# HYBRID FORCE/POSITION CONTROL OF REDUNDANT MOBILE MANIPULATORS<sup>1</sup>

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Abstract: This paper presents a novel force control scheme for redundant mobile manipulators. Based on a decoupled and linearized dynamic model for integrated mobile platform and on-board manipulator, robotic tasks involving both position and output force control are discussed. Take the advantage of the kinematic redundancy of mobile manipulators, explicit force and position control at the same task direction is discussed based on the decoupled mathematical model. The force planning is also discussed based on a nonholonomic cart pushing task. The proposed force/position control approach has been implemented and tested on a mobile manipulator consisting of a Nomadic XR4000 and a Puma 560 robot arm.

## 1. INTRODUCTION

The research of mobile manipulators has drawn researchers' interests due to their dexterous manipulation capability and large motion space. Most applications of the mobile manipulator require the robot to interact with its environment dynamically while providing motion, such as pushing, pulling, cutting, excavating. Implementation of these tasks demands the mobile manipulator to provide both output force control to overcome the resistance and friction of the objects, and motion control to track certain trajectory. The force control schemes, such as external force control, hybrid position/force control and impedance control, can be applied to the force control of mobile manipulators (Antonelli et al., 1999), (Perrier et al., 1997), (Umeda and Nakamura, 1999). However, these force schemes are generally designed to establish and maintain stable contact with static environments. This paper discusses a force control scheme in the case of the end effector interacting with a moving object, in other words, the mobile manipulator needs to provide both motion of the mobile manipulator and output force as the control input for the object along the same task direction. The kinematic redundancy of mobile manipulator makes it possible to decouple the force control loop and motion control loop along the same task direction.

The paper first discusses the dynamic model of a mobile manipulator consisting of a holonomic mobile platform and a PUMA 560 arm. The dynamic model is linearized and decoupled in an augmented output task space. Force control schemes are then revisited and the output force/position control of mobile manipulator is proposed. The output force planning for the control of nonholonomic cart is discussed in section 4.

## 2. DYNAMIC MODEL AND FEEDBACK CONTROL

A mobile manipulator usually consists of a mobile platform and a robot arm. Figure 1 shows the

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associated coordinate frames of both the platform and the manipulator, the world frame  $\Sigma$ , a moving frame  $\Sigma_b$  attached on the mobile platform and a virtual moving frame  $\Sigma'_b$  attached on the mobile platform which always parallels to  $\Sigma$ . The dynam-

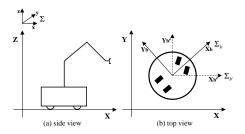


Fig. 1. Mobile manipulators

ics of a mobile manipulator with 6 DOF robot arm and 3 DOF mobile platform can be described by (Tan and Xi, 2001):

$$M(p)\ddot{x} + c(p,\dot{p}) + g(p) = \tau$$

where  $\tau$  is the generalized input torques, M(q)is the positive definite mobile manipulator inertia matrix,  $c(p, \dot{p})$  is the centripetal and coriolis torques, g(p) is the vector of gravity term. The vector  $p = \{q_1, q_2, q_3, q_4, q_5, q_6, x_b, y_b, \theta_b\}^T$  is the joint variable vector of the mobile manipulator, where  $\{q_1, q_2, q_3, q_4, q_5, q_6\}^T$  is the joint variable of the robot arm and  $\{x_b, y_b, \theta_b\}^T$  is the configuration of the platform in frame  $\Sigma$ . The augmented system output vector x is defined as  $x = \{x_1, x_2\}, \text{ where } x_1 = \{p_x, p_y, p_z, O, A, T\}^T$ is the end-effector position and orientation, and  $x_2 = \{x_b, y_b, \theta_b\}$  is the configuration of the mobile platform. Here  $\{O, A, T\}^T$  denotes an orientation representation(Orientation, Attitude, Tool angles). The parameters of the mobile manipulator dynamics can be determined based on the dynamics of the mobile base and the robot arm considering the reaction forces in between. The mobile platform is considered as a holonomic platform, as also shown in Figure 5. A detailed description of the model is referred to (Tan and Xi, 2001).

