15th Triennial World Congress, Barcelona, Spain

INFERENTIAL PROBLEMS FOR A CLASS OF DISCRETE-TIME HYBRID SYSTEMS PART I: PROCESS MODELING

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Abstract: This paper, the first part of our three-part contribution, proposes a process modelling framework that captures integral continuous variable dynamics in discrete-time; the framework may be called discrete-time version of the Phase Transition Model of Hybrid Systems (Alur *et al.*, 1993). The modelling of the system phases is performed through Activity States. The characterisation of Activity States is derived from the physical system variables for easy modelling. The contributions of this paper are as follow. Transitions among activity states are time constrained with upper and lower time bounds relative to an external global clock. Composition of component models, essential for modelling of complex systems, is discussed in detail. The model proposed in this paper is used in two companion papers for solving the inferential problems of state estimation and fault diagnosis.

Keywords: Hybrid Systems, Discrete-time Systems, Systems Modelling, State Estimation, Fault Diagnosis

1. INTRODUCTION

Research on Discrete Event Systems started with Finite State Machine (FSM) models (P.J.G. and W.M., 1989). The model has been subsequently extended to include time and process variables (Mukhopadhay *et al.*, 2000*a*). As a natural extension and in view of the existence of a rich theory of continuous variable dynamics, the FSM model has been further enhanced to encompass hybrid systems (Henzinger *et al.*, 1993) (Zad, 1999). In this work a hybrid extension of the work (Mukhopadhay *et al.*, 2000*a*),(Bhowal *et al.*, 2000),(Mukhopadhay *et al.*, 2000*b*) is proposed.

In (Mukhopadhay *et al.*, 2000*a*), inferential problems such as, state estimation and fault detection and diagnosis (FDD), were discussed for a class of timed discrete event systems. The process model was similar to the Timed Transition Model (TTM) (Ostroff and Wonham, 1990). In this approach (Mukhopadhay *et al.*, 2000*a*), the Reachability Graph (RG) was first constructed. Measurement restriction was applied on the RG and then the FDD mechanism was formulated on top of it. Though the approach is appropriate for many industrial discrete event system, it has the following short comings

- RG becomes very large with the increase value of timing parameters.
- due to the presence of continuous variable, it the initial value is not a point, if can produce infinite number of RG.
- In the TTM, all the dynamics are presented in terms of discrete event transitions. Accordingly differential and difference equations needed for continuous dynamics can not be captured.

The modeling framework presented in this work can capture the timed dynamics of both discrete and continuous variables, and hence can model a class of hybrid system. However, unlike classical hybrid system models, time is discrete here to capture implementation on computer naturally.

This paper is organised as follows. In section 2, the activity state based process model was introduced. Composition of process model was discussed in section 3. Section 4 concludes the paper. The process model and composition are discussed with an example of a heating system

2. ACTIVITY STATE BASED PROCESS MODEL

The *model M* of a discrete time hybrid system is defined in terms of activity states as

$$M = < V, X, t, \Im, \theta > \quad (1)$$

where $V = \{v_1, v_2, ..., v_n\}$ is a finite set of data variables, X is a finite set of activity states (similar to control locations (Alur *et al.*, 1993)), t is a clock variable, $\Im$ is a set of transitions and θ is the initial condition.

$V = V_c \cup V_d$, where V_c is the set of continuous valued variables of type real, V_d is the set of discrete valued variables which can be of type real or type integer or enumerated type.

Data state: A data state σ is an interpretation of all variables in V. The data state σ can have two components, a discrete data state (σ_d) and a continuous data state (σ_c), i.e., $\sigma = < \sigma_c, \sigma_d >$.

Data space: The set of all data states is termed as data space and is denoted by Σ_D. Correspondingly we can write $\Sigma_D = < \Sigma_d \times \Sigma_c >$ where Σ_d, Σ_c are the discrete and continuous data space respectively.

2.1 *Activity states*

The concept of activity states forms the basis of the proposed activity state based model. An activity state is defined by a continuous dynamics (described by a set of difference equations) and a discrete data state. An activity state transition occurs if there is a change in the continuous dynamics or a change in the discrete data state. In the present model it is assumed that the system is having a finite number of activity state.

With each activity state x, a predicate b_x (defined over the continuous variables) is associated. This predicate defines the boundary conditions of the continuous variables. We call this predicate as boundary predicate for an activity state.

