15th Triennial World Congress, Barcelona, Spain

APPLICATION OF GENERALIZED SAMPLED-DATA HOLD FUNCTIONS TO DECENTRALIZED CONTROL STRUCTURE MODIFICATION

Amir G. Aghdam and Edward J. Davison

Department of Electrical and Computer Engineering, University of Toronto
10 King's College Road, Toronto, Ontario M5S 3G4 Canada
amir@control.toronto.edu, ted@control.toronto.edu

Abstract: This paper investigates the decentralized control of LTI continuous-time plants using Generalized Sampled-data Hold Functions (GSHF). GSHFs can be used to modify the structure of the digraph of the resultant discrete plant, by removing certain interconnections in the equivalent discrete-time model to form a hierarchical system model of the plant. This is a new application of discretization, and has as its motivation, the design of decentralized controllers using centralized methods.

Keywords: Decentralized control, Large-scale system, Generalized Sampled-data Hold Functions.

1. INTRODUCTION

Sampled-data system applications are widely found in industry, mostly in order to take advantage of the application of computers as digital controllers. This is usually accomplished by using a simple zero-order hold. The idea of using Generalized Sampled-data Hold Functions (GSHF) instead of a simple zero-order hold (or first-order hold) in control systems, was first introduced by Chammas and Leondes (Chammas and Leondes 1979) and Kabamba (Kabamba 1987). Kabamba investigated several advantages in using GSHFs in control systems, and pointed out that by using GSHF, one can obtain many of the advantages of state feedback controllers, without the requirement of using state estimation procedures (Kabamba 1987), (Kabamba and Yang 1991); in particular he showed that GSHFs can significantly improve the performance of the closed-loop system. In addition, it was shown in (Aghdam and Davison 1996) that digital controllers can have a significant effect on improving the overall performance of certain classes of decentralized control systems, and in (Aghdam and Davison 1999*a*), the application of GSHFs in high-performance decentralized controller design was discussed.

In a discretized model, each transfer function in the resulting transfer matrix is a function of the system parameters, and also the sampling period and type of hold function (zero-order hold, first-order hold, ...). The question arises: using generalized sampled-data hold functions, to what extent can one modify these transfer functions so "something desirable" happens?

In this paper it will be shown how one can use generalized sampled-data hold functions to convert a continuous-time plant to an equivalent discrete-time system with a certain desired structure, in particular a hierarchical structure, which can directly be used to simplify the design of decentralized control systems for the plant.

2. DIGRAPHS AND SYSTEM STRUCTURE

Consider the following strictly proper continuous-time decentralized LTI system with m control agents:

$$\dot{x}(t) = Ax(t) + \begin{bmatrix} b_1 & \dots & b_m \end{bmatrix} \begin{bmatrix} u_1(t) \\ \vdots \\ u_m(t) \end{bmatrix}, \quad (1a)$$

$$\begin{bmatrix} y_1(t) \\ \vdots \\ y_m(t) \end{bmatrix} = \begin{bmatrix} c_1 \\ \vdots \\ c_m \end{bmatrix} x(t), \quad (1b)$$

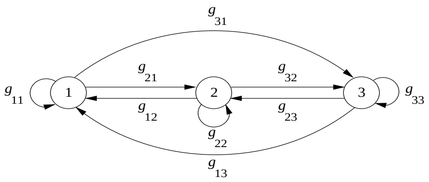

Fig. 1. Digraph of a continuous-time LTI system with 3 control agents.

