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DELAY AND DELAY-DERIVATIVE DEPENDENT ROBUST AND RELIABLE STABILIZATION FOR UNCERTAIN STATE-DELAYED SYSTEMS

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Abstract: This paper focuses on the synthesis problem of delay- and its time-derivative dependent robust and reliable stabilization for linear time-delay systems with norm-bounded parameter uncertainty in the state and delayed-state matrices and also with actuator failures among a pre-specified subset of actuators. An LMI (Linear Matrix Inequality) method is given for the delay and its time-derivative dependent memoryless state feedback synthesis problem to quadratically stabilize the given systems. *Copyright ©2002 IFAC*

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1. INTRODUCTION

Dynamic systems with time-delay are common in chemical processes, electrical heater and long transmission lines in pneumatic, hydraulic, and rolling mill systems. Such time-delays can be a source of instability or may induce degradation on performance. The research on time-delay systems has attracted many researchers for long years. Some of the results have been successfully extended to systems including bounded uncertainty. Many research results have been given in the field of robust control (Xie and Souza, 1992; Wang *et al*., 1998; Cao *et al.,* 1998). In practical application, actuators are very important in transforming the controller output to the plant. Actuator failures may be encountered sometimes. Furthermore, how to preserve the closed-loop control system performance in the case of actuator failures will be more tough and more meaningful. It attracts more and more research interests in recent years (Veillette *et al*., 1992; Seo and Kim, 1996; Suyama, 2001).

In this paper, attention is focused on the robust and reliable stabilization synthesis of linear uncertain systems with delayed state and actuator failures. It is assumed that perfect information of plant states is available for feedback. The synthesis problem addressed here is to design a delay-dependent memoryless state feedback control law such that the closed-loop system with actuator failures is quadratically stable.

This paper is organized as follows: Section 2 will propose system formulation, concept of actuator failures, control objectives, some necessary lemma and definition. Section 3 will introduce the results on synthesizing the delay- and its derivative-dependent robust and reliable state feedback controller via LMI. Conclusion will be given shortly in section 4.

Throughout this paper, let R^n be any real n-dimensional linear vector space. The matrix I denotes an identity matrix with appropriate dimensions. $W > 0(< 0)$ denotes a positive definite (negative definite) symmetric matrix.

2. SYSTEM AND DEFINITION

Consider linear uncertain systems with delayed state described by the following differential equation:

$$\begin{aligned} \tfrac{d}{dt}x(t) &= \overline{A}(t)x(t) + \overline{A}_1(t)x(t-\boldsymbol{t}(t)) + Bu(t) \\ &= (A+\Delta A(t))x(t) + Bu(t) \\ &\quad + (A_1 + \Delta A_1(t))x(t-\boldsymbol{t}(t)) \\ 0 \le \boldsymbol{t}&(t) \le h, \quad \dot{\boldsymbol{t}}(t) \le d < 1 \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input vector. $\overline{A}(t)$, $\overline{A}_1(t)$, $\Delta A(t)$ and $\Delta A_1(t)$ are uncertain real-valued matrices with appropriate dimensions, A and A_1 are known constant real-valued matrices with appropriate dimensions, $\boldsymbol{t}(t)$ denotes time-varying time-delay.

Suppose the uncertain structures of the system (1) are given by,

$$\Delta A(t) = M_1 F_1(t) N_1 \quad \Delta A_1(t) = M_2 F_2(t) N_2 \tag{2}$$

$$F_i^{\mathrm{T}}(t) F_i(t) \le I \qquad i = 1,2 \tag{3}$$

The meomryless state feedback control law is considered in this paper:

$$u(t) = -Kx(t) = -\frac{1}{2\boldsymbol{e}} B^{\mathrm{T}} P x(t) \tag{4}$$

In the following part, concepts of actuator failures are discussed. Actuators of a given system can be classified into two sets. One set is susceptible to failures, denoted by $\Omega \subseteq \{1,2,\cdots,m\}$ hereafter. Actuators in this set may occasionally fail. It is redundant in terms of the stabilization of the system, while it may be necessary for improving control system performance. The other set is robust to failures, denoted by $\overline{\Omega} = \{1,2,\cdots,m\} - \Omega$. For simplicity, assume that actuators in this set never fail, and some of them are required to stabilize a given system, whether or not set $\overline{\Omega}$ can be the minimum set. Introduce a decomposition

$$B = B_{\Omega} + B_{\overline{\Omega}} \tag{5}$$

where B_{Ω} and $B_{\overline{\Omega}}$ are formed from B by zeroing out columns corresponding to Ω and $\overline{\Omega}$ respectively.