In essence, an activity state is characterised by a discrete data state σ_{dx}, a change function Δ_x and the predicate b_x.

Change Function: Here we are considering the class of systems which can be modeled as piecewise linear system. For all **continuous variables** $v_c \in V_c$, the change function $\Delta_x^{v_c}$, associated with activity state x, is defined as the rate of change of the variable with time, i.e., $\frac{\Delta v_c}{\Delta t}$ and is assumed to be constant.

Activity Description Table (ADT): The set $\{< \sigma_{dx}, \Delta_x, b_x > | \forall x \in X\}$ i.e. the discrete data state along with change variables and boundary predicates, is described in a tabular form, forms the **Activity Description Table (ADT)**. Please note that the set of variables of a compponent, excluding the input variables are designated as V_p. This input variables are excluded as any component does not influence their input variables.

Clock Variable: In addition to the data variables and activity states, the model has one special variable, a clock variable t with $type(t) = \mathcal{N}$, the set of all natural numbers. The clock variable represents time on a global clock, external to the system M.

System State: A **system state** q is defined as an ordered tuple $< x, \sigma >$, where σ is a data state belonging to Σ_D

Recall that X is the set of all activity states. Thus $Q \subseteq X \times \Sigma_D$ denotes the set of all possible system states. Because of the presence of continuous variables, Q is infinite.

Timed State: Once we include the value of t with the system state, we get the *timed state*, denoted as s. A timed state s is a tuple $< x, \sigma, t >$. The set of all timed states is denoted as S. Naturally S is also infinite.

2.2 *Transitions*

Besides the Activity Description Table (ADT), we need the transitions among the activity states to be defined for capturing the system behavior. $\Im$ is a finite set of transitions. A transition $\tau \in \Im$ from an activity state x to another activity state x^+ is an ordered six-tuple, $\tau = < x, x^+, e_\tau, h_\tau, l_\tau, u_\tau >$;

where,

x is the present activity state of the transition
x^+ is the next activity state of the transition
e_τ is the enabling condition of the transition τ, more specifically $e_\tau : \Sigma_D \rightarrow \{true, false\}$, that is, e_τ is a boolean function and can be conveniently represented by a set of elementary clauses connected by $\wedge$ and $\vee$ where each elementary clause is a linear inequality involving $\leq, \geq$ or $=$.
h_τ is the transformation function, that transforms data variables during the transition τ, from an activity state x to x^+. Thus, $h_\tau : \Sigma_D \rightarrow \Sigma_D$
l_τ is the lower time bound to elapse, for the transition to occur after e_τ becomes true.
u_τ is the upper time bounds of the transition. It indicates that the transition must take place on or before the upper time bound, if e_τ remains true.

It is always the case that $l_\tau \leq u_\tau$.

No transition can take place if its enabling condition is false in an activity state x. If an enabling condition is true, the transition can take place from x to x^+ any time, within its upper and lower time bounds. A set $V_\tau \subseteq V$ is said to be the *target set* (of variables) of a transition τ, iff $\forall v \in V_\tau, h_\tau(v) \neq v$.

A transition τ from x to x^+ is denoted as $\tau \ :< x, x^+ \ >$ for brevity when its other components are clear from the context. Transitions are represented in a tabular form, called *Timed Transition Table (TTT)*.

2.2.1. *Tick transitions* The *tick transition* or simply *tick*, denoted as η, is defined as

$$\eta = (x, x, true, h_\tau, 0, \infty) \tag{2}$$

where, for h_τ, $\Sigma_D \rightarrow \Sigma_D$ is identity and $t = t + 1$. Thus a tick increments the clock variable t by 1, leaving all other data variables unchanged. In fact, a tick is the only transition that changes the value of t. Tick occurs infinitely often and is not explicitly included in $\Im$.

2.2.2. *Semantics of τ* If $\sigma_x \in \Sigma_D$ be the data state associated with an activity state x at a particular instant, then a transition τ from x to x^+ is enabled, at that instant, if $e_\tau(\sigma_x)$ became true.

Let $< x, \delta_x, (t-1) >$ and $< x, \delta'_x, t >$ be two consecutive timed states. Let $< x, x^+, e_\tau, h_\tau, l_\tau, u_\tau >$ be a transition form x to x^+ and $e_\tau(\delta_x)$ be false and $e_\tau(\delta'_x)$ be true; that is e_τ is enabled at time instant t. The time instant t is designated as *choice point* of τ, denoted as $(t_{c\tau})$.