Applying the following nonlinear feedback control

$$\tau = M(p)u + c(p, \dot{p}) + g(p)$$

where  $u = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9\}^T$  is a linear control vector, the dynamic system can then be linearized and decoupled as

$$\ddot{x} = u \tag{1}$$

The system is decoupled in the augmented task space in frame  $\Sigma$ . Given  $x^d = \{p_x^d, p_y^d, p_z^d, O^d, A^d, T^d, x_b^d, y_b^d, \theta_b^d\}^T$  as the desired position and orientation of the mobile manipulator, the linear feedback control for model (1) can be designed as:

$$u = \ddot{x}^{d} + k_{d}(\dot{x}^{d} - \dot{x}) + k_{p}(x^{d} - x).$$

It is easy to prove that the linearized system is asymptotically stable. However, the task of a mobile manipulator is generally given in the form of end effector position and orientation  $\{p_x^d, p_y^d, p_z^d, O^d, A^d, T^d\}^T$ . The mobile platform position and orientation  $\{x_b^d, y_b^d, \theta_b^d\}^T$  are redundant in the frame  $\Sigma$ . The kinematic redundancy can be used to avoid the singular configuration of the arm by properly positioning the mobile platform, *i.e.*, to maximize the manipulation capability of the arm. The kinematic redundancy can also be employed to supply decoupled force and motion control along the same task direction.

#### 3. OUTPUT FORCE CONTROL

Force control schemes such as hybrid force/position control, impedance force control, explicit force control and many others have been proposed in the context of fixed base manipulator. The force control schemes are highly dependent on the environments of the robot. In many cases, the environment is assumed as static and the directions for force control and position control are separated. However, some applications may require both force and position control along the same task direction, such as pushing, excavating, scraping, etc. In this paper, the kinematic redundancy of the mobile manipulator is utilized to decouple the force control loop and motion control loop in the same task direction. To compare with, the force control schemes based on a decoupled nonredundant robot model is revisited first.

#### 3.1 Force Control Schemes Revisit

Hybrid force/position control was first proposed by Raibert and Craig (Raibert and Craig, 1981). The workspace is divided into two orthogonal subspaces as shown in Figure 2. A selection matrix Sdetermines the subspaces for which force or position are to be controlled. The control laws for position and force control can be designed independently to satisfy different control requirements of force and position. Generally, the force control law is designed to interact with static environment. However, the motion of the environment should also be considered when the robot interacts with a moving object. To improve the performance of the force control law, Schutter (Schutter, 1988) proposed an approach to feed forward the object motion parameters such as object velocity  $\dot{x}_o$  and acceleration  $\ddot{x}_o$ , as shown by the dotted lines in Figure 2. The desired output force  $f^d$  along the motion can be tacked. However, it can been seen that the motion control along the same direction is open loop.

For an object or environment, it is assumed that the end effector position and the contact force

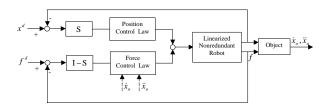


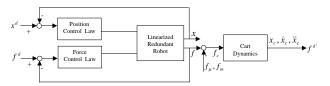
Fig. 2. Hybrid Force/Position Control

with the environment along one DOF can not be controlled independently. The force can then be regulated by controlling the impedance, or compliance of the robot (Hogan, 1985). The basic idea of this approach is to design a control law which will function in accordance with  $f = M_I \ddot{x} + B_I \dot{x} +$  $K_I x$ , where the constant matrices  $M_I, B_I, K_I$ represent inertia, damping and stiffness matrices of the interactive system respectively. Since the robot may encounter different environments for various applications, control gains of the robot should be tuned in accordance with the environmental characteristics. This scheme also has a slow response to force perturbations and the performance of the implicit force control is restricted by the bandwidth of the position controller (Vukobratović, 1997).

# 3.2 Output Force/Position Control of Redundant Robots

For a nonredundant robot arm, the directions for force control and position control have to be orthogonal(Khatib, 1987), as shown in Figures 2. Therefore the force and the position can not be controlled along the same task direction. It is caused by the equal number of control input and desired system outputs. For the hybrid force/position control scheme, the force and position control directions are generally separated by a selection matrix S. For many cases of the impedance control scheme, the control law is essentially a modified position controller. And the performance of the implicit force control is restricted by the bandwidth of the position controller. However, the force control loop and position control loop of a redundant robot along the same task direction can be decoupled. Since the force control is immensely environmental related, the hybrid force/position control of a redundant robot is discussed from two aspects, the capability of the robot and the constraints the environment.