Define set of actual actuator failures of the given system (1) as $\boldsymbol{w}$, which is a subset of Ω, that is, $\boldsymbol{w} \subseteq \Omega$. Introduce a decomposition similar to (5),

$$B = B_w + B_{\overline{w}} \tag{6}$$

where B_w and $B_{\overline{w}}$ are formed from B by zeroing out columns corresponding to $\boldsymbol{w}$ and $\overline{\boldsymbol{w}}$ respectively. Thus the following equalities can be easily got,

$$\begin{aligned} B_{\Omega} B_{\Omega}^{\mathrm{T}} &= B_w B_w^{\mathrm{T}} + B_{\Omega-w} B_{\Omega-w}^{\mathrm{T}} \\ B_{\overline{\Omega}} B_{\overline{\Omega}}^{\mathrm{T}} &= B_{\overline{w}} B_{\overline{w}}^{\mathrm{T}} - B_{\Omega-w} B_{\Omega-w}^{\mathrm{T}} \end{aligned} \tag{7}$$

Consider actual actuator failure case, (1) and (4) may be rewritten respectively as,

$$\tfrac{d}{dt}x(t) = \overline{A}(t)x(t) + \overline{A}_1(t)x(t-\boldsymbol{t}(t)) + B_{\overline{w}}u(t) \tag{8}$$

$$u(t) = -K_{\overline{w}}x(t) = -\frac{1}{2\boldsymbol{e}} B_{\overline{w}}^{\mathrm{T}} P x(t) \tag{9}$$

Denote

$$\overline{A}_0(t) = \overline{A}(t) - B_{\overline{w}} K_{\overline{w}}, \ A_0 = A - B_{\overline{w}} K_{\overline{w}} .$$

Introduce some important Lemma and Definition:

Lemma 1[Cao *et al.*, 1998]: Let A, M, N, and F be real matrices of appropriate dimensions with constraint $F^{\mathrm{T}} F \le I$. Then the following inequalities will be true.

(a) For any matrix $Q > 0$ with appropriate dimensions and any scalar $b > 0$, we have

$$MFN + N^{\mathrm{T}} F^{\mathrm{T}} M^{\mathrm{T}} \le b^{-1} M Q^{-1} M^{\mathrm{T}} + b N^{\mathrm{T}} Q N$$

(b) For any matrix $P > 0$ with appropriate dimensions and any scalar $b > 0$ satisfying $bI - NPN^{\mathrm{T}} > 0$, we have

$$\begin{aligned} &(A + MFN)P(A + MFN)^{\mathrm{T}} \le APA^{\mathrm{T}} \\ &+ APN^{\mathrm{T}}(bI - NPN^{\mathrm{T}})^{-1} NPA^{\mathrm{T}} + bMM^{\mathrm{T}} \end{aligned}$$

(c) For any matrix $P > 0$ with appropriate dimensions and any scalar $b > 0$ satisfying $P - bMM^{\mathrm{T}} > 0$, we have

$$(A+MFN)^{\mathrm{T}}P^{-1}(A+MFN)$$
$$\le A^{\mathrm{T}}(P-\beta MM^{\mathrm{T}})^{-1}A+\beta^{-1}N^{\mathrm{T}}N$$

Lemma 2[Kim, 2001]: Let

$$w(t)=\int_{a(t)}^{b(t)}\int_{t-\theta}^{t} f(s)\,ds\,d\theta$$

Then, the following is satisfied:

$$\tfrac{d}{dt}w(t)=(b-a)f(t)-(1-\dot b)\int_{t-b}^{t-a} f(s)ds$$
$$+(\dot b-\dot a)\int_{t-a}^{t} f(s)ds$$

Lemma 3[Kim, 2001]: Let $a(t)\le b(t)$, then the following inequality is satisfied:

$$\left\|\int_a^b f(s)ds\right\|^2\le (b-a)\int_a^b \|f(s)\|^2 ds$$

The following Definition can be regarded as an extension of existing definition in Khargonekar *et al*. (1990) to actuator failure case.

Definition 1: The system (1) (with $u(t)=0$) is said to be quadratically stable with actuator failures if there exist a positive definite symmetric matrix P and a positive constant α such that for any admissible uncertainty and actuator failures corresponding to any $\omega\subseteq\Omega$ the derivative of the Lyapunov function candidate $V(x(t),t)$ with respect to time t satisfies $dV(x(t),t)/dt\le-\alpha\|x\|^2$ for all pairs $(x(t),t)\in R^n\times R$. Closed-loop system of (1) and (4) is said to be quadratically stabilizable with actuator failures via linear state feedback if there exists a state feedback control such that the closed-loop system is quadratically stable with actuator failures.