Though enabled, the transition is prevented from occurring till the lower time bound l_τ elapses after the choice point. If the transition continues to be enabled from the choice point to the current instant, then the transition must occur before the upper time bound u_τ elapses, unless the occurrence of some other transition causes $e_\tau(\sigma_x)$ to become false. Therefore the transition can only take place at $t \ \ th$ tick when $(l_\tau + t_{c\tau}) \leq t \leq (u_\tau + t_{c\tau})$ provided the activity state is x and $e_\tau(\sigma_x)$ is true $\forall t \ (l_\tau + t_{c\tau}) \leq t \leq (u_\tau + t_{c\tau})$. Significance of time bounds are as follows:

When the time bound is $(0, 0)$, the transition is called instantaneous. When the time bound is $(0, \infty)$, the transition is called spontaneous. When l_τ has some non-zero value, the transition is called delayed.

2.3 *Initial condition*

An initial condition θ is a satisfiable assertion over the variables V and X, characterizing the initial system states, at $t = 0$. Initialization may be considered as resetting of the system and the system after reset always starts from a $q_o =< x_0, \sigma_0 >$.

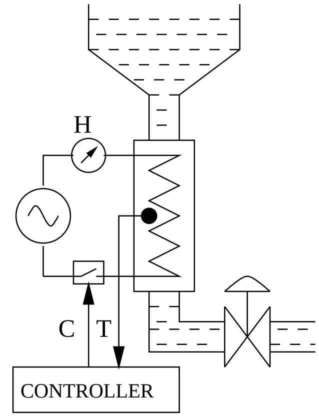

Fig. 1. Temperature control system

2.4 *Activity transition graph*

The process model M can alternatively be represented by a graph, called **Activity Transition Graph (ATG)** $=< X, \Im - \{tick\} >$. The set of activity states X is the set of nodes of ATG and the set of transitions $\Im - \{tick\}$ is the set of directed arcs. Since the set of activity states X is finite, ATG is also finite. ATG has some initial states $X_i \subseteq X$ such that $\forall x_i \in X_i, \ \ \theta(x_i) = true$.

It is assumed that Zeno computation (Ostroff and Wonham, 1990) does not arrise in the models.

2.5 *Example: Heating System*

The definitions stated above, are illustrated through by the example of a heating system. The system is shown in **Fig. 1**.

For modularity, the models of the individual components are specified first. The model for the entire system, is then built by composing the component models. In this example, the failure of the controller and sensors have not been considered for simplicity. These failures can however be included in the model, in a similar manner.

2.5.1. *Component models* **Heating system:** The Heating system has four activity states; namely *heater off in good condition* (H_F), *heater on in good condition*(H_N), *heater off in bad condition*(H_{SF}) and *heater on in bad condition*(H_{SN}). Heater can go to *bad on* condition only from *good on* condition and *bad off* condition from *good off* condition. The model is explained below.

Activity States: $X_H = \{H_F, H_N, H_{SF}, H_{SN}\}$, where H_F: Heater OFF; H_N: Heater ON; H_{SF}: Heater STUCK OFF; H_{SN}: Heater STUCK ON.

Data Variables: $V_H = \{H, S, T, C\}$, where **H:** heater, type discrete, domain {F,N}, where F:OFF and N:ON; **S:** status, type discrete, domain {G,B}, where G:GOOD and B:BAD; **T:** temperature, type

x	Description	V_P			
		σ_{dx}		Δ_x	b_x
		H	S	$\frac{\Delta T}{\Delta t}$	T
H_F	OFF & Good	F	G	-1	$0 \leq T \leq 7$
H_N	ON & Good	N	G	+1	$0 \leq T \leq 7$
H_{SF}	OFF & Bad	F	B	-1	$0 \leq T \leq 7$
H_{SN}	ON & Bad	N	B	+1	$0 \leq T \leq 7$

Table 1. Activity description table of heating system

τ	x	x^+	e_τ	h_τ	l_τ	u_τ
τ_{H1}	H_F	H_N	$C = H$	$\frac{\Delta T}{\Delta t} = +1 \wedge H = N$	0	0
τ_{H2}	H_N	H_F	$C = L$	$\frac{\Delta T}{\Delta t} = -1 \wedge H = F$	0	0
τ_{H3}	H_F	H_{SF}	$True$	$S = B$	0	∞
τ_{H4}	H_N	H_{SN}	$True$	$S = B$	0	∞