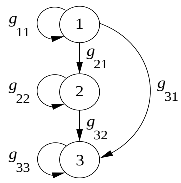

Fig. 2. Digraph of a continuous-time hierarchical LTI system with 3 control agents.

where $x(t) \in \mathbb{R}^n$ is the state vector, and $u_j(t) \in \mathbb{R}^{s_j}$, and $y_j(t) \in \mathbb{R}^{r_j}$, $j \in \bar{\mathrm{m}} = \{1, ..., m\}$ are the control vector and output vector of agent $\#j$ respectively, and A, b_j, and c_j are matrices of appropriate dimensions. The transfer matrix relating the inputs and outputs of this system can be written as:

$$\begin{bmatrix} Y_1(s) \\ \vdots \\ Y_m(s) \end{bmatrix} = \begin{bmatrix} g_{11}(s) & \dots & g_{1m}(s) \\ \vdots & & \vdots \\ g_{m1}(s) & \dots & g_{mm}(s) \end{bmatrix} \begin{bmatrix} U_1(s) \\ \vdots \\ U_m(s) \end{bmatrix},$$

where:

$$g_{ij}(s) := c_i(sI - A)^{-1} b_j, \quad i, j \in \bar{\mathrm{m}}.$$

Here $g_{ij}(s)$ represents the directed arc from node j to node i in the digraph of this system. For example, the digraph of a system with 3 input-output agents is depicted in Figure 1.

In the control of decentralized systems, the structure of the interconnections generally has a significant role to play, and in some decentralized control design procedures, it is often assumed that there exists a bound on the interconnections between different nodes. For example, in decentralized adaptive control methods, such an assumption is often made, or alternatively, certain structural constraints on the interconnections are often assumed in order to assure the stability of the overall system (Gavel and Šiljak 1989), (Shi and Singh 1992), and (Ioannou 1986). In the special case for systems which have a hierarchical structure, one can directly apply centralized control methods to each subsystem (represented by each node in the digraph of the system), with no assumptions required to be made on the interconnections, since it is guaranteed that signals coming from a higher level subsystems to a lower level subsystems will always be bounded, once the higher level subsystems are stabilized. For example, assume that the transfer functions $g_{12}(s)$, $g_{13}(s)$, $g_{12}(s)$ and $g_{23}(s)$ in Figure 1 are all equal to zero; then the corresponding system will have the hierarchical structure shown in Figure 2, and assuming that the system has no unstable decentralized fixed modes, one can design a centralized stabilizing controller for each subsystem separately. In this case, subsystem $\#1$ which has the highest level in the hierarchical structure, will be internally stable and since there is no input signal coming into this subsystem other than its control input which is bounded, this implies that all output signals coming out of this subsystem will be stable. The output signals of subsystem $\#1$ includes the interconnection signal going to subsystem $\#2$ and subsystem $\#3$ through $g_{21}(s)$ and $g_{31}(s)$ respectively. Now subsystem $\#2$, which has the second highest level will also be internally stable, and since all input signals coming to this subsystem, including the control input and interconnection signal from subsystem $\#1$ are bounded, this implies that all output signals of this subsystem will also be bounded. Thus, all interconnection signals coming from the higher level subsystems to subsystem $\#3$ will now be bounded and since this subsystem will also be internally stable, using a similar argument, one can conclude that the input and output signals of subsystem $\#3$ are also bounded. Thence, it can be concluded that the complete system will be internally stable. This implies that the decentralized controller design problem for a hierarchical system can be carried out in a centralized way for each subsystem.

Note that the transfer matrix representing the hierarchical system of Figure 2 has the following lower-triangular form:

$$\begin{bmatrix} Y_1(s) \\ Y_2(s) \\ Y_3(s) \end{bmatrix} = \begin{bmatrix} g_{11}(s) & 0 & 0 \\ g_{21}(s) & g_{22}(s) & 0 \\ g_{31}(s) & g_{32}(s) & g_{33}(s) \end{bmatrix} \begin{bmatrix} U_1(s) \\ U_2(s) \\ U_3(s) \end{bmatrix}. \quad (2)$$

In the general case, the transfer matrix of a hierarchical system always has a lower-triangular form or can be transformed to such a form by renumbering the subsystems appropriately (exchanging the number of subsystem $\#i$ with subsystem $\#j$ is equivalent to exchanging the i^{th} row with the j^{th} row, and the i^{th} column with the j^{th} column in the transfer matrix).

