

## HYBRID SLIDING MODE CONTROLLER WITH FUZZY COMPENSATION IN THE SLIDING SURFACE

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**Abstract:** A new form of a sliding mode controller is presented to deal with pH control. It is based on a combination of Sliding Mode Control and Fuzzy Logic. The conventional sliding surface (calculated from an FOPDT model) is modified using a set of fuzzy rules, which are similar to that of a Fuzzy Logic Controller (FLC). This confers to the controller robustness and flexibility to deal with the highly nonlinear behavior found in a neutralization reactor. The new controller performance is compared with a conventional sliding mode controller and PID. *Copyright © 2002 IFAC*

**Keywords:** Process Control, Sliding Mode Control, Fuzzy Control, Fuzzy Hybrid Systems, pH control.

### 1. INTRODUCTION

pH control is a common operation in many industrial areas such as wastewater treatment, pharmaceutical products, fermentation processes, and food processes. In all these processes it is necessary to maintain a precise control over the pH level of the system, some times to satisfy environmental regulations, some others to obtain products according with the quality requirement.

Most challenges in pH control are faced when the operating condition is between 6 and 8. The difficulty is due to the highly nonlinear behavior of the pH. This is particularly evident when neutralization between strong acids and strong bases is studied. The nonlinear behavior observed in the titration curve induces radical changes in the process gain; quite a few orders of magnitude in most of cases (Henson and Seborg, 1994; Shinsky, 1996; Lakshmi,1998). Nonlinearities also affect the dominant time constant and the dead time, mainly

because of changes in flow and reaction rate. Many approaches have been developed to face the pH control problem such as conventional PID, adaptive control, linearized controllers, gain scheduling control; and intelligent control approaches such as fuzzy logic and neural networks (Kavsek-Biasizzo, et al, 1997).

In order to be successful solving this problem, it is necessary to have a flexible and robust control strategy. However, most of the times the path to stability leads to sluggish behavior beyond design conditions. A flexible controller should be able to keep a reasonable compensation rate at every operating condition. This strategy leads to a two-phase controller: a control mode focused on error compensation and a speed mode that takes care of speeding up or slowing down the controller response to fit a predetermined desired behavior.

It is difficult for a single strategy to fulfil all the requirements in pH control. This paper considers the combination of Sliding Mode Control (SMC) with

Fuzzy Logic as an efficient way to obtain a controller able to perform appropriately in pH control problems. SMC is a procedure to design robust controllers for nonlinear processes. The usual approach to design a SMC controller (SMCr) has two disadvantages: (1) it requires a model of the process, and (2) usually the controllers designed using traditional SMC are complex, with many parameters to be tuned (Camacho and Smith, 2000). Nevertheless, Camacho and Smith (2000) have proved that it is possible to develop SMC based on a First-Order Plus Dead Time (FOPDT) model for nonlinear chemical process. The strength of Fuzzy Logic resides in its capacity to express in a mathematical form, the subjective knowledge based on experiences and analogies (Menzl, 1996). Thus, Fuzzy Logic allows incorporating “intelligence” and “experience” from the expert into the control strategies. The use of linguistic variables and rules to design the fuzzy adjustment of the sliding surface provides a combination of robustness and flexibility.

## 2. NEUTRALIZATION REACTOR MODEL

The model used in this work is based on the neutralization reactor presented by Henson and Seborg (1994). Some equations and steady state values have been modified as it is indicated later. Figure 1 shows a simplified process scheme.

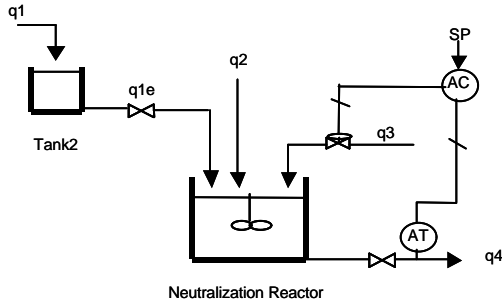
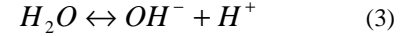
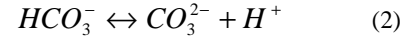
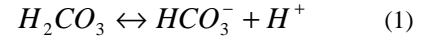


Fig. 1. Neutralization Reactor Configuration.

