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PURCHASE ALLOCATION AND BIDDING IN DUAL ELECTRIC POWER MARKETS WITH RISK MANAGEMENT

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Abstract: Purchase allocation problem is one of the most important problems faced by an electric energy service provider under new market environment. The optimal purchase allocation and bidding among dual electric energy markets are discussed in paper. First the deterministic system demand is considered and then case of stochastic system demand is studied. The optimal purchase is derived with respect to the statistical characteristics of the stochastic market prices and demand. *Copyright© 2002 IFAC*

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1. INTRODUCTION

Electric power industries around the world are undergoing restructuring (Hao, 1999, Overbye, 2000, Mielczarski, *et al.*, 1999, Guan and Luh, 1999). It is generally believed that opening the power industry to competition will benefit consumers with lower prices and better service. Under restructuring, generation suppliers compete to sell power in electric power markets and the power is purchased by energy service providers to meet the forecasted needs of their customers. Many challenging issues arise under the new competitive market structure. Instead of centralized decision-making in a monopoly environment as in the past, many parties with different goals are now involved and competing in the market(Gross and Finlay, 1996, Yamin, and Shahidepour, 1999, Guan, *et al.,*2001). The information available to a party may be limited, regulated, and received with time delay, and decisions made by one party may influence the decision space and well-being of others.

Although the power mechanisms worldwide may quite differ in kinds it is common that there would exist multiple markets for long term forward demand and short-term spot demand. Therefore one common problem faced by energy service providers is how to allocate purchase in different market to minimize the total cost while keeping the risk low. Since one's demand may be uncertain and prices on different market may also be uncertain and volatile, purchase allocation would be a stochastic optimization problem.

The optimal purchase allocation problem among dual electric energy markets is discussed in paper. First the deterministic system demand is considered and then case of stochastic system demand is studied. The optimal purchase is derived according to the statistical characteristics of the stochastic market prices and demand.

2. DETERMINISTIC DEMAND

2.1 Problem Formulation

Suppose an energy provider can buy electric energy from two markets: long term forward market and real time market. In general, the price on the real time market is more volatile than that on the long-term market. Define the long-term market as first market and the real time market as the second market. Suppose there is no market power, i.e., no participant's biding strategy can dominate the formation of the market price. Therefore the market price can be considered as a random variable with known statistical characteristic estimated from the historical data.

When the one's demand is deterministic, the problem becomes how to allocate the purchase quantity to meet total demand while minimizing total purchase cost and risk.

Let λ_1 be the price of the first market with expectation $\overline{\lambda}_1$, and variance σ_1^2; λ_2 the price of the second market with expectation $\overline{\lambda}_2$, and variance σ_2^2. Let D be the total demand. The purchase quantity of energy in the first market can be presented as $D_1 = xD$, where x is the percentage of purchase allocation in the first market. As a result, the purchase quantity in second market is $D_2 = D - D_1 = (1-x)D$. Then, the total purchase cost is $C = D_1\lambda_1 + D_2\lambda_2 = D[x\lambda_1 + (1-x)\lambda_2]$. Risk is reflected by variance of the cost $R^2 = E[C - E(C)]^2$, where $E(C)$ is the expected cost. The object function can be the expected cost plus the risk

$J = E(C) + qR^2$

where $q \geq 0$ is a risk factor.

Hence the purchase allocation problem with deterministic demand is formulated as:

$$\begin{aligned} &\min \quad J = E(C) + qE[C - E(C)]^2 \\ &s.t. \quad D_1 + D_2 = D \end{aligned} \tag{1}$$

2.2 Optimal Purchase Solution

Assume the prices of the two markets λ_1 and λ_2 are independent each other, then the expectation of the total cost and risk can be obtained as follows:

$$\overline{C} = E(C) = D[x\overline{\lambda}_1 + (1-x)\overline{\lambda}_2] \tag{2}$$

and

$$R^2 = E[(C - \overline{C})^2] = D^2[x^2\sigma_1^2 + (1-x)^2\sigma_2^2] \tag{3}$$

respectively. Solving the problem in (1) is to find the optimal purchase allocation percentage. Since

$\dfrac{\partial J}{\partial x} = 2qD[x(\sigma_1^2 + \sigma_2^2) - \sigma_2^2] + (\overline{\lambda}_1 - \overline{\lambda}_2)$,

When $q > 0$ let $\dfrac{\partial J}{\partial x} = 0$, x is obtained as

$x = \dfrac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2} - \dfrac{\overline{\lambda}_1 - \overline{\lambda}_2}{2qD(\sigma_1^2 + \sigma_2^2)}$.