Our goal now is to determine if and how one can modify the structure of a system by using generalized sampling. It is observed that if certain interconnections of the system can be eliminated in the equivalent discrete-time model, then many decentralized control problems can be easily solved by applying centralized design methods to the individual subsystems of the resultant discrete-time system.

3. MAIN RESULT

Consider the continuous-time LTI system represented by (1). It is desired to discretize the system by applying generalized sampled-data hold functions $f_j(t)$, $j \in \bar{m}$, with a sampling period T, to each control agent. This problem can be formulated as follows. Let:

$$u_j(t) = f_j(t)\tilde{u}_j[k], \quad j \in \bar{m}, \quad t \in [kT, (k+1)T), \quad k = 0, 1, ... \tag{3a}$$

$$f_j(t + T) = f_j(t). \tag{3b}$$

As discussed in (Kabamba 1987) and (Aghdam and Davison 1999*a*), the equivalent discrete-time model is thence described by:

$$x[k+1] = A_d x[k] + \begin{bmatrix} b_{d_1} & \dots & b_{d_m} \end{bmatrix} \begin{bmatrix} u_1[k] \\ \vdots \\ u_m[k] \end{bmatrix}, \tag{4a}$$

$$\begin{bmatrix} y_1[k] \\ \vdots \\ y_m[k] \end{bmatrix} = \begin{bmatrix} c_{d_1} \\ \vdots \\ c_{d_m} \end{bmatrix} x[k], \tag{4b}$$

where:

$$A_d = e^{AT}, \tag{5a}$$

$$b_{d_j} = \int_{t_k}^{t_{k+1}} e^{A(T-\tau)} b_j f_j(\tau) d\tau, \quad j = 1, 2, ..., m \tag{5b}$$

$$c_{d_j} = c_j, \quad j = 1, 2, ..., m. \tag{5c}$$

It is desired now to determine if one can choose a set of sampled-data hold functions $f_j(t)$, $j \in \bar{m}$, and a sampling period T, so that certain elements of the transfer matrix corresponding to the equivalent discrete-time model, become equal to zero. The following result is obtained.

Theorem 1. Consider the system (1). There exists a sampled-data hold function $f_p(t)$, for each agent $p \in \bar{m}$, and a sampling period T, so that the equivalent discrete-time model has a hierarchical structure, if and only if there exist distinct integers $i_1, ..., i_m \in \bar{m}$, a positive scalar $h > 0$, and a nonzero vector x_{i_j}, $j = 2, ..., m$ contained in the null-space of $\begin{bmatrix} c_{i_1} \\ \vdots \\ c_{i_{j-1}} \end{bmatrix} (zI - e^{Ah})^{-1}$ which belongs to the controllability subspace of (A, b_{i_j}).

Proof of Theorem 1 . The proof follows from Lemma 1 in (Aghdam and Davison 1999*b*) which guarantees for the sampling period $T = h$, there exists a sampled-data hold function f_{i_j}, so that in the equivalent discrete-time model (4),(5), $b_{d_{i_j}} = x_{i_j}$. Details of the proof may be found in (Aghdam and Davison 2001).

Note that Theorem 1 gives necessary and sufficient conditions for a system to have a hierarchical discrete-time structure. In the next step, sufficient conditions are given, based on the concept of controllability and observability, which are very easy to check.

Theorem 2. Consider the system (1). There exists a sampled-data hold function $f_p(t)$, for each agent $p \in \bar{m}$ and a sampling period $T > 0$, so that the equivalent discrete-time model (4),(5) has a hierarchical structure, if the following two conditions both hold:

a) there exist distinct integers $l_1, l_2 \in \bar{m}$, so that the pair $\left(\begin{bmatrix} c_1 \\ \vdots \\ c_{l_2-1} \\ c_{l_2+1} \\ \vdots \\ c_m \end{bmatrix}, A \right)$ is not observable.

b) the pair (A, b_j) is controllable for every $j \in \bar{m}$, $j \neq l_1$.