Remark 1: If an uncertain system is quadratically stable with actuator failures, it is straightforward to verify that for any admissible uncertainty and actuator failures corresponding to any $\omega\subseteq\Omega$, the resulting realization of the uncertain system will be asymptotically stable.

3. ROBUST AND RELIABLE STABILIZATION

In this section, one method will be presented to design robust and reliable stabilization for ensuring that closed-loop system of (1) and (4) will be quadratically stable with actuator failures.

Here the main stabilization result is given as the following theorem:

Theorem 1: For actuator failures corresponding to any $\omega\subseteq\Omega$, there exist positive definite symmetric matrices P, P_1, P_2 satisfying the following matrix inequality (LMI)

$$\begin{bmatrix} W & H_0 & H_1 & H_2 & H_3 & H_4\\ H_0^{\mathrm{T}} & -J_0 & 0 & 0 & 0 & 0\\ H_1^{\mathrm{T}} & 0 & -J_1 & 0 & 0 & 0\\ H_2^{\mathrm{T}} & 0 & 0 & -J_2 & 0 & 0\\ H_3^{\mathrm{T}} & 0 & 0 & 0 & -J_3 & 0\\ H_4^{\mathrm{T}} & 0 & 0 & 0 & 0 & -J_4 \end{bmatrix}<0 \quad (10)$$

where

$$S=P^{-1}$$
$$W=S(A+A_1)^{\mathrm{T}}+(A+A_1)S-\varepsilon^{-1}B_{\bar\Omega}B_{\bar\Omega}^{\mathrm{T}}$$
$$+\beta_3M_1M_1^{\mathrm{T}}+\beta_4M_2M_2^{\mathrm{T}}+A_1P_1A_1^{\mathrm{T}}$$
$$+A_1P_2A_1^{\mathrm{T}}+\beta_5M_2M_2^{\mathrm{T}}+\beta_6M_2M_2^{\mathrm{T}}$$
$$H_0=B_{\bar\Omega}B_{\bar\Omega}^{\mathrm{T}}\quad J_0=\tfrac{h^2}{2\varepsilon^2}(P_1-\beta_1M_1M_1^{\mathrm{T}})$$
$$H_1=\begin{bmatrix}SN_1^{\mathrm{T}} & SN_2^{\mathrm{T}}\end{bmatrix}\quad J_1=diag\begin{bmatrix}\beta_3I & \beta_4I\end{bmatrix}$$
$$H_2=\begin{bmatrix}A_1P_1N_2^{\mathrm{T}} & A_1P_2N_2^{\mathrm{T}}\end{bmatrix}$$
$$J_2=diag\left[(b_5I-N_2P_1N_2^{\mathrm{T}})\quad(b_6I-N_2P_2N_2^{\mathrm{T}})\right]$$
$$H_3=\begin{bmatrix}\sqrt{2}hSA^{\mathrm{T}} & \tfrac{h}{1-d}SA_1^{\mathrm{T}}\end{bmatrix}$$
$$J_3=diag\left[(P_1-b_1M_1M_1^{\mathrm{T}})\quad(P_2-b_2M_2M_2^{\mathrm{T}})\right]$$
$$H_4=\begin{bmatrix}hSN_1 & \tfrac{h}{1-d}SN_2\end{bmatrix}\quad J_4=diag\begin{bmatrix}\beta_1I & \beta_2I\end{bmatrix}$$

for positive constants ε, $\beta_i, i=1,2,\cdots,6$. Then closed-loop system of (1) and (4) is quadratically stabilizable with actuator failures. Furthermore, if a solution to the LMI optimization problem exists, a suitable robust and reliable stabilization control law for the system (1) is given by

$$u(t)=-Kx(t) \quad (11)$$

where

$$K=\frac{1}{2\varepsilon}B^{\mathrm{T}}S^{-1}=\frac{1}{2\varepsilon}B^{\mathrm{T}}P \quad (12)$$