When $q = 0$, it is the case without risk consideration:

if $\overline{\lambda}_1 > \overline{\lambda}_2$, $\dfrac{\partial J}{\partial x} > 0$, then $x = 0$,

if $\overline{\lambda}_1 < \overline{\lambda}_2$, $\dfrac{\partial J}{\partial x} < 0$, then $x = 1$,

if $\overline{\lambda}_1 = \overline{\lambda}_2$, any $x \in [0,1]$, can meet the equation (1). The above result simply means that if without risk consideration, all the purchase will be allocated in the market with lower price.

Because of $q \geq 0$, $0 \leq x \leq 1$, by combining the above results, the analytic solution of (1) for $\overline{\lambda}_1 > \overline{\lambda}_2$ can be expressed as:

$$x = \begin{cases} \dfrac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2} - \dfrac{\overline{\lambda}_1 - \overline{\lambda}_2}{2qD(\sigma_1^2 + \sigma_2^2)} & \text{if} \quad q > \dfrac{\overline{\lambda}_1 - \overline{\lambda}_2}{2D\sigma_2^2} \\ 0 & \text{if} \quad 0 \leq q \leq \dfrac{\overline{\lambda}_1 - \overline{\lambda}_2}{2D\sigma_2^2} \end{cases} \tag{4}$$

Obviously, the case with $\overline{\lambda}_1 < \overline{\lambda}_2$ is symmetric. When $\overline{\lambda}_1 < \overline{\lambda}_2$,

$$x = \begin{cases} \dfrac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2} + \dfrac{\overline{\lambda}_2 - \overline{\lambda}_1}{2qD(\sigma_1^2 + \sigma_2^2)} & \text{if} \quad q > \dfrac{\overline{\lambda}_2 - \overline{\lambda}_1}{2D\sigma_1^2} \\ 1 & \text{if} \quad 0 \leq q \leq \dfrac{\overline{\lambda}_2 - \overline{\lambda}_1}{2D\sigma_1^2} \end{cases} \tag{5}$$

Therefore it is only necessary to discuss the case $\overline{\lambda}_1 > \overline{\lambda}_2$. The relationship between the allocation percentage x versus the risk weighting factor q is shown in Figure 1.

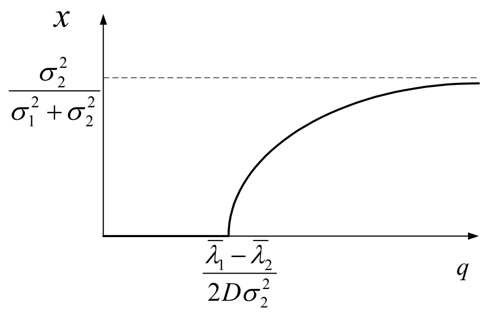

Figure 1 Allocation percentage versus risk weighting factor when $\overline{\lambda}_1 > \overline{\lambda}_2$

It is important to select a suitable risk factor, since it impacts the purchase strategy directly. From Figure 1, it is seen that when $0 \leq q \leq \dfrac{\overline{\lambda}_1 - \overline{\lambda}_2}{2D\sigma_2^2}$, the purchase allocation on the first Market is 0. This implies that if risk is not in consideration, we will only allocate purchase on the market with lower price. When $q > \dfrac{\overline{\lambda}_1 - \overline{\lambda}_2}{2D\sigma_2^2}$, the risk factor begins to influence the purchase allocation. With q increasing, the prices play less important role in determining purchase allocation, and the ratio of the price variances on the two markets becomes more influential. When the risk factor is large enough, the purchase allocation is solely determined by $\dfrac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}$.

If the prices on the two markets are the same, that is, $\overline{\lambda}_1 = \overline{\lambda}_2$, the purchase allocation is also determined by the risk, that is $x = \dfrac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}$.

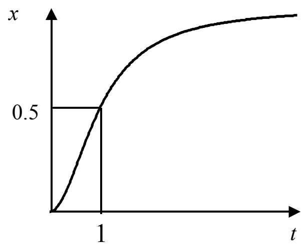

Figure 2 The relationship between x and t.

Let $t=\dfrac{\sigma_2}{\sigma_1}$ reflecting the ratio of the price fluctuation ranges of the two markets, then allocation percentage $x=\dfrac{t^2}{1+t^2}$. The relation between x and t is shown in Figure 2. It is seen that the allocation on the first market increases as the price on the second market is more volatile and thus more risky.

3 UNCORRELATED UNCERTAIN DEMAND AND PRICES

3.1 Problem formulation

In the power system operation, the future system demand or load is generally forecasted based on the historical loads, whether forecasting information, etc. Naturally, there exists uncertainty in forecasting and therefore it is therefore more realistic to consider the demand as a random variable. In this case, the purchase problem is more complicated.