Proof of Theorem 2 . The proof follows from Lemma 1 in (Aghdam and Davison 1999*b*), and from the fact that the equivalent discrete-time model of an unobservable continuous system is also unobservable for any sampling period. Details may be found in (Aghdam and Davison 2001).

Theorem 1 and Theorem 2 provide the conditions under which the digraph of a system can be modified to a hierarchical structure by using sampling. In the next step, the conditions under which the resulting discretized model does not have any decentralized fixed modes will be discussed.

Theorem 3. Given (1), assume that the pair (A, b_j) is controllable for all $j \in \bar{m}$. Then $e^{\lambda T} \in \mathrm{sp}(A_d)$ is not a decentralized fixed mode of the equivalent discrete-time model (4), with respect to the block diagonal gain matrix $K = \mathrm{diag}(K_1, \ \dots \ , K_m)$, $K_j \in \mathbb{R}^{s_j \times r_j}$, if the following three conditions all hold:

i) for every $\lambda_{l_1}, \ \lambda_{l_2} \in \mathrm{sp}(A)$, the relation $\mathrm{Re}\,(\lambda_{l_1}) = \mathrm{Re}\,(\lambda_{l_2})$ implies that $\mathrm{Im}\,(\lambda_{l_1} - \lambda_{l_2}) \neq \frac{2k\pi}{T}, \ l_1, l_2 \in \{1, ..., n\}, \ k = \pm 1, \pm 2, ...$

ii) $\int_0^T e^{-\lambda t} f_j(t) dt \neq 0, \ \lambda \in \mathrm{sp}(A)$

iii) the pair $\left(\begin{bmatrix} c_1 \\ \vdots \\ c_m \end{bmatrix}, A \right)$ is observable.

Proof of Theorem 3. $e^{\lambda_l T} \in \mathrm{sp}(A_d)$ is a decentralized fixed mode of (4) if and only if any one of the following conditions hold (Davison and Chang 1990):

1) $\mathrm{rank} \left(\begin{bmatrix} A_d - e^{\lambda T} I \\ c_1 \\ \vdots \\ c_m \end{bmatrix} \right) < n$

2) $\mathrm{rank} \left(\begin{bmatrix} A_d - e^{\lambda T} I & b_{d_1} & \dots & b_{d_m} \end{bmatrix} \right) < n$

3) $\text{rank}\left(\begin{bmatrix} A_d - e^{\lambda T} I & b_{d_{i_1}} \\ c_{i_2} & 0 \\ \vdots & \vdots \\ c_{i_m} & 0 \end{bmatrix}\right) < n$
for some $i_j \in \bar{\text{m}}$, $j = 1, ..., m$ such that $\{i_1, ..., i_m\} = \bar{\text{m}}$

4) $\text{rank}\left(\begin{bmatrix} A_d - e^{\lambda T} I & b_{d_{i_1}} & b_{d_{i_2}} \\ c_{i_3} & 0 & 0 \\ \vdots & \vdots & \vdots \\ c_{i_m} & 0 & 0 \end{bmatrix}\right) < n$
for some $i_j \in \bar{\text{m}}$, $j = 1, ..., m$ such that $\{i_1, ..., i_m\} = \bar{\text{m}}$

m+1) $\text{rank}\left(\begin{bmatrix} A_d - e^{\lambda T} I & b_{d_{i_1}} & b_{d_{i_2}} & \cdots & b_{d_{i_{m-1}}} \\ c_{i_m} & 0 & 0 & \cdots & 0 \end{bmatrix}\right) < n$
for some $i_j \in \bar{\text{m}}$, $j = 1, ..., m$ such that $\{i_1, ..., i_m\} = \bar{\text{m}}$