Table 2. Timed transition table of heating system

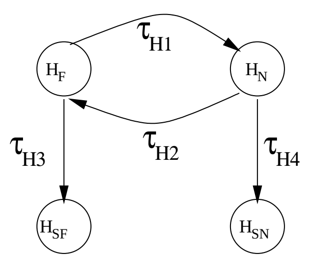

Fig. 2. ATG of heating system

real, domain $(0 \leq T \leq 7)$ [1] ; **C:** controller, type discrete, domain {H,L}, where H:HIGH and L:LOW.

Initial Condition: $(H = F \wedge S = G \wedge T = 0)$

Note: Instead of a single initial system state $< x, \sigma >$, as in this case, we may get a set of initial system state depending on the definition of θ.

The ADT of the heating system is shown in **Table 1**. The Timed Transition Table (TTT) is shown in **Table 2**. The Activity Transition Graph (ATG) is shown in **Fig.2**.

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Controller: Controller has two activity states; namely *Control output LOW* (C_L) and *Control output HIGH* (C_H). The controller toggles between these two states, depending on the heater temperature. The model is explained below.

Activity states: $X_C = \{C_L, C_H\}$, where C_L: Controller output low; C_H: Controller output high.

Data variables: $V_C = \{C, T\}$, where **C:** controller, type discrete, domain $\{L, H\}$, where L:LOW and H:HIGH; **T:** temperature, type continuous, domain $(0 \leq T \leq 7)$.

Initial condition: $(C = L)$

The Activity Description Table (ADT) is shown in **Table 3**. The Timed Transition Table is shown in **Table 4**. The Activity Transition Graph is shown in **Fig. 3**

[1] Here we have considered arbitary boundary conditions 0 and 7. The actual value shall depend on the system under investigation

X	Description	V_P
		σ_{dx}
		C
C_L	Output LOW	L
C_H	Output HIGH	H

Table 3. Activity description table of Controller

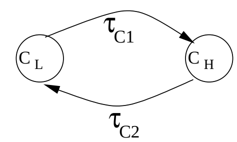

Fig. 3. ATG of Controller

τ	x	x^+	e_τ	h_τ	l_τ	u_τ
τ_{C1}	C_L	C_H	$T \leq 3$	C=L	0	0
τ_{C2}	C_H	C_L	$T \geq 5$	C=H	0	0

Table 4. Timed transition table of controller

3. COMPOSITION

A system typically consists of many components operating concurrently and coordinating with each other. Models for such systems can be constructed by parallel composition of the individual component models. The composition is defined below for two components; the definition can be extended in a natural way for more than two components.

3.1 *Composition of two component models*

Let the process models of two systems M_1 and M_2 be $M_1 =< V_1, X_1, t, \Im_1, \theta_1 >$ and $M_2 =< V_2, X_2, t, \Im_2, \theta_2 >$.

The process model of the composite system is obtained by parallel composition of two components M_1 and M_2 (denoted as $M = M_1 || M_2$) and is defined as the five-tuple

$$M = (V, X, t, \Im, \theta).$$

where

$V = V_1 \cup V_2$ is the set of data variables in the composite model and $V_1 \cap V_2 \neq \phi$ for interacting systems

$X \subseteq X_1 \times X_2$ is the set of activity states of the composite model.

t is the global time,

$\Im = \Im_1 || \Im_2$ is the set of transitions of composite model

$\theta = \theta_1 \wedge \theta_2$ is the initial condition of the composite model.

The transformations $\{h_\tau\}$ and the enabling conditions $\{e_\tau\}$ of transitions for both the machines are suitably composed so that they apply on the entire data variable set V. This can be accomplished by the following steps.

In M_1, $\forall \tau \in \Im_1, \quad \forall v \in V - V_1, \quad h_\tau(v) = v$ and $e_\tau(v) = true$; $\forall x_1 \in X_1, b_{x_1,v} = true$; In M_2, $\forall \tau \in \Im_2, \quad \forall v \in V - V_2, \quad h_\tau(v) = v$ and $e_\tau(v) = true; \forall x_2 \in X_2, b_{x_2,v} = true.$

Construction of $\Im$, b_x and Δ_x are explained in the follows subsections

3.2 *Transitions of the composite model*

Transitions of the composed model are determined in the following manners. Let $< x_1, x_2 > \in X$ and there be transitions τ_1 in M_1 and τ_2 in M_2 such that, $\tau_1 =< x_1, x_1^+, e_{\tau_1}, h_{\tau_1}, l_{\tau_1}, u_{\tau_1} >$ and $\tau_2 =< x_2, x_2^+, e_{\tau_2}, h_{\tau_2}, l_{\tau_2}, u_{\tau_2} >$. The different possible cases are described follows.

