

PERIODIC DISTURBANCE REJECTION USING TWO DOF REPETITIVE CONTROL FOR A MOTOR/GEAR TRANSMISSION SYSTEM

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Abstract: In this paper, we design and analyse a two degree-of-freedom (TDOF) repetitive control system for a motor/gear transmission system with actuator saturation. Modifications are made for our previous design to incorporate the performance-limiting element, i.e. actuator saturation, to acquire lower order controller, and to achieve faster computation. Furthermore, structured singular value (μ) is used to evaluate and compare the nominal/robust performance and robust stability of our designs. The simulation results are presented to show the feasibility and effectiveness of the new design for periodic and non-periodic disturbance reduction for motor/gear transmission velocity regulation.

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Keywords: Repetitive control, disturbance rejection, electrophotographic process control

1. INTRODUCTION

A motor/gear transmission system is defined as one in which an electric motor drives a gear train to achieve objectives like torque transmission or velocity reduction. Such system has been seen to prevail in many office and home products, e.g. copiers and printers. For example, the transmission system of a printer is usually composed of a brushless dc (BLDC) or stepper motor and a gear train with pre-designed gear ratio to achieve the required moving velocity. While constant velocity is important in a motor/gear-train system, transmission errors are usually unavoidable due to manufacturing tolerance. Common disturbance sources are gear eccentricity and tooth-to-tooth profile error. A well-known issue arising from those disturbance sources is periodic velocity ripples or fluctuations. Since the majority velocity variation is of known and constant periods that are related to the transmission gearing, repetitive control is an obvious approach to compensate for these periodic disturbances.

The repetitive controller is one of the control algorithms based on the Internal Model Principle (Francis and Wonham, 1976) and has been widely implemented in various applications. A repetitive control based system has been shown to work well for tracking periodic reference commands or for rejecting periodic disturbances in regulation applications. Although the idea has been verified as early as 1981 (Inoue, et al., 1981), a rigorous analysis and synthesis of repetitive controllers for continuous-time systems was not proposed until 1989, see (Hara, et al., 1989). Almost at the same time, Tomizuka, et al. (1989) addressed the analysis and synthesis of discrete-time repetitive controller considering the fact that digital implementation of a

repetitive controller is simpler and more straightforward. Since then, repetitive control has gained its popularity in various applications where periodic disturbances rejection or repetitive tracking are demanded. These include controls of disk drive servo (Tomizuka, et al., 1989; Guo, 1997; Moon, et al., 1998), rejection of load disturbances in steel casting process (Manayathara, et al., 1996), motor speed ripple reduction (Godler, et al., 1995), and eccentricity compensation in rolling (Garimella and Srinivasan, 1996). Note that in those applications, the repetitive controller can be put into either the feedback loop or the feedforward path though it is known that it is better to use the feedforward design to avoid instability if there is uncertainty in the process time-delay. Recent repetitive control applications have been seen to combine with other linear or nonlinear schemes, e.g. adaptive control (Manayathara, et al., 1996). In Manayathara's work, he incorporated an algorithm to identify the disturbance frequency on-line for use by the repetitive controller. Most recently, the concept of H_∞ loop shaping design has been incorporated into the design of repetitive controllers by many researchers (Lee and Smith, 1998; Weiss and Hafele, 1999). The immediate advantage comes from flexibility of the linear fractional formulation and availability of existing software tools. As pointed out by Guo (1997), the loop shaping technique can be used to reduce the sensitivity amplification within certain frequency region, which is typical of a repetitive controller. It is worth notice that lower order models can be used instead of the full delay model when implementing a feedback-loop type repetitive controller (Lee and Smith, 1998). Another new type of repetitive control based system that incorporates two degree of freedom (TDOF) H_∞