Assume now that conditions (i), (ii), and (iii) are all satisfied. Conditions (i) and (ii) imply that the resulting discretized system will not lose controllability and observability (corresponding to any controllable or observable pairs in the continuous-time model) (Middleton and Freudenberg 1995). Thus, the pair (A_d, b_{d_j}) is controllable for all $j \in \bar{\text{m}}$, and the pair $\left(\begin{bmatrix} c_1 \\ \vdots \\ c_m \end{bmatrix}, A_d\right)$ is observable. This implies that the rank conditions (1) to $(m+1)$ for the existence of a decentralized fixed mode do not hold. ■

Discussion. The results of Theorem 1 and Theorem 2 are interesting in the sense that they introduce a completely new application for sampling in control. The results can be very useful in the design of decentralized controllers, when the structure of the original system does not permit one to directly apply centralized controller design methods to decentralized systems. In this case, if the conditions given in Theorem 1 and Theorem 2 are met, one can find a set of GSHFs to modify the structure of the system in the equivalent discrete-time model (4),(5), so that centralized digital control methods can now be directly applied to each interconnected subsystem. In particular, the results obtained may be applied to the decentralized adaptive control problem discussed in (Aghdam and Davison 1999*b*). In this case, one applies sampling to obtain a hierarchical discrete-time model, and thence directly applies digital adaptive controllers to each of the subsystems.

Remark 1. Note that conditions (i) and (ii) in Theorem 3 ensure non-pathological sampling for generalized sampled-data hold functions. In the case of a zero-order hold, condition (i) is sufficient to guarantee non-pathological sampling (Chen and Francis 1995). In addition, if the eigenvalues

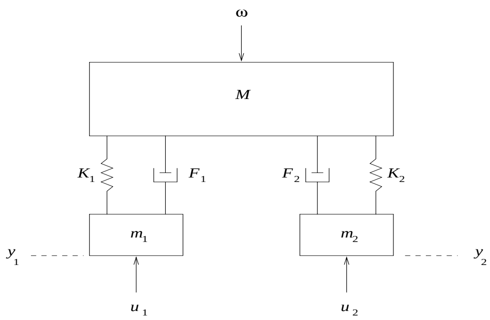

Fig. 3. The 2-input, 2-output mass-spring system of Example 1.

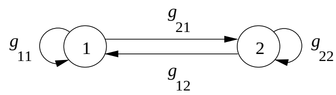

Fig. 4. Digraph of the mass-spring system of Example 1.

of the continuous-time system are all real, condition (i) in Theorem 3 will be met.

Remark 2. It can be easily seen that asymptotic stability of the equivalent discrete-time model will imply asymptotic stability of the corresponding continuous-time model (Kabamba 1987). Therefore, if one designs a digital controller to stabilize the plant's discretized model, it will result in stability for the original continuous-time system.

Remark 3. It is to be noted that a disadvantage of generalized sampled-data hold functions is that they are prone to robustness difficulties in the continuous time domain, e.g. see (Feuer and Goodwin 1994), (J. S. Freudenberg and Braslavsky 1997).

4. NUMERICAL EXAMPLE

Example 1. Consider the 2-input, 2-output mass-spring system of Figure 3 and assume that the measured outputs are $y_{m1} := \dot{y}_1$ and $y_{m2} := y_2 + \dot{y}_2$. For $m_1 = m_2 = 1$, $M = 10$, $K_1 = K_2 = 1$, $F_1 = F_2 = 0.1$ and $\omega = 0$ this system is described by the following system matrices:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -0.2 & -0.02 & 0.1 & 0.01 & 0.1 & 0.01 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0.1 & -1 & -0.1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0.1 & 0 & 0 & -1 & -0.1 \end{bmatrix}, b_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, b_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{6a}$$

$$c_1 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \; c_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}. \tag{6b}$$

