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INPUT DESIGN: A METHOD FOR ACHIEVING PERSISTENCY OF EXCITATION IN NONLINEAR PARMETRIZED SYSTEMS

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Abstract: In this work sufficient time-varying conditions were established for assuring local persistent excitation for identification of a large class of nonlinearly parametrized model structures under integral cost functionals of arbitrary degree. The conditions are algebraic in nature. They can be set up off-line in a symbolic form and evaluated on-line. The potential benefit of these conditions mainly reclines in the design of excitations for nonlinear system identification. A simple method for optimal input design is presented. Examples illustrate the features of the approach.

Keywords: Input Design, Optimal Excitation, Parameter Estimation, Cost Functional

1. INTRODUCTION

In many technical areas physical model structures are common in the modelling of dynamic systems from first principles, e.g., in chemical processes, hydrodynamic and mechanical systems. They are required in control, optimization and diagnosis among other areas, specially when accuracy cannot be reached by black-box or semiphysical models. However phenomenological relations are often characterized by a high degree of interaction among parameters and variables involving transcendent and trigonometric functions.

In parameter identification or adaptive control laws based on optimization of a cost functional, these interactions can lead to a nonconvex search problem upon a nonlinear parameterization in time domain.

Nonconvex functionals are often characterized by the presence of moving local minima and slots in the parameter space. This makes generally the search of the global minimum troublesome and imposes the adaptive law to exhibit certain ability to sort eventual local minima. To this goal tensorial information to construct adaptive laws is necessary (Bambill and Jordán, 1999*a*), (Bambill and Jordán, 1999*b*).

On the other side, it is well known that for global convergence of parameters, not only global identifiability with respect to a nonlinear model structure is required, but also the property of persistent excitation in a finite period of measures (Kreisselmeier and Rietze-Augst, 1990). The concept of persistent excitation in a nonlinear sense is not directly inferred from the well understood homologous concept in Linear Estimation, (Dasgupta *et al.*, 1991). For the stationary state this is more closer connected to frequency content and multilevel amplitude in some unclear manner. Some approaches are given for triangular model structure and convex/concave parametrization in, (Skantze *et al.*, 2000).

When stating these conditions differentially for a general analytic nonlinear system, it is not straightforward to established connections between the excitation and the regressor, i.e., between the richness of the input and the persistency

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of excitation of the regressor, due to the complexity of the model structure. If, on the contrary, it is done algebraically, more simple relations could be obtained.

The aim of this paper is to find sufficient conditions that ensure persistency of excitation in a nonlinear sense and to relate these with the input design for parameter estimation. The approach is based on pure algebraic conditions that can be tested on-line. Additionally links of the conditions with the cost functional are found. Examples depict the features of the proposed approach.

2. PROBLEM STATEMENT

The problem handled in this work concerns the design of input signals for a general class of nonlinear systems in order to allow a specified estimator to find the values of the unknown system parameters in an exponential or asymptotic form. As this problem is closely joined with the identification problem, let us in this section relate the identifiability concept together with the property of appropriate system excitation.

2.1 *System description*

Let a nonlinear time-invariant system of order n be described in input-output form

$$y^{(n)} = \phi(y^{(n-1)}, ..., y, u^{(n-\rho)}, ..., u; \theta^*), \quad (1)$$

with u the system input, y the system output, $u^{(i)}$and $y^{(i)}$ their respective derivatives, $\rho > 0$ the relative degree and θ^* an unknown parameter vector in a connected compact set $\mathcal{D}_\theta \subset \Re^{n_0}$. The vector function ϕ is supposed Lipschitz-continuous on $\mathcal{D}_\theta$, but not necessary in the variables. This last relaxation enables the use of piecewise continuous excitations. In the general case, ϕ is characterized in nonalgebraic form, e.g., by means of transcendent functions.

In the next, it is supposed that $\left\{u^{(i)}\right\}$ with $i = 1, ..., n - \rho$, and $\left\{y^{(i)}\right\}$ with $i = 0, ..., n$, are measured in continuous time and bounded in a finite time interval $[0, T]$, $T > 0$, i.e., they belong to $\mathcal{L}_{\infty e}$ on $[0, T]$. This assumption does not exclude unstable modes of (1), but only eventual finite scape times must be located outside $[0, T]$.