Proof: Consider actuator failures corresponding to $\omega\subseteq\Omega$, the linear uncertain system (1) with controller (4) may be represented in the form as system (8) with controller (9). The candidate Lyapunov function for closed-loop system of (8) and (9) is chosen as follows,

$$V(x(t),t) = x^{\mathrm{T}}(t)Px(t) + V_1(x(t),t) + V_2(x(t),t) \tag{13}$$

where

$$V_1(x(t),t) = h\int_0^h \int_{t-q}^t x^{\mathrm{T}}(s)\Omega_1(P_1, \boldsymbol{b}_1)x(s)ds d\boldsymbol{q}$$

$$V_2(x(t),t) = \tfrac{h}{(1-d)^2}\int_t^{h+t}\int_{t-q}^t x^{\mathrm{T}}(s)\Omega_2(P_2, \boldsymbol{b}_2)x(s)ds d\boldsymbol{q}$$

$$\Omega_1(P_1, \boldsymbol{b}_1) = \boldsymbol{b}_1^{-1}N_1^{\mathrm{T}}N_1 + A_0^{\mathrm{T}}(P_1 - \boldsymbol{b}_1 M_1 M_1^{\mathrm{T}})^{-1}A_0$$

$$\Omega_2(P_2, \boldsymbol{b}_2) = \boldsymbol{b}_2^{-1}N_2^{\mathrm{T}}N_2 + A_1^{\mathrm{T}}(P_2 - \boldsymbol{b}_2 M_2 M_2^{\mathrm{T}})^{-1}A_1$$

It is obvious that the following equality is true,

$$x(t-\boldsymbol{t}(t)) = x(t) - \int_{t-t(t)}^t \tfrac{d}{dq}x(\boldsymbol{q})d\boldsymbol{q} = x(t) - \int_{t-t(t)}^t \{\overline{A}_0(\boldsymbol{q})x(\boldsymbol{q}) + \overline{A}_1(\boldsymbol{q})x(\boldsymbol{q}-\boldsymbol{t}(\boldsymbol{q}))\}d\boldsymbol{q}$$

then there will be

$$\tfrac{d}{dt}x(t) = \{\overline{A}_0(t) + \overline{A}_1(t)\}x(t) - \overline{A}_1(t)\int_{t-t(t)}^t \{\overline{A}_0(\boldsymbol{q})x(\boldsymbol{q}) + \overline{A}_1(\boldsymbol{q})x(\boldsymbol{q}-\boldsymbol{t}(\boldsymbol{q}))\}d\boldsymbol{q}$$

By using Lemma 1 and Lemma 3, the derivative of the Lyapunov function (13) with respect to time t will be

$$\begin{aligned}
\tfrac{d}{dt}V &\le x^{\mathrm{T}}\{[\overline{A}_0(t)+\overline{A}_1(t)]^{\mathrm{T}}P + P[\overline{A}_0(t)+\overline{A}_1(t)]\}x \\
&+ \left\|x^{\mathrm{T}}P\overline{A}_1(t)P_1^{1/2}\right\|^2 + \left\|\int_{t-t(t)}^t P_1^{-1/2}\overline{A}_0(q)x(q)dq\right\|^2 \\
&+ \left\|x^{\mathrm{T}}P\overline{A}_1(t)P_2^{1/2}\right\|^2 + \left\|\int_{t-t(t)}^t P_2^{-1/2}\overline{A}_1(q)x(q-t(q))dq\right\|^2 \\
&+ \tfrac{d}{dt}V_1 + \tfrac{d}{dt}V_2 \\
&\le x^{\mathrm{T}}\{[\overline{A}_0(t)+\overline{A}_1(t)]^{\mathrm{T}}P + P[\overline{A}_0(t)+\overline{A}_1(t)]\}x \\
&+ \left\|x^{\mathrm{T}}P\overline{A}_1(t)P_1^{1/2}\right\|^2 + \left\|x^{\mathrm{T}}P\overline{A}_1(t)P_2^{1/2}\right\|^2 + \tfrac{d}{dt}V_1 + \tfrac{d}{dt}V_2 \\
&+ t(t)\int_{t-t(t)}^t \left\|P_1^{-1/2}\overline{A}_0(q)x(q)\right\|^2 dq \\
&+ t(t)\int_{t-t(t)}^t \left\|P_2^{-1/2}\overline{A}_1(q)x(q-t(q))\right\|^2 dq
\end{aligned}$$

From Lemma 1 and Lemma 2, there will be,

$$\int_{t-t(t)}^t \left\|P_1^{-1/2}\overline{A}_0(\boldsymbol{q})x(\boldsymbol{q})\right\|^2 d\boldsymbol{q} \le \int_{t-h}^t x^{\mathrm{T}}(\boldsymbol{q})\Omega_1(P_1, \boldsymbol{b}_1)x(\boldsymbol{q})d\boldsymbol{q}$$

$$\int_{t-t(t)}^t \left\|P_2^{-1/2}\overline{A}_1(\boldsymbol{q})x(\boldsymbol{q}-\boldsymbol{t}(\boldsymbol{q}))\right\|^2 d\boldsymbol{q} \le \frac{1}{1-d}\int_{t-t(t)-h}^{t-t(t)} x^{\mathrm{T}}(\boldsymbol{q})\Omega_2(P_2, \boldsymbol{b}_2)x(\boldsymbol{q})d\boldsymbol{q}$$

