser and Lampe 2000*b*) given by the equations

$$
\begin{aligned}
&\frac{\mathrm{d}x}{\mathrm{d}t} = Ax + B_1 w + B_2 u \\
&y = C_2 x, \quad z = C_1 x \\
&y_k = y(kT) \qquad\qquad (1) \\
&\Phi_0 \psi_{k+q} + \ldots + \Phi_q \psi_k = \Gamma_0 y_{k+q} + \ldots + \Gamma_q y_k \\
&u(t) = \mu(t - kT)\psi_k, \quad kT < t < (k+1)T
\end{aligned}
$$

where $x,\ z,\ y,\ w,\ u$ are vectors of the dimensions $p \times 1,\ r \times 1,\ n \times 1,\ \ell \times 1,\ m \times 1,$ and

$A,\ B_1,\ B_2,\ C_1,\ C_2,\ \Phi_i,\ \Gamma_i$ are constant matrices of appropriate dimensions. Hereby, $\det \Phi_0 \neq 0$, and at least one of the matrices $\Phi_q,\ \Gamma_q$ is not a zero matrix. Furthermore, $T > 0$ is the sampling period and $\mu(t)$ is a piecemeal smooth function giving the shape of the control impulses. Consequently, the vectors $x(t)$ and $y(t)$ are continuous functions.

2. Under the condition $w(t) = 0$, with respect to the known results of (Floquet 1883, Yakubovich and Starzhinskii 1975), consider the solutions of the homogeneous equation (1) of the form

$$\begin{aligned} x(t) &= t^\gamma \, \mathrm{e}^{\lambda t} \, x_1(t) \\ y(t) &= t^\gamma \, \mathrm{e}^{\lambda t} \, y_1(t) \\ \psi_k &= k^\gamma \, \mathrm{e}^{\lambda k T} \, \psi_0 \end{aligned} \tag{2}$$

where $x_1(t) = x_1(t+T)$ and $y_1(t) = y_1(t+T)$ are valid. Moreover, ψ_0 is a constant vector, λ is a constant and γ is a non-negative integer. If there exists a solution of the form (2), then the constant λ is called a *characteristic index* of the system (1), and the value $\mathrm{e}^{\lambda T}$ its *multiplier* according to the characteristic index λ. In what follows, a solution of the form (2) is called a *Floquet solution.*

3. Consider the polynomial matrix

$$Q(z) = \begin{bmatrix} zI_p - \mathrm{e}^{AT} & O_{pm} & -\mathrm{e}^{AT} M(A) B_2 \\ -C_2 & I_n & O_{nm} \\ O_{mn} & -\Gamma(z) & \Phi(z) \end{bmatrix} \tag{3}$$

where

$$M(A) = \int_0^T \mathrm{e}^{-At} \, \mu(t) \, \mathrm{d}t \tag{4}$$

$$\begin{aligned} \Phi(z) &= \Phi_0 z^q + \Phi_1 z^{q-1} + \ldots + \Phi_q \\ \Gamma(z) &= \Gamma_0 z^q + \Gamma_1 z^{q-1} + \ldots + \Gamma_q \, . \end{aligned} \tag{5}$$

Hereby, $O_{ik},\ I_n$ denote the $i \times k$ zero matrix and the $n \times n$ identity matrix, respectively.

3. RESULTS

Theorem 1. The set of multipliers of the systems (1) coincides with the set of non-vanishing eigenvalues of the matrix (3).

If the matrix (3) has no vanishing eigenvalues, then the system (1) is called a system of *Floquet type.*

Theorem 2. For a system of Floquet type every solution of the homogeneous system is a linear combination of its Floquet solutions.