The stream  $q_1(t)$  is an aqueous solution of  $HNO_3$ , which is introduced in the tank 2 in order to eliminate significant flow variation to the reactor. The exit flow from the surge tank,  $q_{1e}(t)$ , is a function of the height in tank 2 and the hydrodynamic conditions in the discharge pipe. Stream  $q_2(t)$  is the buffer stream, an aqueous solution of  $NaHCO_3$ . Stream  $q_3(t)$  is a basic solution, an aqueous solution of  $NaOH$  and  $NaHCO_3$ . The purpose of the process is to neutralize the acid stream  $q_1(t)$  by manipulating the basic stream flow  $q_3(t)$ , while  $q_2(t)$  remains constant. Thus, for this paper the main disturbance is the acid flow, while the manipulated variable is the basic flow. The three streams are introduced to the neutralization reactor, where perfect mixture is assumed. It is also assumed constant density and complete solubility of the ions involved.

The following chemical reactions take place inside the reactor:



Chemical equilibrium is modeled using the definition of two reaction invariants,  $W_a$  and  $W_b$ . The first invariant,  $W_a$ , is a charge related quantity, while  $W_b$ , is the concentration of the  $CO_3^{2-}$  ion. Unlike pH, these invariants are conserved quantities. These invariants are expressed as:

$$W_{ai} = [H^+]_i - [OH^-]_i - [HCO_3^-]_i - 2[CO_3^{2-}]_i \quad (4)$$

$$W_{bi} = [H_2CO_3]_i + [HCO_3^-]_i + [CO_3^{2-}]_i \quad (5)$$

where  $i$  represents the streams involved in the process, from 1 to 4. The invariant balances include the complete system dynamic in the accumulation term, complementing the original model (Henson and Seborg, 1994). Such balances are:

$$q_{1e}(t)W_{a1}(t) + q_2(t)W_{a2}(t) + q_3(t)W_{a3}(t) - q_4(t)W_{a4}(t) = A_1 \frac{d(h_1(t)W_{a4}(t))}{dt} \quad (6)$$

and

$$q_{1e}(t)W_{b1}(t) + q_2(t)W_{b2}(t) + q_3(t)W_{b3}(t) - q_4(t)W_{b4}(t) = A_1 \frac{d(h_1(t)W_{b4}(t))}{dt} \quad (7)$$

pH is calculated from the following equation:

$$pH'(t) = -\log[H^+](t) \quad (8)$$

where,

$$[H^+](t) = W_b \frac{\frac{K_{a1}}{[H^+](t)} + \frac{2K_{a1}K_{a2}}{[H^+](t)^2}}{1 + \frac{K_{a1}}{[H^+](t)} + \frac{K_{a1}K_{a2}}{[H^+](t)^2}} + W_a + \frac{K_w}{[H^+](t)} \quad (9)$$

The pH transmitter is modelled as a first order transfer function:

$$t_{T1} \frac{d(c(t))}{dt} + c(t) = K_{T1} pH(t) \quad (10)$$

where  $c(t)$  is the sensor output, and  $\tau_{T1}$  and  $K_{T1}$  are the time constant and sensor gain respectively. Additionally, because the pH transmitter is located downstream, it is necessary to consider a variable transport delay  $t_0(t)$  in the measurement:

$$t_0(t) = \frac{LAp}{q_4(t)} \quad (11)$$

where L and Ap, are the distance from the bottom of the reactor to the measurement point, and the pipe cross-section, respectively.

The reason why pH is such a difficult variable to control is because of the highly nonlinear behavior present in process parameters. In order to observe this effect, an empirical model will be fitted using plant (simulation) response. The most common model used in self-regulating chemical processes is the First Order Plus Dead Time (FOPDT) model. The main reason why it is still used is because it leads to good PID tuning in most cases (most PID tuning equations used in process industry are based on FOPDT model identification) (Smith and Corripio, 1997; Marlin, 2000; Shinsky, 1996). Recent developments in PID Auto-tuning (Luo, R., et.al., 1998) and in Sliding Mode Control (Camacho and Smith, 2000) also use the FOPDT model. The transfer function for the FOPDT model is:

$$G_p(s) = \frac{C(s)}{M(s)} = \frac{K_p e^{-t_0 s}}{t s + 1} \quad (12)$$

If process identification tests are run at different operating conditions (valve positions), the nonlinear behavior of the fitted model parameters is demonstrated. Figure 2 illustrates the behavior of the three parameters in the FOPDT model: gain ( $K_p$ ), time constant ( $\tau$ ), and dead time ( $t_0$ ).