$$\tfrac{d}{dt}V_1 = h^2 x^{\mathrm{T}}(t)\Omega_1(P_1, \boldsymbol{b}_1)x(t) - h\int_{t-h}^t x^{\mathrm{T}}(\boldsymbol{q})\Omega_1(P_1, \boldsymbol{b}_1)x(\boldsymbol{q})d\boldsymbol{q}$$

$$\tfrac{d}{dt}V_2 \le \frac{h^2}{(1-d)^2}x^{\mathrm{T}}(t)\Omega_2(P_2, \boldsymbol{b}_2)x(t) - \frac{h}{1-d}\int_{t-h-t}^{t-t} x^{\mathrm{T}}(s)\Omega_2(P_2, \boldsymbol{b}_2)x(s)ds$$

then,

$$\begin{aligned}
\tfrac{d}{dt}V \le\; & x^{\mathrm{T}}(t)\{[\overline{A}_0(t) + \overline{A}_1(t)]^{\mathrm{T}}P \\
&+ P[\overline{A}_0(t) + \overline{A}_1(t)] + P\overline{A}_1(t)P_1\overline{A}_1^{\mathrm{T}}(t)P \\
&+ P\overline{A}_1(t)P_2\overline{A}_1^{\mathrm{T}}(t)P + h^2\Omega_1(P_1, \boldsymbol{b}_1) \\
&+ \frac{h^2}{(1-d)^2}\Omega_2(P_2, \boldsymbol{b}_2)\}x(t)
\end{aligned}$$

Thus if there exist positive definite symmetric matrices P, P_1, P_2 for positive constants $\boldsymbol{e}$, $\boldsymbol{b}_i, i = 1,2,\cdots,6$ and the following inequality (14) is satisfied,

$$\begin{aligned}
R(t) \overset{\Delta}{=} & [\overline{A}_0(t) + \overline{A}_1(t)]^{\mathrm{T}}P \\
&+ P[\overline{A}_0(t) + \overline{A}_1(t)] + P\overline{A}_1(t)P_1\overline{A}_1^{\mathrm{T}}(t)P \\
&+ P\overline{A}_1(t)P_2\overline{A}_1^{\mathrm{T}}(t)P + h^2\Omega_1(P_1, \boldsymbol{b}_1) \\
&+ \frac{h^2}{(1-d)^2}\Omega_2(P_2, \boldsymbol{b}_2) < 0
\end{aligned} \tag{14}$$

there must be some positive constant c such that $dV/dt \le -c\|x\|^2 < 0$. From Definition 1, the closed-loop system is quadratically stabilizable with actuator failures.

From Lemma 1, (2), (3), (7) and some rearrangements, (10) implies (14) by pre-multiplying (14) and post-multiplying it with P^{-1}, and using Schur complement techniques. Thus we complete the proof. □

Remark 2: The LMI given in (10) is linear with respect to matrices S, $P_i, i=1,2$ and positive real constants $b_i, i=1,2,\cdots,6$. Through mature optimization method (Boyd *et al*., 1994), the desired parameters S, $P_i, i=1,2$ and $b_i, i=1,2,\cdots,6$ can be easily got, and the procedure does not require any parameter tuning. In other words, the robust and reliable stabilization problem can be reduced to the solvability of LMI.

Algorithm: Step 1: Given matrices A, A_1, B, M_1, N_1, M_2, N_2, time-delay constraints h, d, and set Ω of actuator failures;

Step 2: Decompose B into B_Ω and $B_{\overline{\Omega}}$ according to set Ω, calculate the associated dimensions n, m;

Step 3: Choose positive parameter e;

Step 4: If (10) with constraints $b_i > 0, i=1,2,\cdots,6$, $S > 0$, $P_i > 0, i=1,2$ have feasible solutions, go to Step 5; Otherwise, tune parameter e and repeat Step 4;

Step 5: The desired memoryless state feedback controller will be got as (11) and (12).

Remark 3: The results in this section can be easily extended to multi-delay case.

4. CONCLUSIONS

The robust and reliable stabilization synthesis problems are discussed for the linear time-delay systems including time-varying parameter uncertainties and actuator failures. Based on the notion of quadratic stabilization, sufficient conditions for the solvability of the robust and reliable stabilization problem are obtained to ensure the quadratic stabilization with actuator failures. Pre-defined controller structure for the linear uncertain systems with delayed state is used to construct the desired delay- and its derivative-dependent robust and reliable memoryless state feedback controller. The controller gain matrix can be got via the solvability of LMI without the need of tuning parameters.

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