The problem of finding a solution for the polynomial equation

$$\begin{aligned} \det & \begin{bmatrix} zI_p - \mathrm{e}^{AT} & O_{pm} & -\mathrm{e}^{AT} M(A) B_2 \\ -C_2 & I_n & O_{nm} \\ O_{mn} & -\Gamma(z) & \Phi(z) \end{bmatrix} \\ & = d(z) \end{aligned} \tag{6}$$

is called the *modal control problem* for the sampled-data system (1). In (6) the quantity $d(z)$ is a given polynomial, and the polynomial matrices $\Phi(z),\ \Gamma(z)$ are the demanded unknowns. If there exists a solution of equation (6) for any $d(z)$, then the system (1) is called *completely modal controllable.*

Theorem 3. Let the pair (A, B_2) be completely controllable, and the pair (A, C_2) completely observable. Furthermore, let the set of different eigenvalues $\lambda_1, \ldots, \lambda_h$ of the matrix A satisfy the conditions

$$\begin{aligned} & \lambda_r - \lambda_s \neq \frac{2\kappa\pi \mathrm{j}}{T} \\ & \qquad (r \neq s;\ r, s = 1, \ldots, h;\ \kappa = 0, \pm 1, \ldots\,) \end{aligned} \tag{7}$$

and also

$$\int_0^T \mathrm{e}^{\lambda_i t} \, \mu(t) \, \mathrm{d}t \neq 0, \qquad (i = 1, \ldots, h)\,. \tag{8}$$

Then, the system (1) is completely modal controllable.

Remark Practical methods for the solution of polynomial equations of the form (6) are presented in (Rosenwasser and Lampe 2000*a*). The solution consists of two steps. At first, a basic controller $(\tilde{\Phi}(z), \tilde{\Gamma}(z))$ is determined that makes the matrix in (6) unimodulare. For instance, this problem can be numerically solved by the algorithm proposed in (Yakubovich 1984). With the help of the basic controller, in a second step, the parameterized set of all stabilizing controllers, or of all controllers with a given characteristic polynomial can be easily constructed.

Theorem 4. Let at least one of the following conditions hold:

(1) The pair (A, B_2) is not completely controllable.
(2) The pair (A, C_2) is not completely observable.
(3) For at least one $\nu,\ 1 \leq \nu \leq h$ the relation

$$\int_0^T \mathrm{e}^{\lambda_\nu t} \, \mu(t) \, \mathrm{d}t = 0 \tag{9}$$

takes place.

Then, the system (1) is not completely modal controllable.

The proofs for the Theorems 3 and 4 can be given by using the methods presented in (Rosenwasser and Lampe 2000*a*). The main point is to show that certain pairs of polynomial matrices are irreducible. ■

In the following, the proofs for the Theorems 1 and 2 are given.

4. GENERATING FUNCTIONS AND $Z-$TRANSFORMS

1. The sequence of complex numbers

$$\{c_k\} = \{c_0, c_1, c_2, \ldots\} \tag{10}$$

is called a *Taylor sequence* if there exist positive numbers $L > 0$ and $R > 0$ such that

$$|c_k| < \frac{L}{R^k} \,. \tag{11}$$

For a Taylor sequence $\{c_k\}$ the sum

$$c^*(\zeta) = \sum_{i=0}^{\infty} c_i \zeta^i \tag{12}$$

converges for $|\zeta| < R$, and for $|z| > R$ the Laurent series

$$c_*(\zeta) = \sum_{i=0}^{\infty} c_i z^{-i} \tag{13}$$

converges. The sum of the sequence (12) is called the *generating function* of the Taylor sequence (10), and the sum of the sequence (13) its $z-$transform (Ogata 1987). By construction we have

$$c^*(\zeta) = c_*(z)|_{z=\zeta^{-1}} \,. \tag{14}$$

2. The Taylor sequence $\{c_k(\eta)\}^{(r)}$ with

$$c_k(\eta) = k^r \eta^k c_0 \tag{15}$$

where η, c_0 are complex numbers, and r is a non-negative integer, will be called a *Floquet sequence of order* r. The quantity η is named the *parameter* of the sequence $\{c_k(\eta)\}^{(r)}$.