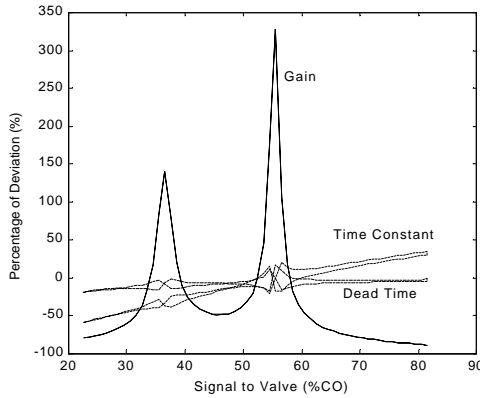


Fig. 2. Process Characteristics Deviation from Design condition vs. Controller output.

Figure 2 shows the deviation of process characteristics values with respect to their value at the design condition. The design condition calls for the valve to be 52% open. At this point no deviation (0%) is observed for all three parameters. It is also observed that the parameter with the most significant nonlinear behavior is the process gain. The process gain, relating how much the pH increases due to an increase in the base flow, shows a reduction of 80% from its design value when the valve is about 80%

open. It also shows an increment of about 300% from its design value when the valve is about 56% open.

Significant changes in the gain lead to controller misbehavior especially when the gain used to design the controller is much smaller than gain values in some other operating conditions. The main reason is because any controller's aggressiveness is an inverse function of the process gain. In other words, the more sensitive the process is to input changes, the less sensitive the controller should be to error changes.

### 3. SMC FOR NONLINEAR CHEMICAL PROCESSES

SMC is a technique derived from Variable Structure theory to design controllers. Such controllers have the capacity to handle nonlinear and time-varying systems without a dramatic change in their behavior (Camacho and Smith, 2000). All these characteristics are required in order to succeed in a neutralization reactor control loop.

The SMC technique first defines a surface along which the process can slide to achieve its desired final value (Camacho and Smith, 2000). Then, it defines a reaching function to force the controller move towards the sliding surface. The sliding surface,  $S(t)$ , is a function of the order of the process model, as it is expressed in the equation proposed by Slotine (1991):

$$S(t) = \left( \frac{d}{dt} + \mathbf{I} \right)^n \int_0^t e(t) dt \quad (13)$$

where  $e(t)$  is the error,  $\lambda$  is a surface tuning parameter, and  $n$  is the order of the process model. Equation 13 shows that when the model is a higher order one  $S(t)$  also becomes a high order equation with many parameters to tune. Camacho and Smith (2000) have demonstrated that using an FOPDT empirical model it is possible to obtain a useful and versatile controller, with all the necessary characteristics of robustness to face highly nonlinear systems. The SMCr developed by Camacho and Smith (2000) is defined by:

$$m(t) = \bar{m} + \left( \frac{t_0 t}{K_p} \right) \left[ \frac{c(t) - \bar{c}}{t_0 t} + \mathbf{I}_0 e(t) \right] + K_D \frac{S(t)}{|S(t)| + \mathbf{d}} \quad (14)$$

with a surface defined by:

$$S(t) = \text{sign}(K_p) \left( -\frac{d(c(t))}{dt} + \mathbf{I}_1 e(t) + \mathbf{I}_0 \int_0^t e(t) dt \right) \quad (15)$$

where  $m(t)$  is the controller output to the final control element.  $K_D$ ,  $\mathbf{I}_0$ ,  $\mathbf{I}_1$ , and  $\mathbf{d}$  are tuning parameters of the SMCr.  $c(t)$  is the pH sensor output. The second term on the right side of the Equation 14 represents the sliding mode. This term is responsible for keeping the system going towards steady state. It is also called the continuous mode. The third term is

called the reaching mode, or discontinue mode. This part of the controller is responsible for leading the system onto the sliding surface.