design can be seen in the papers of Peery (1993), Li (1999) and Chen (2001ab). Peery and Ozbay formulated their problem in continuous-time domain and thus a non-convex optimization problem was posed due to the nonlinear delay term (e^{sT}) imposed by repetitive controllers though robust performance was taken into consideration. Li and Tsao conducted the TDOF design in discrete-time domain. The pros are that the nonlinear delay term (e^{sT}) in continuous-time domain is translated into a linear delay term (z^{-N}) in discrete-time domain and simultaneous design of both controllers that account for the inherent repetitive control structure is possible. Besides, since the H_∞ controller design is now a convex optimization problem, it can be cast into a linear matrix inequality (LMI) framework (Gahinet, 1996) and solved by existing optimization software, e.g. LMI Toolbox in MATLAB. As implied from Guo, the effect of the q-filter can actually be taken into consideration by lumping it into the design of the 2- DOF controller. Our previous work (Chen and Chiu, 2001b) showed that a TDOF controller that achieves robust performance could actually be synthesized without using a low pass q-filter in the positive feedback loop of the delay taps. However, search of a stabilizing or robust controller for the proposed TDOF control structure usually demands huge amount of computation due to the inherent large delay model and a very high order controller is also unavoidable when the order of the periodic signal generator is huge.

This paper is a continuation of the work of Chen and Chiu (2001b). The new results to be described here include the following: First, to achieve fast computation and obtain lower order controller, kernel of the repetitive controller, i.e. the delay taps along with a low pass filter (see Fig. 8), can be replaced with a fictitious unity norm bounded uncertainty. Note that the low pass filter can also be lumped into the overall controller, as done in our previous work, whereas in this new configuration it is more convenient to isolate the low pass filter from the augmented plant due to the non-causality of the one, i.e. the zero-phase q-filter to be used. Secondly, the control structure is determined by accounting for all possible controller locations and different degree of freedom controls. It is argued that given specified performance and uncertainty weights, the least degree of freedom control achieving the best robust performance is two. This can be verified by comparison among different degree of freedom controls using structured singular values (μ). Finally, one of the known nonlinearities, actuator saturation, is taken into consideration by modeling the saturation element as real parametric uncertainty to the gain of the plant. Comparison of μ between the two designed control systems (with and without saturation) indicates the limiting effect of actuator saturation on the system performance. The proposed repetitive control based system is applied to a motor/gear transmission system in order to reduce sensitivity of the velocity regulation to both periodic and non-periodic disturbances and manufacturing

uncertainties or nonlinearities. Effectiveness of the proposed control system is verified by simulation results, which shows promising periodic and non-periodic disturbance reduction in the desired frequency range. It is also shown that the nominal system still maintains certain performance level when the actuator saturates due to large disturbances.

The organization of the paper is as follows. Next section will start with the description of the motor/gear transmission system and discussion of the disturbance sources. Section 3 will devote to the determination of the control structure and synthesis of the repetitive control based H_∞ controller. Simulation and discussion of the results are presented in section 4. Conclusion remarks and future work are summarized in section 5.

2. PROBLEM STATEMENT

2.1 System Description

The motor/gear transmission system in our study is shown in Fig. 1. The system is composed of the main BLDC motor with its onboard driver and the gear train that connects to the load. The main task of the BLDC motor is to supply torque to preserve constant angular velocity of the load. The driver controls the velocity of the motor by adjusting the amount of currents flowing through the armature windings of the motor. The motor shaft drives the gear train that connects with the gear at the output stage (GAOS). Phase locked loop (PLL) and pulse width modulation (PWM) are two of the most popular methods utilized in marketed BLDC drivers for achieving the desired velocity regulation. The BLDC motor for the motor/gear transmission system usually includes a rotor made of permanent magnets, a stator where the armature windings are attached and three Hall-effect sensors for detecting rotor position. The armature coils consist of concentrated windings, which are y-connected or delta-connected and fixed on a printed circuit board.