Lemma 1. For the sequence $\{c_k\}$ to be presentable in the form

$$\{c_k\} = \sum_{i=1}^{h} \sum_{q=0}^{\nu_i} \{c_k(\eta_i)\}^{(q)} a_{iq} \tag{16}$$

where a_{iq} are constants with $a_{i,\nu_i} \neq 0$, it is necessary and sufficient that its generating function $c^*(\zeta)$ allows the irreducible presentation

$$c^*(\zeta) = \sum_{i=1}^{h} \frac{a_i(\zeta)}{(\zeta - \eta_i)^{\nu_i + 1}} \tag{17}$$

where $a_i(\zeta)$ is a polynomial with $\deg a_i(\zeta) \leq \nu_i$, i.e. $c^*(\zeta)$ is a strictly proper rational function that is analytical in $\zeta = 0$.

The proof follows from known properties of the $z-$transforms and equation (14). ■

In what follows, a sequence of the form (16) will be called a *sequence of Floquet type.*

5. DISCRETE MODELS OF SAMPLED-DATA SYSTEMS

1. The system of difference equations

$$\begin{aligned} x_{k+1} &= \mathrm{e}^{AT} x_k + L(T)\psi_k \\ y_k &= C_2 x_k \\ \Phi_0 \psi_{k+q} + \ldots + \Phi_q \psi_k &= \Gamma_0 y_{k+q} + \ldots + \Gamma_q y_k \end{aligned} \tag{18}$$

where

$$L(T) = \mathrm{e}^{AT} M(A) B_2 \tag{19}$$

will be called the discrete model of the sampled-data system (1). The whole of equations (18) together with the relations

$$\begin{aligned} x(kT+\varepsilon) &= \mathrm{e}^{A\varepsilon} x_k + L(\varepsilon)\psi_k \\ &\quad 0 \leq \varepsilon \leq T;\; k = 0, 1, 2, \ldots \\ y(kT+\varepsilon) &= C_2 x(kT+\varepsilon) \end{aligned} \tag{20}$$

with

$$L(\varepsilon) = \int_0^{\varepsilon} \mathrm{e}^{A(\varepsilon-\tau)} \mu(\tau) \, \mathrm{d}\tau \tag{21}$$

is called the *modified discrete model* of the sampled-data system (1).

2. Between the solutions of the original system (1) and its modified discrete model exist reversibly unique relations in the following sense. If $x(t)$, $y(t)$, ψ_k is a solution of the equations (1), then the whole of vector sequences $\{x_k\} = \{x(kT)\}$, $\{y_k\} = \{y(kT)\}$, $\{\psi_k\}$ is a solution of the discrete model (18). Reversely, if the whole of vector sequences $\{x_k\}$, $\{y_k\}$, $\{\psi_k\}$ is an arbitrary solution of the discrete model (18), then there exist functions $x(t)$ and $y(t)$ that are determined by (20), and a sequence $\{\psi_k\}$ that are together a solution of the original system (1).

6. PROPERTIES OF DISCRETE MODELS

Lemma 2. Let $\det \Phi_0 \neq 0$. Then, any solution of the discrete model (18) is a Taylor sequence.

Proof

a) The formal transfer in (18) to the $z-$transform leads to

$$\begin{bmatrix} x_*(z) \\ y_*(z) \\ \psi_*(z) \end{bmatrix} = zQ^{-1}(z)B(z) \stackrel{\triangle}{=} v_*(z) \qquad (22)$$

where $Q(z)$ is the matrix (3), $B(z)$ is a polynomial vector depending on the initial conditions x_0, $\psi_0, \psi_1, \ldots, \psi_{q-1}$; and $v_*(z)$ is the $z-$transform of the vector sequence $\{v_k\}$ with $v'_k = \begin{bmatrix} x'_k \; y'_k \; \psi'_k \end{bmatrix}$, where the prime denotes the transpose.