Shared transitions: τ_1 and τ_2 are said to be shared transition if they are specified by the user to take place simultaneously. A shared transition requires that at the time of transition both transitions are enabled simultaneously . If this condition is satisfied the shared transition can be defined as

$$\tau_s =<< x_1, x_2 >, < x_1^+, x_2^+ >, e_{\tau_s}, h_{\tau_s}, l_{\tau_s}, u_{\tau_s} >$$

where,

enabling condition: $e_{\tau_s} = e_{\tau_1} \wedge e_{\tau_2}$

transformation function: $h_{\tau_s} = h_{\tau_1} \circ h_{\tau_2}$, linear composition of the component's transition functions. In order to define such composition the condition $V_{\tau_1} \cap V_{\tau_2} = \phi$ must hold.

lower time bound: $l_{\tau_s} = max(l_{\tau_1}, l_{\tau_2})$

upper time bound: $u_{\tau_s} = min(u_{\tau_1}, u_{\tau_2})$

Non shared transitions:

If the transitions τ_1 and τ_2 do not satisfy the condition of shared transition, then

(1) $< x_1^+, x_2 > \in X$ and $< x_1, x_2^+ > \in X$ and
(2) $\tau_{s1}, \tau_{s2} \in \Im_1 || \Im_2$ where
$\tau_{s1} =<< x_1, x_2 >, < x_1^+, x_2 >, e_{\tau_1}, h_{\tau_1} \circ id_2, l_{\tau_1}, u_{\tau_1} >$,
$\tau_{s2} =<< x_1, x_2 >, < x_1, x_2^+ >, e_{\tau_2}, id_1 \circ h_{\tau_2}, l_{\tau_2}, u_{\tau_2} >$

$id_1 : \Sigma_{D_1} \rightarrow \Sigma_{D_1}$ and $id_2 : \Sigma_{D_2} \rightarrow \Sigma_{D_2}$ are the identity functions over the data state spaces of the components M_1 and M_2.

τ_{s1} and τ_{s2} are called *non-shared transitions* and capture the situation where either of the components (and not both) is making the transition.

3.3 *Change functions of composite activity states*

We first define the set of change functions of the composite state (x) and then define the predicate b_x, representing the valid data space of x. The set of change functions, denoted as Δ_x, of a composite activity state $x =< x_1, x_2 >$, can be constructed as

Case I: Non shared variables

$$\forall v \in V_{x_1}, \;\; and \;\; \notin V_{x_2}, \;\; \Delta_x^v = \Delta_{x_1}^v$$
$$\forall v \in V_{x_2}, \;\; and \;\; \notin V_{x_1}, \;\; \Delta_x^v = \Delta_{x_2}^v$$

Case II: Continuous shared variables ($\forall v_c \in V_{x_1} \cap V_{x_2}$)

$$\Delta_x = \Delta_{x_1} \circ \Delta_{x_2}$$

where, $\circ$ denotes a linear composition. For linear changes superposition theorem may be applied. For example if $\Delta_{x_1}^v = 1$ and $\Delta_{x_2}^v = 2$, then Δ_x^v may be defined as $\Delta_{x_1}^v + \Delta_{x_2}^v = 3$, where $\Delta_{x_i}^{v_j}$ denotes the change function of variable v_j in activity state x_i.

3.4 *Composition of b_x*

If b_{x_1} and b_{x_2} are the invariant predicates of activity states x_1 and x_2 of model M_1 and M_2 respectively, then the Invariant predicate of composite activity state $< x_1, x_2 >$, denoted as $b_{x_1 x_2}$ is

$$b_{x_1 x_2} = b_{x_1} \wedge b_{x_2} \qquad (3)$$

The composition is explained with the help of heating system example.

3.5 *Example: Heating System*

The Composition algorithm is applied for constructing the composite model of heating system of section 2.5, consisting of one controller and one heater. The composition is shown below.