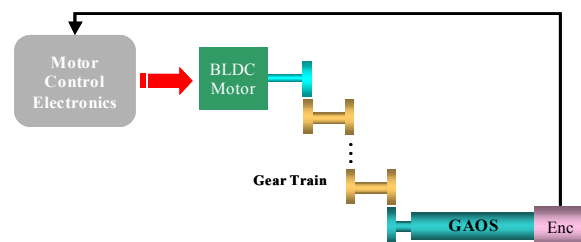


Fig. 1. Motor/Gear transmission system

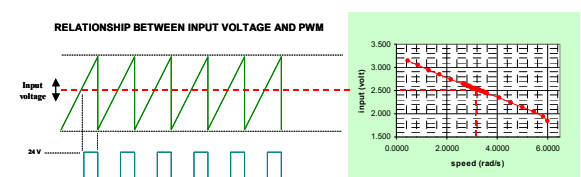


Fig. 2. The linear operating range of the plant is identified by varying the voltage while monitoring the output velocity.

For BLDC motors, PLL is the most common method of velocity control. The PLL is a feedback system

composed of a phase comparator, a low-pass filter, an error amplifier in the forward signal path, and a pulses generator in the feedback path. The desired angular velocity is proportional to the frequency of the reference pulses generated by the oscillator. The motor angular velocity is proportional to the frequency of the pulses generated by the Hall-effect sensors. The phase comparator compares the phase differences between these two signals and generates an error signal proportional to the phase difference. The phase error is then passed through the low-pass filter and the amplifier to set voltage level for the PWM generator, which will modulate the amount of current flowing through the motor phase windings, to adjust the motor velocity (see Fig. 2).

Note that the PLL is to regulate the speed of the motor and not that of the GAOS. Since the motor shaft, the gear train and the GAOS are not rigidly connected, constant motor speed does not guarantee constant velocity of the GAOS. From another point of view, the frequency generating (FG) sensor might not be able to pick up all the disturbances introduced in between the motor and the GAOS. This is demonstrated by observing the signals from the FG sensor and that of an optical encoder mounted on the GAOS (see Fig. 3). From the power spectrums of the two different velocity measurements, it is seen that most periodic disturbances detected in the load velocity do not appear at all in the velocity measurement of the motor.

Therefore, the strategy is to use the angular velocity of the GAOS to control the drive signal input to the BLDC motor, thereby stabilizing the output velocity. Since the motor rotates at a much higher velocity than the load, high-resolution control is possible. To attain this, an optical encoder is mounted on the GAOS to pick up the velocity signal.

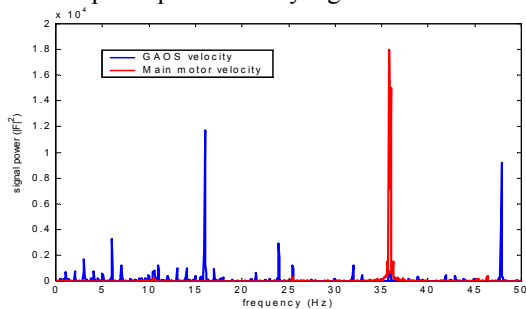


Fig. 3. Power spectrum of the velocity variation at the motor and at the output stage gear

2.2 Disturbance Sources

Although torque ripples have been known to be unavoidable for BLDC motors operating at low velocity and with small inertia, the low pass-filtering effect of the load inertia and the relative high-speed operation make the effect of motor torque ripples negligible in the motor/gear transmission system. On the other hand, the effect of gear train noises such as eccentricity and tooth-to-tooth error are not negligible. The disturbance frequency caused by eccentricity and tooth-to-tooth error can be readily identified from the rotational speed and the gear

geometry. The relationship among the geometric properties and the predicted disturbance frequency for some of the rotating components for a typical monochrome laser printer is summarized in Fig. 4. It is obvious to see that while eccentricity usually induces only low frequency fluctuation, tooth profile errors are major contributors to the mid and high frequency fluctuations. The measured velocity regulation of the GAOS and its power spectrum is shown in Fig. 3. It can be seen that the tooth-to-tooth error associated with the GAOS introduces a disturbance at 16 Hz. Note that mechanical tolerances and contacts among rotating components, other than the gears, which connect directly or indirectly to the GAOS can also generate force disturbances that will result in output velocity variation, e.g. the 48 Hz disturbance contributed by gear 2 or 3. From the velocity variation spectrum, Fig. 3, we see that the disturbance in the system manifest themselves as two types of velocity errors. The first type of disturbances is defined as periodic and has fundamental frequency of 8 Hz. The second type of disturbances is defined as non-periodic and most of them have very low frequency lying below 16 Hz.