Suppose the statistical characteristics of two market prices are same as in the case of deterministic demand. Let D be the forecasted demand, D_p is actual total demand, the expectation of D_p is $\overline{D}_p$, variance $\sigma^2_{D_p}$.

If the purchase allocated in the first market is $D_1 = xD$ based on the forecasted demand, then the energy purchase in the second market is $D_2 = D_p - D_1 = D_p - xD$. The total purchase cost is

$$C = D_1\lambda_1 + (D_p - D_1)\lambda_2 = Dx(\lambda_1-\lambda_2)+D_p\lambda_2 \tag{6}$$

In this case, the purchase allocation problem is defined as:

$$\min_x \quad J = E(C)+qE[C-E(C)]^2 \tag{7}$$

The problem in (7) is similar to (1), but the random variable D_p is included.

3.2 Optimal Purchase Solution

Suppose two markets prices λ_1, λ_2, and actual total demand $\overline{D}_p$ are independent each other, then,

$$\overline{C} = E(C) = Dx(\overline{\lambda}_1-\overline{\lambda}_2)+\overline{D}_p\overline{\lambda}_2 \tag{8}$$

$$\begin{aligned}R^2 &= E[(C-\overline{C})^2] \\ &= D^2x^2(\sigma_1^2+\sigma_2^2)+2xDD_p\sigma_2^2 + \\ &+\sigma_2^2\sigma_{D_p}^2+\overline{D}_p{}^2\sigma_2^2+\sigma_{D_p}^2\overline{\lambda}_2^2\end{aligned} \tag{9}$$

For $q>0$, let $\dfrac{\partial J}{\partial x}=0$, we can get:

$$x=\frac{\sigma_2^2}{\sigma_1^2+\sigma_2^2}\cdot\frac{\overline{D}_p}{D}-\frac{\overline{\lambda}_1-\overline{\lambda}_2}{2qD(\sigma_1^2+\sigma_2^2)},$$

For $q=0$, the optimal allocation is clearly the same as in the case of deterministic demand.

Based on $q\ge 0$ and $0\le x\le 1$, the allocation can be summarized more concisely as

if $\overline{\lambda}_1 > \overline{\lambda}_2$,

$$x=\begin{cases}\dfrac{\sigma_2^2}{\sigma_1^2+\sigma_2^2}\cdot\dfrac{\overline{D}_p}{D}-\dfrac{\overline{\lambda}_1-\overline{\lambda}_2}{2qD(\sigma_1^2+\sigma_2^2)} & if \quad q>\dfrac{\overline{\lambda}_1-\overline{\lambda}_2}{2\overline{D}_p\sigma_2^2}\\ 0 & if \quad 0\le q\le\dfrac{\overline{\lambda}_1-\overline{\lambda}_2}{2\overline{D}_p\sigma_2^2}\end{cases} \tag{10}$$

and if $\overline{\lambda}_1 < \overline{\lambda}_2$,

$$x=\begin{cases}\dfrac{\sigma_2^2}{\sigma_1^2+\sigma_2^2}\cdot\dfrac{\overline{D}_p}{D}-\dfrac{\overline{\lambda}_1-\overline{\lambda}_2}{2qD(\sigma_1^2+\sigma_2^2)} & \\ \quad if \quad q>\dfrac{\overline{\lambda}_2-\overline{\lambda}_1}{2[D(\sigma_1^2+\sigma_2^2)-\overline{D}_p\sigma_2^2]} & \\ \quad and \quad \dfrac{\sigma_2^2}{\sigma_1^2+\sigma_2^2}\cdot\dfrac{\overline{D}_p}{D}<1 & \\ 1 \quad if \quad 0\le q\le\dfrac{\overline{\lambda}_2-\overline{\lambda}_1}{2[D(\sigma_1^2+\sigma_2^2)-\overline{D}_p\sigma_2^2]} & \\ \quad or \quad \dfrac{\sigma_2^2}{\sigma_1^2+\sigma_2^2}\cdot\dfrac{\overline{D}_p}{D}\ge 1 & \end{cases} \tag{11}$$