This SMCr has shown good and robust performance when controlling nonlinear chemical processes (Camacho and Smith, 2000), including processes that have inverse response (Camacho, et al., 1999), or variable dead time (Camacho and Smith, 2000). However, when the system is extremely nonlinear, and process gain varies in a nonmonotonic manner, the SMCr shows a slow response or excessive overshoot. In both cases the result is long stabilization time. It is precisely here where the symbiosis with Fuzzy Logic opens the door to the proposed solution. It is possible to increase the robustness and the “intelligence” of the SMCr through fuzzy rules, making the controller response slower or faster when appropriate. This is the purpose of the present research. The result is the development of a hybrid controller, combining the best features from SMC and Fuzzy Logic, which can be used in pH control in a neutralization reactor.

#### 4. FUZZY LOGIC IN PROCESS CONTROL

The ways in which fuzzy logic is used in process control are three:

- As a controller: Fuzzy Logic Controller (FLC).
- As a PID improvement supervisor: Fuzzy Gain Scheduling (FGS) or Fuzzy Self Tuning (FST).
- As a hybrid controller combined with another strategy: Neuro-fuzzy controllers, Genetic Algorithms (GA) based tuning of FLC's, Fuzzy Dynamic Matrix Control, or Fuzzy Sliding Mode Control.

No matter what strategy is used, the fuzzy system has crisp inputs and crisp outputs, and a fuzzy inference mechanism to relate them. The inference system uses linguistic variables associated in a degree of membership with linguistic values defined using membership functions in the same universe of discourse as the crisp variables. Obtaining linguistic variables with degrees of membership in linguistic values using crisp variable information is called fuzzification. A membership function is a scalar function whose domain is the universe of discourse and its range is the continuous set of degrees of membership to the corresponding linguistic value. The most common membership function shapes are triangular and trapezoidal, although more complex forms are found (Reznick, 1997).

After the input variable is fuzzified, the inference is made using a set of fuzzy rules, which are generally deduced using expert knowledge. The fuzzy rules are a set of if-then statements, which are based on linguistic variables, relating the inputs to outputs. In other words, the fuzzy rules define a set of imprecise dependence between two linguistic variables. Fuzzy variables are processed in an inference engine using a

set of fuzzy rules. The outputs from the engine are fuzzy variables with linguistic values defined over the universe of discourse of the crisp outputs. These fuzzy variables and their degrees of membership are used to obtain crisp outputs. This operation is called defuzzification.

### 5. THE HYBRID SURFACE-BASED FUZZY SLIDING MODE CONTROL

#### 5.1 The Steady-state Compensator.

When the performance of the SMCr is studied, one of the observations is that controller response is slow for set point changes. The main reason for the controller's slow response facing set point changes is that the sliding surface  $S(t)$  has an erratic behavior initially. In Figure 3 (solid line) it is shown how  $S(t)$  varies when the pH set point is reduced by 5%.  $S(t)$  starts to decrease, later reaches a minimum value, then start to grow until reaches its final value. It is necessary to highlight the great difference between the minimum value reached by  $S(t)$  and its final value; from  $-0.15$  to  $0.066$ , a difference of  $0.216$  units. This behavior results in a long stabilization time for the system.

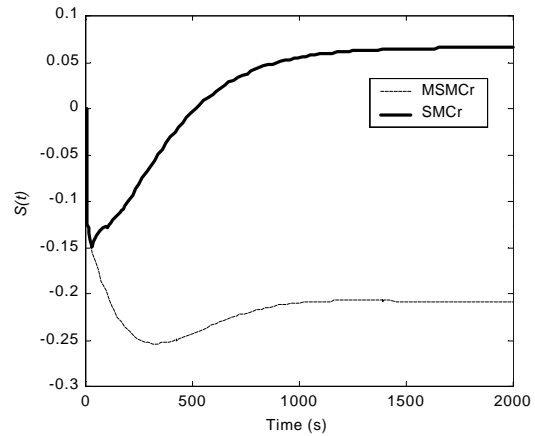


Fig. 3.  $S(t)$  variation with (MSMCr) and without (SMCr) Steady-state compensator.

One way to enhance the SMCr performance is to change the continuous mode in Equation 14, because this term controls the way how the system approaches its new final value.