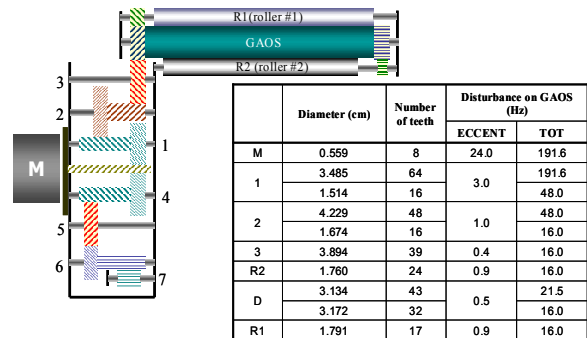


Fig. 4. Gear configuration for a typical laser printer and disturbance frequency prediction

3. REPETITIVE CONTROLLER DESIGN

3.1 Nominal and Uncertainty Models

The frequency response of the system was obtained by injecting sinusoidal input into the system. The result is plotted in Fig. 5 (the cross). A 3rd and 2nd order models (the solid lines in Fig. 5) were found to fit the frequency response of the actual plant. Although the 3rd order model has better fit in the high frequency region, comparison of their multiplicative error with respect to the actual plant (see Fig. 6) indicated that there is no obvious advantage using the 3rd order model instead of the 2nd order model within the frequency region of 0 to 100 Hz. A 2nd order nominal model is found to be

$$P(s) = \frac{26195.5071}{s^2 + 561.3282s + 5581.9398} \quad (1)$$

Based on data collected from different identification experiments, the output multiplicative errors for the 2nd order nominal model are calculated and plotted in Fig. 6. It can be seen that at low frequency the modelling error is about 3% while at high frequency the error can be as high as 10000% for the 2nd order fit and 1000% for the 3rd order fit. The significant amount of uncertainty about the plant can be

contributed to the unmodeled structure dynamics, nonlinearities as well as the uncertainty and measurement limitations of the data acquisition equipment. A filter given below can be used to fit the upper bound of the errors.

$$W_2(s) = k \left(\frac{s/z+1}{s/p+1} \right)^r, \quad (2)$$

where $k \in (0, \infty)$, $r \in \mathbb{Z}^+$, and $z, p \in \mathbb{R}^+$. In spite of the fact that a high order filter can be found to fit the upper bound of all the multiplicative errors, a low order and low bandwidth filter is usually preferred when taking controller order and higher identification error at high frequency into consideration. The solid line in Fig. 6 corresponds to $k=0.03$, $z=350$, $p=7$ and $r=1$. As will be shown later, we are able to seek a controller achieving robust performance with this upper bound. In fact, with the performance weight fixed, any other upper bound that attempts to account for more modelling error at high frequency region, e.g. with larger z or r , leads to either non-existence of stabilizing controller or failure of robust performance test. In practical applications, it is usually unavoidable for control engineers to do trade-off between the desired performance and the amount of uncertainties that the system is capable of overcoming.

The actuator saturation is due to the velocity modulation scheme (PWM) in the BLDC motor. Fig. 2 shows that saturation occurs as the input voltage lies outside the lower and upper limits of the PWM level. Therefore, the motor will reach its maximum angular velocity when the voltage level drops below the lower limit or stop when the voltage level rises above the upper limit. The saturation can be viewed as a performance limiting semi-linear element that limits the amount of available control effort. Fig. 7 suggests a way to model such non-linear element as sector-bounded uncertainty, which comprises a unity gain in parallel with a norm-bounded unstructured uncertainty, i.e. $\Phi = 0.5(1 + \Delta)u$ with $|\Delta|_\infty \leq 1$. Note that this modelling is independent of the input value (u_{\max}, u_{\min}) that triggers saturation so that u_{\max} doesn't need to be equal to u_{\min} . It should also be pointed out that Δ is real. Note that before proceeding to the controller synthesis, the plant design model and the weighting filters are all converted to their discrete-time representations.