Due to the kinematic redundancy, there are more control inputs than desired outputs. For the decoupled system model (1) of the mobile manipulator, the linear control input is a  $9 \times 1$ vector, while the desired end effector position and orientation,  $\{p_x^d, p_y^d, p_z^d, O^d, A^d, T^d\}^T$ , is only a  $6 \times 1$  vector. The redundant degrees of freedom, which correspond the extra linear control inputs, can be utilized to accomplish secondary tasks. For a path tracking task, the redundant degree of freedom can be used to position the mobile platform such that singular configurations of the arm are avoided. For a task to interact with the environment, output force of the end effector can be chosen as secondary tasks. For instance, the desired output position and force of the mobile manipulator can be chosen as  $\{f_x^d, f_y^d, f_z^d, O^d, A^d, T^d, p_x^d, p_y^d, \theta_b^d\}^T$ , where  $f_x^d, f_y^d$ and  $f_z^d$  are the desired output forces. As shown in Figure 3, a selection matrix S is not necessary for the redundant robot. It is worth noting that along x direction of the world frame  $\Sigma$ , both desired position and force,  $p_x^d$  and  $f_x^d$ , are chosen.  $p_y^d$  and  $f_y^d$  are chosen simultaneously along y direction.



## Fig. 3. Hybrid Force/Position Control for Redundant Robots

Since system (1) is decoupled, it can be divided into two subsystems, position control subsystem and force control subsystem. The state variable space of the position control subsystem,  $x_p$ , is a subspace of the state space x of system (1). Let  $x_p = \{p_x, p_y, O, A, T, \theta_b\}^T$  and denote its corresponding linear control input by  $u_p$ . The force control subspace  $x_f$  is chosen as  $x_f = \{x_b, y_b, p_z\}^T$  and the corresponding linear control input is denoted by  $u_f$ . System (1) can therefore be rewritten into two subsystems:

$$\begin{aligned} \ddot{x}_p &= u_p \\ \ddot{x}_f &= u_f \end{aligned} \tag{2}$$

The linear feedbacks for the two subsystems can be designed as

$$u_{p} = k_{pp}(x_{p}^{d} - x_{p}) + k_{pd}(\dot{x}_{p}^{d} - \dot{x}_{p}) + \ddot{x}_{p}^{d}$$
$$u_{f} = \ddot{x}_{f} + k_{fp}(f^{d} - f) + k_{fi} \int_{0}^{t} (f^{d}(\sigma) - f(\sigma)) d\sigma^{(3)}$$

In the controller (3), the force control loop and position control loop along the same task direction are decoupled due to the redundancy of the control inputs. And explicit force control of the end effector can be designed.

From the point view of the mobile manipulator, it is seen from (3) that desired position and force along the same task direction can be tracked simultaneously. This is mathematically true. However, the environment is critical to the force control schemes. The desired output forces  $f_x^d$  and  $f_y^d$ in (3) are related to the environment dynamics. For instance, zero force output should be commanded in the free space, and no motion should be commanded in the constraint direction with rigid contact. For an object, the dynamics should be considered. The force and position of the object is related. For instance,  $f = m\ddot{x}$  relates the position of an object and the force applied onto it. The dynamics of the object and friction determines the desired output force and the output position of the end effector. However, force and position do not have to be related by the mass of the object only. For some tasks, the force and position can be planned separately along the same task directions. For instances, for a cutting task with known resistance on the cutting trajectory, desired output position and desired output force can be considered separately in a task direction. If the robot cooperates with human, the output force of a human,  $f_m$ , should also be considered as dynamic environment. In a multi-robot environment, more than one robot can share a task. The output force for individual robot can be considered independently from its motion, as long as the composed force satisfies the constraints of the environment dynamics. The friction along the moving direction is determined by the roughness of the surface and mass of the object. In brief, the environments will determine the desired output force and position for certain applications. Force and position can be planned independently. As shown in Figure 3, the advantage of hybrid force/position control of a redundant robot lies in that the force control loop can be separated from the position control loop. The interacting force with the dynamic environment can then be regulated explicitly by considering the environmental dynamics. The force does not have to be regulated implicitly as it is done in the implicit force control scheme. This ensures the bandwidth of the force control loop. As an example, the output force planning of the mobile manipulator is discussed for the pushing control of a nonholonomic cart.

#### 4. FORCE AND MOTION PLANNING

Figure 4 shows an application of the mobile manipulator. The mobile manipulator, as shown in Figure 4(a), is used to push a cart with nonholonomic constraint, as shown in Figure 4(b). The output forces of the mobile manipulator,  $f_x$  and  $f_y$ , correspond to the control input force of the cart,  $f_1$  and  $f_2$ , as shown in Figure 4.  $x_c, y_c$  and  $\theta_c$  represent the configuration of the cart. The desired output forces of the mobile manipulators are obtained by computing the control input of the nonholonomic cart. The kinematic and dynamic model need to be considered to derive the control input of the cart. First, the kinematic model of the a cart with nonholonomic constraint can be described by