If the error dependence in the continuous mode is eliminated in Equation 14, the controller equation can be expressed as:

$$m(t) = \bar{m} + K_s [c(t) - \bar{c}] + K_D \frac{S(t)}{|S(t)| + d} \quad (16)$$

Therefore, the continuous mode is transformed into a steady state compensator, where  $K_s$  is a tuning parameter.

The steady state compensator proposed introduces a stable term into the controller equation. This occurs because the error variation does not affect the way by which the system slides on the surface once it has been reached. Furthermore, when a set point change occurs in the system, the MSMCr reacts immediately with a pure proportional response. All these advantages are reflected in the time needed to reach a new steady state when the system is affected by a set point change.

### 5.2 The Hybrid Sliding Surface.

In order to enhance the MSMCr performance, it is necessary to include a new element that confers to the controller enough intelligence to react aggressively or slowly when necessary. This “smart” feature is designed using Fuzzy Logic. The previous section shows how any change that contributes to reach faster the sliding surface leads to a faster and less erratic response. Therefore, the ideal place to introduce intelligence is inside of  $S(t)$ . Hence, Equation 15 is modified to include the fuzzy element,  $\Delta S_F(e(t), \Delta e(t))$  as follows:

$$S_H(t) = \text{sign}(K_p) \left( -\frac{d(c(t))}{dt} + I_1 e(t) + I_0 \int_0^t e(t) dt \right) + \Delta S_F(e(t), \Delta e(t)) \quad (17)$$

where  $S_H(t)$  is the hybrid sliding-fuzzy surface. Equation (17) shows that the surface is a combination of two terms, the classical expression for SMC, plus a term whose value is determined by means of fuzzy rules. This term is a function of the error,  $e(t)$ , and the variation of the error  $\Delta e(t)$ . Thus, the inputs for the fuzzy rules are these variables. Then, the equation for the Hybrid Fuzzy-Sliding Controller (HFSMCR) can be written as:

$$m(t) = \bar{m} + K_S [c(t) - \bar{c}] + K_D \frac{S_H(t)}{|S_H(t)| + \mathbf{d}} \quad (18)$$

The addition of  $\Delta S_F$  to the controller equation provides the intelligence and robustness desired. The fuzzification of the inputs, as well as the defuzzification of the output is achieved by defining five linguistic values through membership functions along the universe of discourse of each input and output. Fig. 4 shows schematically the operation of the fuzzy inference system. Table 1 shows the fuzzy rules used to build the inference system; where PB means Positive Big, Z is zero, NS is Negative Small and so forth. The fuzzy rules were obtained analyzing how the controller must act facing changes in the system. For example, when both the error and  $\Delta e(t)$  are “positive big”, indicating that the system is out from the operation point and it is moving away from it, the controller need to act quickly, sending less signal to the control valve. Now then, regarding to Eq. (18), if the controller needs to send less signal to

the control valve, then  $S_H(t)$  should decrease significantly to do it. In order to decrease this variable (see Eq. (17)),  $\Delta S_F$  should be negative and “big” that is the value shown in the Table 1. All the rules were obtained using similar reasoning.

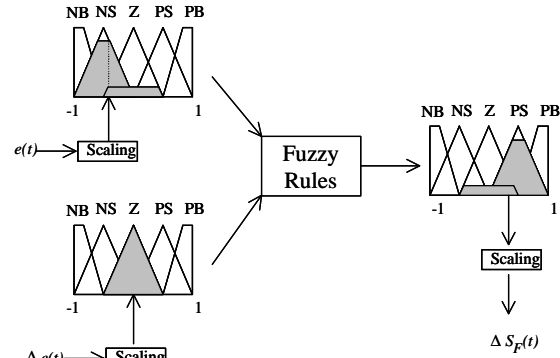


Fig. 4. Mamdani Inference System for  $\Delta S_F(t)$ .