b) We show that under the above assumptions the vector $v_*(z)$ is at most proper. Indeed, for $\det \Phi_0 \neq 0$ the matrix $Q(z)$ is strictly reducible in the sense of (Kailath 1980), and by construction the power of each row of the matrix $zB(z)$ has a power that does not exceed the power of the corresponding row of the matrix $Q(z)$. Therefore, due to the general criterion (Kailath 1980), it follows that the vector $v_*(z)$ is strictly proper or proper.

c) From the above follows that the vector $v_*(z)$ has a $z-$transform. Hence

$$\begin{aligned} x_*(z) &= \sum_{k=0}^{\infty} x_k z^{-k} \\ y_*(z) &= \sum_{k=0}^{\infty} y_k z^{-k} \qquad (23) \\ \psi_*(z) &= \sum_{k=0}^{\infty} \psi_k z^{-k} . \end{aligned}$$

Comparing the coefficients for equal powers of z on the right and left side, we state that the sequences $\{x_k\}$, $\{y_k\}$ and $\{\psi_k\}$ are solutions of the equations (18) for arbitrary initial conditions x_0, $\psi_0, \psi_1, \ldots, \psi_{q-1}$. Since these solutions are uniquely determined, and they describe the whole set of solutions, any solution of the discrete model consists of Taylor sequences and formula (22) determines its $z-$transform. ∎

Lemma 3. Let $\det \Phi_0 \neq 0$. Then

$$\eta = z_\ell \qquad (24)$$

where z_ℓ is an arbitrary non-vanishing root of the equation

$$\det Q(z) = 0 \qquad (25)$$

is a necessary and sufficient condition for the existence of a solution of Floquet type for the discrete model (18) with parameter η of the form

$$\{v_k\} = \sum_{q=0}^{\nu} \{c_k(\eta)\}^{(q)} a_q \qquad (26)$$

where a_q are constant and $\nu \geq 0$.

Proof Due to Lemma 2 any solution of the discrete model has a $z-$transform given by formula (22). Besides from the properties of the $z-$transforms (Ogata 1987) follows that the sequence (26) can be the original of the $z-$transform (22) in that and only in that case, when the parameter η is a non-vanishing pole of the matrix (22). But from (22) follows that the poles of the vector $v_*(z)$ are located in the set of eigenvalues of the matrix $Q(z)$. Thus the necessity was shown.

Directly can be seen that if z_ℓ is a non-vanishing eigenvalue of the matrix $Q(z)$, then the discrete model (18) has the solution of Floquet type

$$x_k = z_\ell^k x_0, \quad y_k = z_\ell^k y_0, \quad \psi_k = z_\ell^k \psi_0 \qquad (27)$$

where x_0, y_0 and ψ_0 are constant vectors determined as non-vanishing solution of the linear system

$$Q(z_\ell) \begin{bmatrix} x_0 \\ y_0 \\ \psi_0 \end{bmatrix} = 0\,. \qquad (28)$$

Such a solution exists due to $\det Q(z_\ell) = 0$. The sufficiency is shown. ∎

7. PROOF OF THEOREM 1

Let relation (2) be valid, where λ is a certain characteristic index of the system (1). Then, the Floquet sequences

$$\begin{aligned} x_k &= x(kT) = k^\gamma \, \mathrm{e}^{k\lambda T} \, x_1(0) \\ y_k &= y(kT) = k^\gamma \, \mathrm{e}^{k\lambda T} \, y_1(0) \qquad (29) \\ \psi_k &= k^\gamma \, \mathrm{e}^{k\lambda T} \, \psi(0) \end{aligned}$$

determine a solution of the discrete model with the parameter $\eta = \mathrm{e}^{\lambda T}$. Therefore, by Lemma 3 the quantity $\eta = \mathrm{e}^{\lambda T}$ proves to be a non-vanishing eigenvalue of the matrix $Q(z)$. Conversely, let z_ℓ be a non-vanishing eigenvalue of the matrix $Q(z)$.