4 CORRELATED UNCERTAIN DEMAND AND PRICES

In the power market practice, the bidding strategies of the market participants generally play an important role in determining the market clearing prices. The prices in different markets are often correlated since the bidders would tend to respond to the market prices along the time horizon. Let r be the correlation coefficients between λ_1 and λ_2. For the deterministic demand, the optimal purchase allocation is obtained by following the similar derivation as:

$$x_r=\frac{\sigma_2^2-\sigma_1\sigma_2 r}{\sigma_1^2+\sigma_2^2-2\sigma_1\sigma_2 r}-\frac{\overline{\lambda}_1-\overline{\lambda}_2}{2qD(\sigma_1^2+\sigma_2^2-2\sigma_1\sigma_2 r)}$$

$$\text{if} \quad q > \frac{\bar{\lambda}_1 - \bar{\lambda}_2}{2D(\sigma_2^2 - \sigma_1\sigma_2 r)}$$

$$x_r = 0 \qquad \text{if} \quad 0 \le q \le \frac{\bar{\lambda}_1 - \bar{\lambda}_2}{2D(\sigma_2^2 - \sigma_1\sigma_2 r)} \tag{12}$$

In comparison of (12) with (4), we can obtain

$$x_r - x = (\sigma_2^2 - \sigma_1^2 - \frac{\bar{\lambda}_1 - \bar{\lambda}_2}{qD})\frac{\sigma_1\sigma_2 r}{(\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2 r)(\sigma_1^2 + \sigma_2^2)}$$

Clearly if $\sigma_2^2 > \sigma_1^2 + \frac{\bar{\lambda}_1 - \bar{\lambda}_2}{qD}$, $x_r > x$. This means that when the uncertainty in the second market is large enough, the purchase allocation in the first market for the correlated case should be more than that of the uncorrelated case.

For the uncertain demand, let r_1 and r_2 be the correlation coefficients between λ_1 and D_p, λ_2 and D_p respectively. The optimal allocation is derived as

$$x_r = \begin{cases} 0 & \text{if } t < 0 \\ t & \text{if } 0 \le t < 1 \\ 1 & \text{if } t > 1 \end{cases} \tag{13}$$

where

$$t = \frac{\sigma_2^2 \bar{D}_p + \bar{\lambda}_2 \sigma_{D_p} \sigma_2 r_2 + \sigma_{D_p} \sigma_2^2 r_2}{D(\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2 r)} - \frac{\bar{D}_p \sigma_1\sigma_2 r + \bar{\lambda}_2 \sigma_{D_p} \sigma_1 r_1 + \sigma_{D_p} \sigma_1 \sigma_2 \sqrt{r r_1 r_2}}{D(\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2 r)} - \frac{\bar{\lambda}_1 - \bar{\lambda}_2}{2qD(\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2 r)} \tag{14}$$

5 PURCHASE BID GENERATION

A power purchase bid is piece-wise linear or staircase amount-price function. The essence to obtain a bid is to generate a set of power amount and price pairs. The results of optimal purchase allocation obtained in Section 3 and 4 can be applied to generate the purchase bid curve.

Consider generating the purchase bid on the first market.

Suppose the system demand and the average price of the second market are known by forecasting, the amount-price pair can then be generated as:

$$D_1^i = \begin{cases} D - \dfrac{\lambda_1^i - \bar{\lambda}_2^i}{2q\sigma_2^{i2}} & \text{if} \quad 2qD\sigma_2^{i2} > \lambda_1^i - \bar{\lambda}_2^i \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

where $\bar{\lambda}_2^i$ is the average price of the second market for the given price λ_1^i at the first market with {λ_1^i, D_1^i} as a pair on the bid curve. Figure 3 shows a typical purchase bid curve.

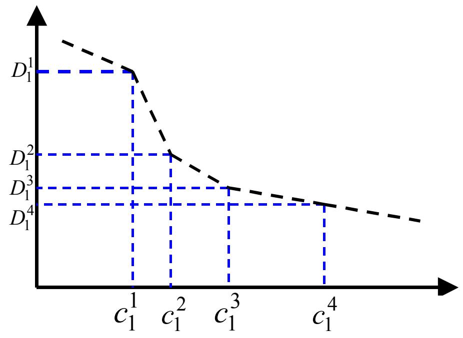

Figure 3 A typical purchase bid curve

6 NUMERICAL SIMULATION

The numerical simulation is performed based on the actual data of the U.S. California power market for the week of May 27 2000 to June 3 2000. The hourly prices of PX (Power Exchange) day-ahead market and ISO (Independent System Operator) real-time market (for NP15 and SP15 areas) in this period are shown in Figure 4. The price range in the PX power market is between \$17.81/MWh and \$88.26/MWh with the mean price \$55.76/MWh and for the ISO real time market, the price range is from -\$163/MWh to \$125.80/MWh with the mean of price \$67.25/MWh. The negative prices are caused by the some must-run generating resources such as nuclear units that need to dump energy.