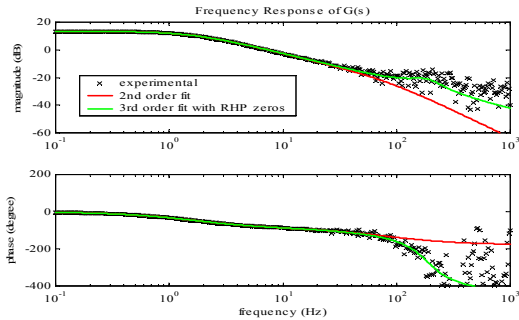


Fig. 5. Frequency response for the motor/gear transmission system

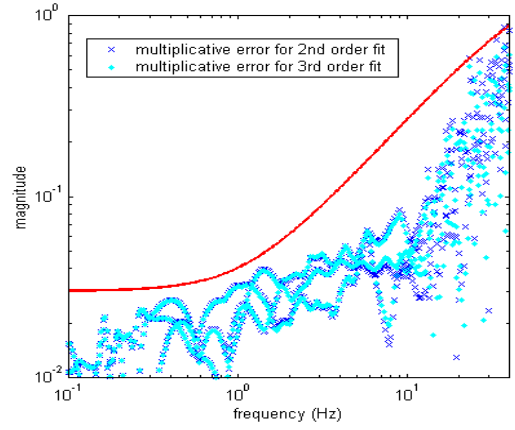


Fig. 6. Output multiplicative model uncertainty

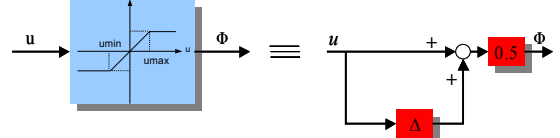


Fig. 7. Model the saturation element as gain uncertainty.

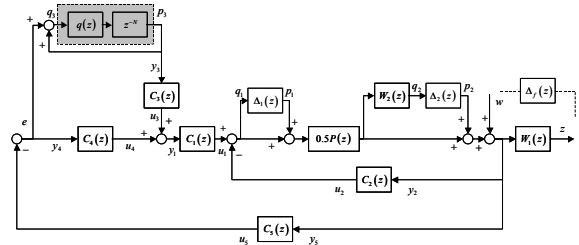


Fig. 8. Multi-degree-of-freedom digital repetitive control system configuration

3.2 Control Structure and Objective

Consider the five-degree of freedom (MDOF) repetitive control system as shown in Fig. 8. Using simple block diagram algebra, the number of DOF control can actually be reduced to two. That is, a new C_3 and C_4 can be defined as $C_3 \triangleq C_5 C_3 C_1$ and $C_4 \triangleq C_5 C_4 C_1 C_2$ while setting $C_2 = 0$ and $C_1 = C_5 = 1$. Hence, a TDOF controller should be able (sufficient though not necessary) to provide the same control performance achievable by other higher DOF controllers. On the other hand, it has already been proved by Li [20] that the TDOF control with C_3 and C_4 will provide better performance than the 1-DOF control with C_1 . This paper will mainly focus on the TDOF design using C_3 and C_4 .