Then, the discrete model has a solution of Floquet type (27). With the notation

$$\lambda = \frac{1}{T} \log z_\ell \tag{30}$$

and after substituting (27) in (20), we get

$$\begin{aligned} x(t) &= \mathrm{e}^{\lambda t}\, x_1(t),\ x_1(t) = x_1(t+T) \\ y(t) &= \mathrm{e}^{\lambda t}\, y_1(t),\ y_1(t) = y_1(t+T) \end{aligned} \tag{31}$$

where the presentation (31) does not depend on the selection of the possible values of the logarithm in relation (30). The theorem is proven. ■

8. PROOF OF THEOREM 2

a) Let

$$v^*(\zeta) = \zeta^{-1} Q^{-1}(\zeta^{-1}) B(\zeta^{-1}) \tag{32}$$

be the generating function of the solution of the discrete model that is received from (22) by substituting ζ^{-1} for z. We will show that under the conditions of Theorem 2 the matrix (32) is strictly proper. Indeed, it follows from the above that the matrix $Q(z)$ has no vanishing eigenvalues, such that the matrix $Q^{-1}(z)B(z)$ is analytical in the origin. Hence, with respect to (22) we obtain

$$v_*(z)\,\big|_{z=0} = O_{p+n+m,1}\,. \tag{33}$$

Therefore, using (14) we receive that the vector $v^*(\zeta)$ is strictly proper.

b) From the strictly properness of the vector (32) and Lemma 1 follows that under the conditions of Theorem 2 all solutions of the discrete model are sequences of Floquet type.

c) Let us denote by

$$\begin{aligned} x(s) &\triangleq x^*(\zeta)\,\big|_{\zeta=\mathrm{e}^{-sT}} \\ \psi(s) &\triangleq \psi^*(\zeta)\,\big|_{\zeta=\mathrm{e}^{-sT}} \end{aligned} \tag{34}$$

the discrete Laplace transforms of the sequences $\{x_k\}$ and $\{\psi_k\}$. Since $x^*(\zeta)$ and $\psi^*(\zeta)$ are strictly proper, there exist the limits

$$\begin{aligned} \ell^-[x(s)] &\triangleq \lim_{\mathrm{Re}\, s\to-\infty} x(s) = O_{p1} \\ \ell^-[\psi(s)] &\triangleq \lim_{\mathrm{Re}\, s\to-\infty} \psi(s) = O_{m1}\,. \end{aligned} \tag{35}$$

d) Multiplying the k-th equation in (20) by e^{-ksT} and summing leads to

$$\mathcal{D}_x(T,s,\varepsilon) = \mathrm{e}^{A\varepsilon}\, x(s) + L(\varepsilon)\psi(s) \tag{36}$$

where the notation

$$\begin{gathered} \mathcal{D}_x(T,s,\epsilon) \triangleq \sum_{k=-\infty}^{\infty} x(kT+\varepsilon)\,\mathrm{e}^{-ksT} \\ 0 \le \varepsilon \le T \end{gathered} \tag{37}$$

was used. Formula (37) defines the modified discrete Laplace transform of the vector $x(t)$.

e) On the basis of (36) we construct the discrete Laplace transform of the vector $x(t)$ in continuous time (Rosenwasser and Lampe 2000*b*), that is determined by the relation

$$\begin{gathered} \mathcal{D}_x(T,s,t) \triangleq \sum_{k=-\infty}^{\infty} x(kT+t)\,\mathrm{e}^{-ksT} \\ -\infty < t < \infty\,. \end{gathered} \tag{38}$$

For the calculation of the vector $\mathcal{D}_x(T,s,t)$ the method of continuation of the transform $\mathcal{D}_x(T,s,t)$ with respect to the time t is applied, that is described in (Rosenwasser and Lampe 2000*b*).