Table 1. Fuzzy Rules to Obtain  $\Delta S_F(t)$  from system behavior

$e(t)$	$\Delta e(t)$				
	NB	NS	Z	PS	PB
NB	NB	NB	NB	NB	NS
NS	NB	NS	NS	Z	PS
Z	NS	NS	Z	PS	PS
PS	NS	Z	PS	PS	PB
PB	PS	PB	PB	PB	PB

## 6. SIMULATION RESULTS AND CONCLUSIONS

Simulations were developed to study the reactor behavior when PID, SMC, and HFSMC control strategies are implemented. The tuning parameters for the PID used were:  $K=3.5$ ,  $t_i=30$  s,  $t_d=15$  s. For the SMC;  $\lambda_0 = 0.001$ ,  $\lambda_1 = 0.9$ ,  $K_D = 1900$ ,  $\delta = 45$ . While for HFSMC;  $K_e = 0.7$ ,  $K_{De} = 0.7$ ,  $K_D = 2100$ ,  $K_S = 0.004$ .

Figure 5 shows how the system reacts to a set point change (both increase and decrease its value by 5%). It can be noticed that the hybrid controller provides both faster response and reduced stabilization time. This behavior is a result of the steady state compensator (fast response), and the fuzzy adjustment made to the sliding surface. Figure 6 shows the system response when a 5% change in the feed (acid) flow occurs. Figure 7 presents the reactor behavior when a 10% change in the buffer stream flow is induced in the system. The proposed controller exhibits better behavior under disturbances for which it was not tuned, indicating robust performance characteristics. Table 2 presents the quantitative measure of the controller’s performance using IAE (Integral Absolute Error).

Table 2. Integral of the Absolute value of the Error (IAE) for every controller in the four tests performed.

IAE	5% SetPoint Increase	5% SetPoint Reduction	5% Change in Acid Flow	10% Change in Buffer Flow
SMC	180	210	86	10
HFSMC	160	180	43	8
PID	170	200	63	15

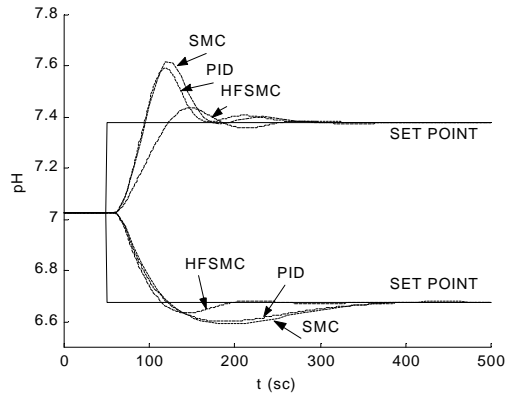


Fig. 5. Reactor behavior to set point changes using PID, SMC, and HFSMC control.

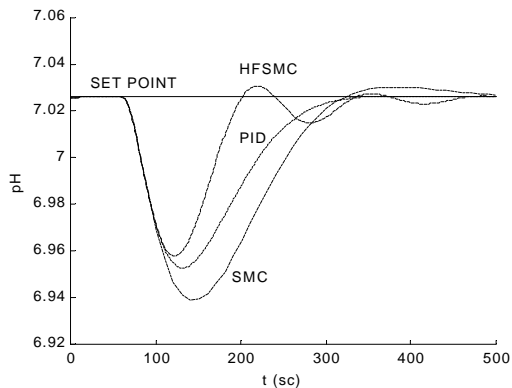


Fig. 6. Reactor behavior to acid flow change using PID, SMC, and HFSMC control.

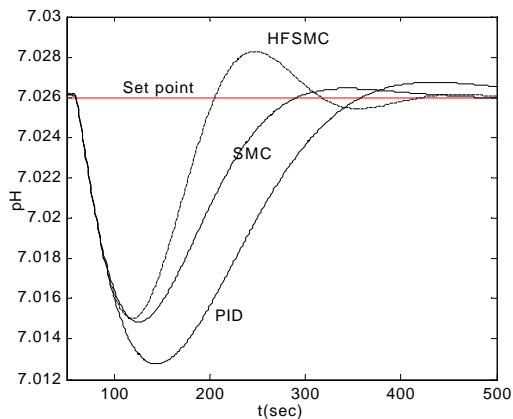


Fig. 7. Reactor behavior to buffer flow change using PID, SMC, and HFSMC control.

Further research will address the following issues:

- Obtain tuning equations relating the new controller parameters with process characteristics.
- A comprehensive stability analysis to determine limits in disturbance change before the controller becomes ineffective.
- The addition of new input (more intelligence) to the FIS to incorporate time-related behavior in the weight of the fuzzy adjustment in the total value of  $S(t)$ .

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