$$\dot{x}_c = v_1 \cos \theta_c \tag{4}$$

$$\dot{y}_c = v_1 \sin \theta_c \tag{5}$$

$$\dot{\theta}_c = v_2, \tag{6}$$

where  $v_1$  and  $v_2$  are the forward velocity and the angular velocity of the cart respectively. The dynamic model of a nonholonomic cart and its nonholonomic constraint can be represented by

$$\ddot{x}_{c} = \frac{\lambda}{m_{c}} \sin \theta_{c} + \frac{f_{1}}{m_{c}} \cos \theta_{c}$$

$$\ddot{y}_{c} = -\frac{\lambda}{m_{c}} \cos \theta_{c} + \frac{f_{1}}{m_{c}} \sin \theta_{c}$$

$$\ddot{\theta}_{c} = \frac{L}{I_{c}} f_{2}$$

$$\dot{x}_{c} \sin \theta_{c} - \dot{y}_{c} \cos \theta_{c} = 0,$$
(7)

where  $f_1, f_2$  are the force and torque applied on the cart.  $f_1, f_2$  are transformed to the desired output force of the end effector,  $f_x^d, f_y^d$ .  $m_c$  and  $I_c$  are the mass and inertia of the cart.  $\lambda$  is a Lagrange multiplier and  $\theta_c$  is the cart orientation. L is the length of the cart.

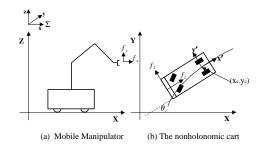


Fig. 4. Control of Nonholonomic Carts

Output stabilization is considered in this paper. Choosing a manifold rather that a particular configuration as the desired system output, the system can be input-output linearized. By choosing  $x_c, y_c$  as the system output, the system can be linearized with respect to the control input  $f_1$ and  $v_2$ . This can be explained by the following derivations. From equation (4) and (5), it is easy to see that  $v_1 = \dot{x}_c \cos \theta_c + \dot{y}_c \sin \theta_c$ . Here  $v_1$  is actually the forward velocity along the x' direction. Considering the velocity along y' direction is  $\dot{x}_c \sin \theta_c - \dot{y}_c \cos \theta_c = 0$ , the following relation can be obtained(7):

$$\dot{v}_1 = \ddot{x}_c \cos \theta_c + \ddot{y}_c \sin \theta_c \\ \dot{v}_2 = \ddot{\theta}_c$$

It is worthy noting that  $f_1 = m_c(\ddot{x}_c \cos \theta_c + \ddot{y}_c \cos \theta_c)$  can be obtained from (7). Suppose the desired output of interest is  $\{x_c, y_c\}$ , the following input-output relation can be obtained by the derivative of the first two equations in (4,5) and (7):

$$\ddot{x}_c = \frac{1}{m_c} \cos \theta_c f_1 - v_1 \sin \theta_c \cdot v_2$$
  
$$\ddot{y}_c = \frac{1}{m_c} \sin \theta_c f_1 + v_1 \cos \theta_c \cdot v_2$$

Considering  $f_1$  and  $v_2$  as the control inputs of the system, the input output can be formulated in a matrix form:

$$\begin{pmatrix} \ddot{x}_c \\ \ddot{y}_c \end{pmatrix} = G \begin{pmatrix} f_1 \\ v_2 \end{pmatrix}$$

where

$$G = \begin{pmatrix} \frac{\cos \theta_c}{m_c} & -v_1 \sin \theta_c \\ \frac{\sin \theta_c}{m_c} & v_1 \cos \theta_c \end{pmatrix}$$

The nonholonomic cart can then be linearized and decoupled as

$$\begin{pmatrix} \ddot{x}_c \\ \ddot{y}_c \end{pmatrix} = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$$

where  $\{w_1, w_2\}^T = G\{f_1, v_2\}^T$ . Given a desired path of the cart,  $x_c^d, y_c^d$ , which satisfy the non-holonomic constraint, the desired control can be designed as

$$w_1 = \ddot{x}_c^d + k_{px}(x_c^d - x_c) + k_{dx}(\dot{x}_c^d - \dot{x}_c)$$
  
$$w_2 = \ddot{y}_c^d + k_{py}(y_c^d - y_c) + k_{dy}(\dot{y}_c^d - \dot{y}_c)$$

The angular velocity  $v_2$  can then be obtained by  $\{f_1, v_2\}^T = G^{-1}\{w_1, w_2\}^T$ . It can be seen that control input  $f_1$  generates the forward motion and  $v_2$  controls the cart orientation such that  $x_c^d$  and  $y_c^d$  are tracked. The control input  $f_2$  can then be computed by the backstepping approach (Khalil, 1996). Defining  $v_2 = \phi(x_c, y_c, \theta_c)$  and  $z = \dot{\theta}_c - \phi$ , then equation of (6) can be transformed into

$$\dot{z} = -\dot{\phi} + \frac{L}{I_c} f_2 \tag{8}$$

The control input  $f_2$  can then be designed as

$$f_2 = \frac{I_c}{L} (-\dot{\phi} + k_\theta (\phi - \dot{\theta}_c)) \tag{9}$$

Certainly, given an initial configuration and final configuration, careful path planning needs to be done to obtain a path which satisfies the nonholonomic constraint of the cart.