Multiplying (36) by $\mathrm{e}^{-s\varepsilon}$ leads to the DPFR (displayed pulse frequency response)

$$\begin{gathered} \varphi_x(T,s,\varepsilon) = \mathrm{e}^{A\varepsilon}\,\mathrm{e}^{-s\varepsilon}\, x(s) + L(\varepsilon)\,\mathrm{e}^{-s\varepsilon)}\,\psi(s) \\ 0 \le \varepsilon \le T\,. \end{gathered} \tag{39}$$

Since

$$\varphi_x(T,s,t) = \varphi_x(T,s,t+T) \tag{40}$$

it follows from (39)

$$\begin{aligned} &\varphi_x(T,s,t) = \\ &\quad = \varphi_1(T,s,t)x(s) + \varphi_2(T,s,t)\psi(s) \end{aligned} \tag{41}$$

where

$$\begin{gathered} \varphi_i(T,s,t) = \frac{1}{T}\sum_{k=-\infty}^{\infty} F_i(s+k\mathrm{j}\omega)\,\mathrm{e}^{k\mathrm{j}\omega t} \\ (i=1,2) \end{gathered} \tag{42}$$

with $\omega = 2\pi/T$ and

$$\begin{aligned} F_1(s) &= \int_0^T \mathrm{e}^{A\varepsilon}\,\mathrm{e}^{-s\varepsilon}\,\mathrm{d}\varepsilon \\ F_2(s) &= \int_0^T L(\varepsilon)\,\mathrm{e}^{-s\varepsilon}\,\mathrm{d}\varepsilon\,. \end{aligned} \tag{43}$$

The matrices (42) are integral functions of the argument s. Multiplying (41) by e^{st} we find

$$\begin{aligned} &\mathcal{D}_x(T,s,t) = \\ &\quad = \mathcal{D}_1(T,s,t)x(s) + \mathcal{D}_2(T,s,t)\psi(s) \end{aligned} \tag{44}$$

where the matrices

$$\mathcal{D}_i(T,s,t) = \varphi_i(T,st)\,\mathrm{e}^{st}, \quad (i=1,2) \quad (45)$$

are integral functions of the argument s.

f) Applying the formula for the inverse transform (Rosenwasser and Lampe 2000*b*) of (38), we find

$$x(t) = \frac{T}{2\pi \mathrm{j}} \int\limits_{d-\mathrm{j}\omega/2}^{d+\mathrm{j}\omega/2} \mathcal{D}_x(T,s,t)\,\mathrm{d}s \quad (46)$$

where d is a real constant, such that all poles of the integrand are located in the halfplane $\mathrm{Re}\, s < d$. Since from (37) and (38) follows

$$\mathcal{D}_x(T,s,\varepsilon + kT) = \mathcal{D}_x(T,s,\varepsilon)\,\mathrm{e}^{ksT} \quad (47)$$

for $t \geq 0$, due to (35) we get

$$\begin{aligned} \ell^-[\mathcal{D}_x(T,s,t)] &= \lim_{\mathrm{Re}\, s \to -\infty} \mathcal{D}_x(T,s,t) \\ &= O_{p1}\,. \end{aligned} \quad (48)$$

As was shown in (Rosenwasser and Lampe 2000*b*), the conditions (48) and (46) allow the representation

$$x(t) = T \sum_i \operatorname*{Res}_{\tilde{s}_i} \mathcal{D}_x(T,s,t) \quad (49)$$

where $\tilde{s}_i$ are the poles (characteristic indices of the system (1)) located in the stripe $-\mathrm{j}\omega/2 \leq \mathrm{Im}\, s < \mathrm{j}\omega/2$. Calculating the corresponding residues and taking into account that (45) are integral functions, we obtain that the vector $x(t)$ is a finite sum of functions of the form (2). Obviously, the same is true for the vector $y(t)$. Moreover, the sequence $\{\psi_k\}$ is the sum of a sequence of the form (2). Therefore, due to the assumptions, the matrix $Q(z)$ has no eigenvalues equal to zero.

The theorem is proven. ■

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