The digraph of this system is depicted in Figure 4. The transfer functions in this figure are given by:

$$g_{11}(s) = \frac{s^4 + 0.12s^3 + 1.201s^2 + 0.02s + 0.1}{s(s^4 + 0.22s^3 + 2.212s^2 + 0.24s + 1.2)},$$
$$g_{12}(s) = \frac{0.001(s^2 + 20s + 100)}{s(s^4 + 0.22s^3 + 2.212s^2 + 0.24s + 1.2)},$$
$$g_{21}(s) = \frac{0.001(s^3 + 21s^2 + 120s + 100)}{s^2(s^4 + 0.22s^3 + 2.212s^2 + 0.24s + 1.2)},$$
$$g_{22}(s) = \frac{s^5 + 1.12s^4 + 1.321s^3 + 1.221s^2 + 0.12s + 0.1}{s^2(s^4 + 0.22s^3 + 2.212s^2 + 0.24s + 1.2)}.$$

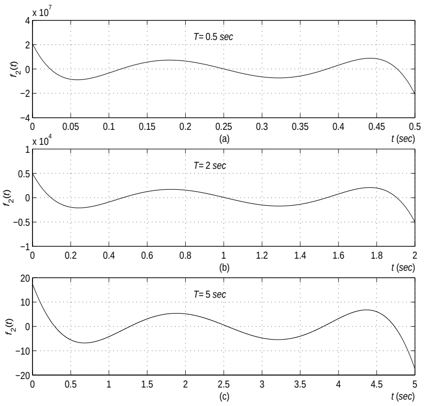

Fig. 5. Sampled-data hold functions for control agent #2 in Example 1. (a) $T = 0.5$ *sec*; (b) $T = 2$ *sec*; (c) $T = 5$ *sec*.

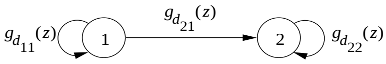

Fig. 6. Digraph of the equivalent discrete-time mass-spring system of Example 1.

Here, (c_1, A) and (c_2, A) are unobservable, while (A, b_1) and (A, b_2) are controllable. Thus, the conditions of Theorem 2 hold for any arrangement of control agents. In other words, one can use sampling with specific sampled-data hold functions to construct two different hierarchical discrete-time models, with either one of subsystems at a higher level. For instance, let us assume that $i_1 = 1$ and $i_2 = 2$ (which means that $l_1 = 1$ and $l_2 = 2$ in Theorem 2), and let us use equation (5b) to obtain a sampled-data hold function for control agent #2, so that the resultant discretized model will be hierarchical with subsystem #1 at the higher level. In this case, it can be shown that for all values of the sampling period T, the vector $[1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0]'$ is a basis for the null-space of $c_1(zI - A_d)^{-1}$ (in fact, the null-space is one dimensional). Solving the minimum energy problem for (5b), the sampled-data hold functions of Figures 5 (a), (b), and (c) are obtained for $T = 0.5$ *sec*, $T = 2$ *sec*, and $T = 5$ *sec* respectively (Aghdam and Davison 2001). Let us choose $T = 5$ *sec*. Applying the sampled-data hold function of Figure 5 (c) to control agent #2, and a simple zero-order hold to control agent #1, the discrete-time hierarchical model of Figure 6 is obtained. The transfer functions $g_{d_{11}}(z)$, $g_{d_{21}}(z)$, $g_{d_{22}}(z)$ are given by:

$$\frac{-0.1627(z^4 - 6.295z^3 + 7.274z^2 - 4.752z + 1.175)}{z^5 - 2.450z^4 + 3.045z^3 - 2.449z^2 + 1.187z - 0.3329},$$
$$\frac{1.398(z^5 - 1.190z^4 + 1.706z^3 - 1.137z^2 + 0.5280z + 0.02318}{z^6 - 3.450z^5 + 5.494z^4 - 5.494z^3 + 3.637z^2 - 1.520z + 0.3329},$$
$$\frac{0.1}{z - 1},$$