The PX day-ahead market is assigned as Market 1 and ISO market as Market 2 with calculated statistical parameters $\bar{\lambda}_1 = \$55/\text{MWh}$, $\sigma_1^2 = 189$, $\bar{\lambda}_2 = \$72/\text{MWh}$, $\sigma_2^2 = 1122$. Assume $\bar{D}_p = 1786.89$MWh, the variance $\sigma_{D_p}^2 = 2.8737 \times 10^3$.

The optimal purchase allocation D_1 for the PX day-ahead market is shown in Table 1 with different risk weighting factors

Table 1 Optimal power purchase allocation

q (10^{-5})	0.338	1.690	2.028	6.084
D_1	1786.9	1786.9	1742.1	1592.8

Generally, the average price in the PX day-ahead market is lower than that of the ISO real-time market. Therefore more than 80% of the purchase is allocated in the PX market. However when the risk is considered, the purchase is not fully allocated in one market. The purchase allocation versus the risk factor is plotted in Figure 5. It is seen that the purchase allocation in the PX market would decrease with the increase of the risk weighting factors.

The purchase bid curve is given Figure 6 for the following given parameters: λ_1^i = {15, 35, 55, 75, 95, 115} in USD/MWhr with the corresponding second market prices {92,90,72, 40, 38, 35} and the same variance 1122. The risk factor is 6.084×10^{-5}, and system demand 1787 MWh.

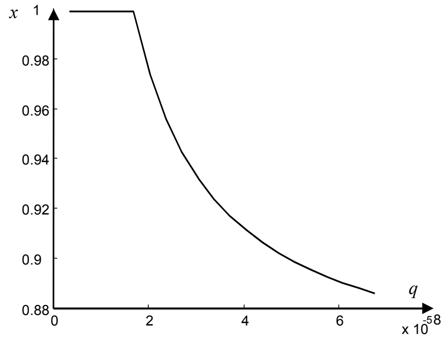

Figure 5 Purchase allocation versus the risk factor

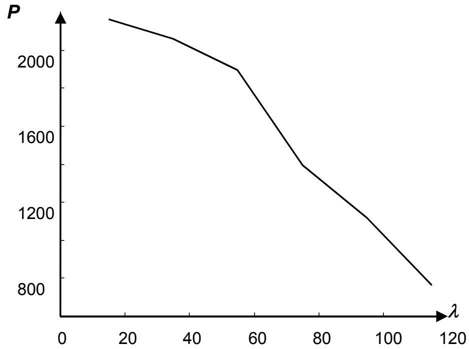

Figure 6 The purchase bid curve

7. CONCLUSIONS

In this paper, the purchase allocation problem for dual electric power markets is discussed. The analytical solutions are obtained for both deterministic and stochastic demand with uncorrelated and correlated prices. The method is discussed for generating purchase bids. The numerical simulation is performed based on the actual data from California power market. The results show that although the average price on the PX day-ahead is lower, the purchase is not fully allocated in one market if risk is considered. The purchase bid generated is also demonstrated.

8. ACKNOWLEDGMENT

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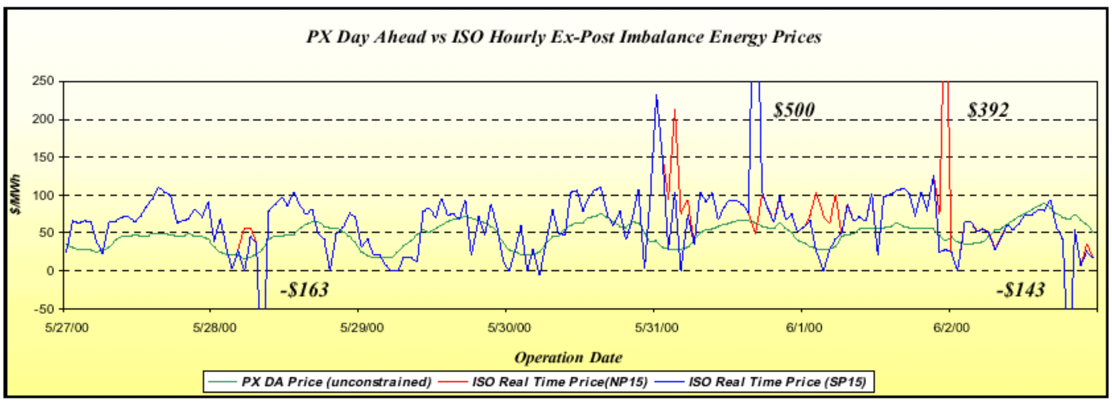

Figure 4 The price figure of PX day-ahead and ISO market from May 27 2000 to June 3 2000