#### 5. EXPERIMENTAL SETUP AND RESULTS

The proposed approaches have been implemented on a mobile manipulator consisting of a Nomadic XR4000 mobile robot and a Puma560 robot arm, as shown in Figure 5. There are two PCs in the mobile platform, one uses Linux as the operating system and runs the mobile robot control software and the other uses a real time operating system QNX and runs the Puma 560 control software. The two computers are connected via an Ethernet connection and communicate at a frequency of 300-500Hz. The sampling period for the Puma 560 control software is 1ms. The end-effector is equipped with a jr3 force/torque sensor. The mobile platform is equipped with laser sensor and the cart direction in the moving frame  $\Sigma_b$  can be detected. The length and weight are 0.89m and 45kg respectively.



Fig. 5. Mobile manipulator setup

#### 5.1 Pushing cart on a straight line

In Figure 6, the results of pushing a nonholonomic cart along a straight line are presented. The desired forces to control the cart,  $f_1$  and  $f_2$ , are computed and transformed to the world coordinates,  $f_x$  and  $f_y$ . The mobile manipulator pushes the cart forward along a straight line parallel to the y direction. Figures 6 (a) and (b) show the output force errors and Figure 6(c) and (d)are the trajectories in x and y directions. The cart is pushed forward for 0.8m as shown Figure 6(d).  $f_2$  maintains the cart on the straight line and  $f_1$  generates the motion of the cart on the line. This experiment has demonstrated that the position and output force of the end effector can be controlled along the same task directions, as illustrated in Figure 6. The force control errors along x and y directions have shown that the errors are approaching zeros. Furthermore, Figures 6 (c) and (d) show that positions along x and y directions are tracking the desired path. Therefore both forces and positions along the same task direction are controlled.

## 5.2 Turning the cart at a corner

Pushing the cart along a straight line is relatively easy since the desired end effector position and output force is easy to plan. The second experiment considered a complex task. The mobile manipulator first pushed the cart forward 0.7min about 20 seconds, made a turn in about 35 seconds, and then pushed the cart forward again

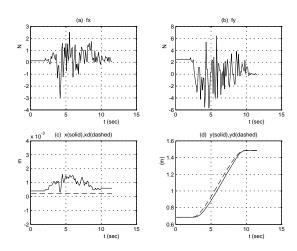


Fig. 6. Pushing the cart forward for 0.8m

for 0.7m along x direction. Figure 7(a) shows the trajectories of the cart,  $x_c$  and  $y_c$ , and the end effector,  $p_x$  and  $p_y$ . The cart trajectory is smooth and satisfies the nonholonomic constraint. Figure 7(b) is the cart orientation  $\theta_c$  with respect to time. The cart started from a configuration parallel to ydirection, and turned to a configuration parallel to x direction. The output force is planned based on the desired trajectory of the cart. Figure 7 (c)(d)are the force applied onto the cart,  $f_x$  and  $f_y$ . It is worth noting that the force are recorded in world frame  $\Sigma$ . It is seen that the force  $f_x$  pushed the cart along x direction in the last 20 seconds, and  $f_y$  pushed the cart along y direction in the first 20 seconds. This experiment has demonstrated that a complex task can be completed by the decoupled position and force control scheme.

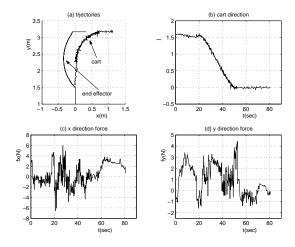


Fig. 7. Turning the cart at an corner

#### 6. CONCLUSION

Based on a linearized and decouple model in an augmented task space, this paper discuss the force/position control of of the mobile manipulator. This paper analyzes the difference between force control of nonredundant robots and redundant robots, and proposed the decoupled force/position control scheme along the same task direction for a redundant robot. Therefore, both the force control and the position control specifications can be satisfied by designing the control laws accordingly. The paper further discusses the environmental constraints in the force control and separate the design of the mobile manipulator controller and the analysis of the environment. Both force and position tracking can be achieved under certain environment constraints. The force planning is then discussed using the cart pushing example and experiment has been done to verify the control schemes.

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