respectively. In addition, it can be easily verified that the conditions of Theorem 3 for the given model, GSHF, and sampling period hold true, which implies that the equivalent discrete-time model has no DFMs. This implies that one can design a decentralized controller for this system by applying a digital controller for each subsystem independently, using centralized methods. For instance, consider the controllers:

$$v_{11}[k+1] = v_{11}[k] + 5(y_{m1}[k] - y_{ref,1}), \quad (7a)$$
$$v_{12}[k+1] = 0.6876v_{12}[k] + y_{m1}[k], \quad (7b)$$
$$u_1[k] = -0.3591y_{m1}[k] - 0.005629v_{11}[k] - 0.04669v_{12}[k], \quad (7c)$$

and:

$$v_{21}[k+1] = v_{21}[k] + 5(y_{m2}[k] - y_{ref,2}), \quad (8a)$$
$$v_{22}[k+1] = -0.8004v_{22}[k] + y_{m2}[k], \quad (8b)$$
$$u_2[k] = -9.393y_{m2}[k] - 0.6006v_{21}[k] - 1.086 \times 10^{-4}v_{22}[k], \quad (8c)$$

which have been designed for subsystem #1 and subsystem #2 respectively, independently of each other, using centralized methods to solve the robust servomechanism problem. Here $y_{ref,1}$ and $y_{ref,2}$ in (7) and (8) denote constant reference signals for control agent #1 and control agent #2 respectively. The eigenvalues of the resultant closed-loop discrete-time system corresponding to subsystem #1 and subsystem #2 are given by:

$$\mathrm{sp}_1 = \{0.9246, 0.2935 \pm 0.7898i, 0.7906 \pm 0.08309i, 0.5514 \pm 0.5753i\},$$

and:

$$\mathrm{sp}_2 = \{-0.8004, 0.5303 \pm 0.2824i\}$$

respectively. The eigenvalues of the overall closed-loop discrete-time system are thus given by:

$$\mathrm{sp} = \mathrm{sp}_1 \cup \mathrm{sp}_2.$$

Assume now that $x[0] = [1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1]'$ and $v_{11}[0] = v_{12}[0] = v_{21}[0] = v_{22}[0] = 0$. Figures 7 (a) and (b) give the outputs of the resultant closed-loop system, for a unit step reference input in control agent #1 and a zero reference input in control agent #2, using the decentralized controller (7), (8). The input signals, corresponding to control agent #1 and control agent #2, are given in Figures 7 (c) and (d), respectively.

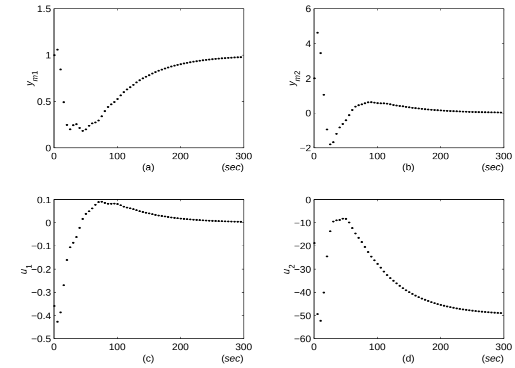

Fig. 7. Closed-loop simulations for Example 1. (a) Samples of the output response in control agent #1; (b) samples of the output response in control agent #2; (c) Samples of the control signal in control agent #1; (d) samples of the control signal in control agent #2.

It is to be noted that if an incorrect polarity of inputs and outputs occurs in the plant to be controlled, due to for example incorrect wiring, the equivalent discrete-time model obtained preserves its hierarchical structure (the corresponding transfer functions may have different signs however). Thus this implies that one can obtain a solution to the decentralized adaptive switching control problem introduced in (Aghdam and Davison 1999*b*) for this system by applying centralized switching control methods to each control agent, and in this case it is guaranteed that each subsystem will be "stabilized in a finite time" (Aghdam and Davison 1999*b*). This observation obtained is very important; for example when the parameters of the mass-spring system are such that the corresponding interconnections are not weak enough to be ignored, this implies that the continuous-time decentralized switching control method proposed in (Aghdam and Davison 1999*b*) may not be applicable, but that its discrete-time counterpart, which employs the proposed sampled-data hold functions described in this paper can still be very effective.