As discussed in the last section, the actual plant can be represented as a saturation element $0.5(1 + \Delta_1)$ with $|\Delta| \leq 1$ followed by a nominal model $P(z)$ with output multiplicative uncertainties $W_2 \Delta_2$. W_2 is the frequency-dependent uncertainty weighting filter mentioned earlier such that $\|\Delta_1\|_\infty \leq 1$. $q(z)z^{-N}$ is the low passed delay model which, with the positive feedback, forms the repetitive controller. Note that the value of N depends on the sampling frequency and the fundamental frequency of the disturbance to be rejected. $q(z)$ can be any low pass filter. Here the

moving average type filter proposed by Tomizuka (1989) is used, which can be expressed as:

$$q(z) = \frac{z^{-1} + a + z}{a + 2}, \quad (3)$$

where a decides the roll-off rate of the q -filter. For example, most literature suggests the use of $a = 4$. Since $q(e^{j\omega}) = (a + 2 \cos \omega)/(a + 2)$, $a = 4$ implies that $|q(e^{j\omega})|$ will decrease monotonically from 1 to 1/3 as ω varying from 0 to the Nyquist frequency. W_1 is the performance weighting for the measured velocity error, which should supply gain (e.g. >1) or larger weighting in the frequency region where non-periodic disturbances locate. It is also picked as a stable low order filter with the following representation:

$$W_1(s) = \left(\frac{z^r s + w_b}{s + w_b/p^r} \right)^r, \quad (4)$$

where $w_b \in (0, \infty)$, $r \in \mathbb{Z}^+$, and $z, p \in \mathbb{R}^+$. To emphasize the non-periodic disturbance reduction below 50 Hz in this case, we can set $w_b = 2\pi \times 50$, $r = 1$, $z = 0.01$, and $p = 1$, respectively. The input and output of the TDOF controller are denoted by (y_3, y_4) and (u_3, u_4) . The exogenous disturbance w includes the periodic disturbances from the gear train and the non-periodic disturbances such as frictions. The measured velocity error e then comes from the exogenous disturbance and plant model uncertainties.

The objective is to simultaneously design a TDOF discrete-time controller, i.e. C_3 and C_4 , for the repetitive control based system to achieve robust performance in the sense that the controller should reduce velocity fluctuations at the plant output while maintain stability of the system as the nominal model is subjected to frequency-dependent output multiplicative error and input saturation.

3.3 Controller Synthesis and Performance Analysis

The TDOF control configuration shown in Fig. 8 can be cast into a general LFT framework. The kernel of the repetitive controller, $q(z)z^{-N}$, is replaced by a fictitious uncertainty Δ_3 . Also another fictitious uncertainty Δ_f is connected between the disturbance input and plant output. The TDOF controller can be obtained by solving the mixed-sensitivity minimization problem given by

$$\gamma_{opt} = \inf_{K \text{ stabilizing}} \left\| \begin{array}{c} W_1(1 + PC_4)^{-1} \\ C_4 P(1 + C_4 P)^{-1} \\ -W_2 PC_4(1 + PC_4)^{-1} \\ 1 - P(1 - C_4 P)^{-1} C_3 \end{array} \right\|_{\infty}, \quad (5)$$

where

$P \triangleq$ Motor/Gear transmission system

$W_1 \triangleq$ Performance weighting

$W_2 \triangleq$ Uncertainty weighting

$K \triangleq [C_3 \quad C_4]$ The TDOF controller

With this design approach, the controller order of K will be significantly less than that with our previous design. In fact, the order of K can be as high as 80 with the old design. Besides, the computation time is also reduced.

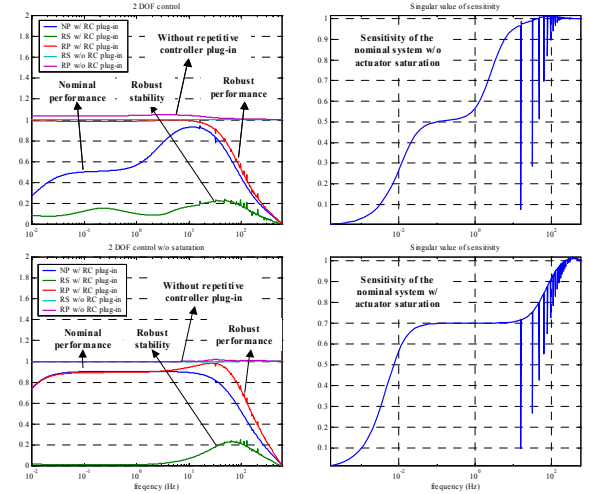


Fig. 9. Structured singular values and sensitivity plots for evaluating the performance and stability of the TDOF repetitive control systems with and without saturation limits