Data variables: $V = V_H \cup V_C = \{H, S, T, C\}$

Transitions: $\Im = \Im_H || \Im_C$

In the example of heating system, there is no shared transition. The composite transition set is shown in **Table 5**.

Initial condition: $\theta = \theta_H \wedge \theta_C$
$\theta = (H = F \wedge S = G \wedge T = 0) \wedge (C = L)$

Activity states: The set X obtain after composition is shown in **Table 6**. The ATG is shown in **Fig. 4**.

4. CONCLUSION

In this paper a process model for hybrid system is propopsed which is an extension over the TTM proposed by (Ostroff and Wonham, 1990). It is argued that the activity state based process modelling proposed here can abstract much os low level complexity.

τ_S	Composition	x	x^+	e_τ	h_τ	l_τ	u_τ
τ_{S1}	τ_{C_1}	$H_F C_L$	$H_F C_H$	$T \leq 3$	$C = H$	0	0
τ_{S2}	τ_{H_3}	$H_F C_L$	$H_{SF} C_L$	$True$	$S = B$	0	∞
τ_{S3}	τ_{H_1}	$H_F C_H$	$H_N C_H$	$C = H$	$\frac{\Delta T}{\Delta t} = +1 \wedge H = N$	0	0
τ_{S4}	τ_{H3}	$H_F C_H$	$H_{SF} C_H$	$True$	$S = B$	0	∞
τ_{S5}	τ_{C_1}	$H_{SF} C_L$	$H_{SF} C_H$	$T \leq 3$	$C = H$	0	0
τ_{S6}	τ_{C_2}	$H_N C_H$	$H_N C_L$	$T \geq 5$	$C = L$	0	0
τ_{S7}	τ_{H_4}	$H_N C_H$	$H_{SN} C_H$	$True$	$S = B$	0	∞
τ_{S8}	τ_{H_2}	$H_N C_L$	$H_F C_L$	$C = L$	$\frac{\Delta T}{\Delta t} = -1 \wedge H = F$	0	0
τ_{S9}	τ_{H_4}	$H_N C_L$	$H_{SN} C_L$	$True$	$S = B$	0	∞
τ_{S10}	τ_{C_2}	$H_{SN} C_H$	$H_{SF} C_L$	$T \geq 5$	$C = L$	0	0

Table 5. Timed transition table of composite model of heating system

x	Composition	V_P				
		σ_{dx}			Δ_{xT}	b_x
		H	S	C	$\frac{\Delta T}{\Delta t}$	b_{xT}
x_1	$H_F C_L$	F	G	L	-1	$0 \leq T \leq 7$
x_2	$H_F C_H$	F	G	H	-1	$0 \leq T \leq 7$
x_3	$H_N C_H$	N	G	H	+1	$0 \leq T \leq 7$
x_4	$H_N C_L$	N	G	L	+1	$0 \leq T \leq 7$
x_5	$H_{SF} C_L$	F	B	L	-1	$0 \leq T \leq 7$
x_6	$H_{SF} C_H$	F	B	H	-1	$0 \leq T \leq 7$
x_7	$H_{SN} C_H$	N	B	H	+1	$0 \leq T \leq 7$
x_8	$H_{SN} C_L$	N	B	L	+1	$0 \leq T \leq 7$

Table 6. Activity description table of composite model of heating system

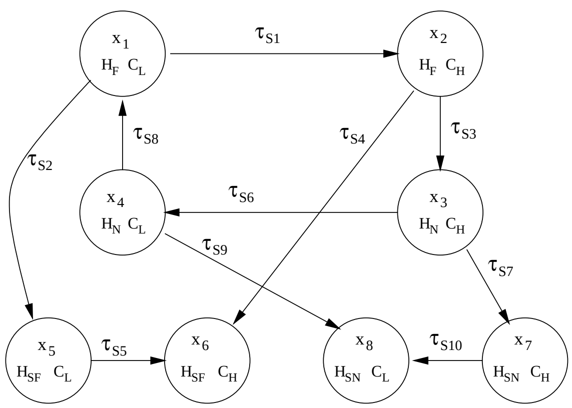

Fig. 4. ATG of composite model of heating system

Composition of such process models are given. The use of such model is demonstrated in the context of the problems of state estimation and fault diagnosis in two companion papers.

Acknowledgement: Thanks are due to the anonymous reviewers for their valuable comments.

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