5. CONCLUSION

A new application for generalized sampled-data system control is introduced in this paper, which has the property that it can be used to simplify the structure of the resulting discretized model. Conditions under which the resultant discretized model can have a hierarchical digraph are discussed in the paper, and a method to synthesis the corresponding sampled-data hold functions for each control agent is presented. The motivation for simplifying the plant structure to become hierarchical, is that controller design, particularly for decentralized control, can be greatly simplified. For example, in decentralized control problems, one can design a decentralized controller for the system by applying centralized methods directly to each control agent for such a hierarchical discrete system. Simulation results are given in the paper to show how such a discretization procedure can result in a simple hierarchical digraph, and thence simplify the decentralized controller design problem in obtaining a solution to the robust servomechanism problem.

REFERENCES

Aghdam, A. G. and E. J. Davison (1996). Decentralized control of systems with approximate decentralized fixed modes. In: *Proceedings of IFAC Symposium in Large Scale Systems,* Belfort, France. Vol. 2. pp. 49–54.

Aghdam, A. G. and E. J. Davison (1999*a*). Decentralized control of systems, using generalized sampled-data hold functions. In: *Proceedings of the 38'th IEEE Conference on Decision and Control*. pp. 3912–3913.

Aghdam, A. G. and E. J. Davison (1999*b*). Decentralized switching control using a family of controllers approach. In: *Proceedings of the 1999 American Control Conference*. pp. 47–52.

Aghdam, A. G. and E. J. Davison (2001). Application of generalized sampled-data hold functions to decentralized control structure modification. *Univ. of Toronto Report No. 0103.*

Chammas, A. B. and C. T. Leondes (1979). On the finite time control of linear systems by piecewise constant output feedback. *International Journal of Control* **30**(2), 227–234.

Chen, T. and B. A. Francis (1995). *Optimal Sampled-Data Control Systems*. Springer-Verlag.

Davison, E. J. and T. N. Chang (1990). Decentralized stabilization and pole assignment for general proper systems. *IEEE Transactions on Automatic Control* **AC-35**(6), 652–664.

Feuer, A. and G. C. Goodwin (1994). Generalized sample hold functions-frequency domain analysis of robustness, sensitivity, and intersample difficulties. *IEEE Transactions on Automatic Control* **AC-39**(5), 1042–1047.

Gavel, D. T. and D. D. Šiljak (1989). Decentralized adaptive control: Structural conditions for stability. *IEEE Transactions on Automatic Control* **AC-34**(4), 413–426.

Ioannou, P. A. (1986). Decentralized adaptive control of interconnected systems. *IEEE Transactions on Automatic Control* **AC-31**(4), 291–298.

J. S. Freudenberg, R. H. Middleton and J. H. Braslavsky (1997). Robustness of zero shifting via generalized sampled-data hold functions. *IEEE Transactions on Automatic Control* **AC-42**(12), 1681–1692.

Kabamba, P. T. (1987). Control of linear systems using generalized sampled-data hold functions. *IEEE Transactions on Automatic Control* **AC-32**(9), 772–783.

Kabamba, P. T. and C. Yang (1991). Simultaneous controller design for linear time-invariant systems. *IEEE Transactions on Automatic Control* **36**(1), 106–111.

Middleton, R. H. and J. S. Freudenberg (1995). Non-pathological sampling for generalized sampled-data hold functions. *Automatica* **31**(2), 315–319.

Shi, L. and S. K. Singh (1992). Decentralized adaptive controller design for large-scale systems with higher order interconnections. *IEEE Transactions on Automatic Control* **AC-37**(8), 1106–1118.