The robust performance of the designed control system is evaluated by looking at the structure singular value of $F_l(F_u(M, R), K)$ with respect to uncertainty block $\Delta = \text{diag}(\Delta_1, \Delta_2, \Delta_f)$, i.e. $\mu_{\Delta}(F_l(F_u(M, R), K))$. Note that $R(z) = q(z)z^{-N}$ is the kernel of the repetitive controller. It should be also pointed out that a controller stabilizing the nominal system is not guaranteed if γ_{opt} is greater than or equal to 1. However, due to its conservatism, most of time the controller solution to the above minimization problem will still stabilize the system after we plug-in the repetitive kernel.

4. SIMULATION RESULTS

The aforementioned synthesis problem is reformulated into the LMI framework and solved using existing CAD software (e.g. MATLAB LMI Toolbox). Nominal performance (NP), robust stability (RS), and robust performance (RP) are evaluated based on the definition of the structured singular value (μ) and can be calculated using existing numerical tools (e.g. MATLAB μ -synthesis Toolbox). Related mathematical and analytical details will not be presented here for concision. Fig. 9 summarizes the results of performance and stability for the control systems with and without actuator saturation. For the design without actuator saturation, it is actually possible for us to raise the performance weights (W_1) by a factor of 1.3, which provides larger reduction range for non-periodic disturbances as can be seen from the sensitivity plots on the right hand side of Fig. 9. It is worth notice that the robust stability of system with the controller is pretty good whether there is saturation or not. Also, as can be seen from the figure, performance or stability evaluation for the system without repetitive

controller plug-in tends to overly conservative. Finally, although results not presented here, a comparison between 3-DOF and TDOF design is made, which actually shows no significant improvement using 3-DOF control structure.

Feasibility of the TDOF control design is verified by injecting real disturbances collected from the encoder mounted on the GAOS into the simulated control system. The output disturbance magnitude spectrums before and after compensation are shown in Fig. 10. It can be seen that the proposed controller not only reduces the non-periodic disturbances at the low frequency range, but also attenuates the periodic disturbances and its harmonics in the high frequency region.

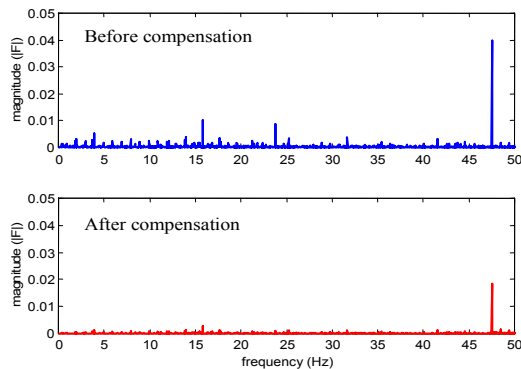


Fig. 10. Simulation results of the TDOF repetitive control system

5. CONCLUSION

In this paper, a two degree-of-freedom (TDOF) repetitive control system is designed for a motor/gear transmission system with actuator saturation. Modifications are made for our previous design to incorporate the performance-limiting element, i.e. actuator saturation, to acquire lower order controller, and to achieve faster computation. It is shown that the controller order for the new design can be as low as 4 when low order nominal model, performance and uncertainty weights are properly chosen. Furthermore, structured singular value (μ) is used to evaluate and compare the nominal/robust performance and robust stability of our designs. It can be seen how actuator saturation poses limit on the nominal/robust performance of the system although robust stability is not significantly affected. The simulation result based on actual disturbance data is presented to show the effectiveness of the new design for periodic and non-periodic disturbance reduction for motor/gear transmission velocity regulation.

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