2.2 *Nonlinear estimation*

Consider the model structure $\mathcal{M}(\theta)$, where $\theta \in \mathcal{D}_\theta$ is a parameter vector variable. Accordingly, a nonlinear regression is

$$y_\theta^{(n)} = \phi(y^{(n-1)}, ..., y, u^{(n-\rho)}, ..., u; \theta), \quad (2)$$

with $y_\theta^{(n)}$ the prediction of $y^{(n)}$.

The estimation of θ^* can be formulated as an unconstrained optimization problem

$$\min_{\theta \in \mathcal{D}_\theta} V_m(u^*(t), \theta), \quad (3)$$

for $t \in (0, T]$ with

$$V_m(u^*(t), \theta) := \frac{1}{mt} \int_0^t \varepsilon^m(\tau, \theta) d\tau, \quad (4)$$

where m is an positive real value, u^* is an optimal excitation and

$$\varepsilon(t, \theta) = y^{(n)}(t, \theta^*) - y_\theta^{(n)}(t, \theta) \quad (5)$$

is the estimation error.

2.3 *Optimal excitation*

Following definitions will be useful for further development..

Definition 1. (Convex lower bound). Let V_m be uniformly locally convex about θ^*. Then V_m is said to have a convex lower bound of degree q_0 in $\mathcal{D}_\theta \subset \Re^{n_0}$ such that $V_m(t, \theta) \geq \sigma \left|\theta - \theta^*\right|^{q_0}$, with $\sigma > 0$, $t \in (0, T]$, where $q_0 \in \{q_i\}$ is the minimal power of the monomials $(\theta_i - \theta_i^*)^{q_i}$ of the Taylor series of V_m about θ^* for $i = 1, ..., n$. ■

Definition 2. (Optimal excitation). Given (1) with $\mathcal{M}(\theta^*)$, then the model structure $\mathcal{M}(\theta)$ is said globally identifiable at θ^* with respect to $\mathcal{D}_\theta$ on $[0, T]$ if there exists at least one excitation signal $u = u^*$ such that $y^{(n)}(u^*) \neq 0$ and

$$\begin{aligned} y^{(n)}(u^*) = y_\theta^{(n)}(u^*),\ \theta \in \mathcal{D}_\theta \\ \Rightarrow \theta^* = \theta, \end{aligned} \quad (6)$$

with $u^* \in \mathcal{D}_{u^*}$, where

$$\begin{aligned} \mathcal{D}_{u^*} = \{u : [0, T] \rightarrow \mathcal{L}_{\infty e} \left| V_m(u(t), \theta) \geq \right. \\ \sigma \left|\theta - \theta^*\right|^{q_0}, \sigma > 0, \theta \in \mathcal{D}_\theta\} \end{aligned} \quad (7)$$

■

3. INPUT DESIGN PROBLEM

This section deals with sufficient conditions for the existence of u^* and with its optimal design.

3.1 *Convex optimal excitation*

For $\Im_0$ the set of nonnegative integers and the zero, $\bar{\sigma} = [\sigma_1, ..., \sigma_{n_0}]^T \in \Im_0^{n_0}$ and $(\theta - \theta^*)^{\bar{\sigma}} := (\theta_1 - \theta_1^*)^{\sigma_1} ... (\theta_{n_0} - \theta_{n_0}^*)^{\sigma_{n_0}}$, the Taylor series for V_m about $\theta = \theta^*$ is

$$V_m(t, \theta) = \sum_{i=0}^{\infty} \frac{(\theta - \theta^*)^{\bar{\sigma}_i}}{\sigma_{i_1}! ... \sigma_{i_{n_0}}!} \left(\frac{\partial^{q_i} V_m(t, \theta)}{\partial \theta_1^{\sigma_{i_1}} ... \partial \theta_{n_0}^{\sigma_{i_{n_0}}}} \right) \Bigg|_{\theta = \theta^*}, \quad (8)$$

with θ_i, θ_i^* components of θ and θ^*, and

$$q_i = \alpha_{i_1} + ... + \alpha_{i_{n_0}} = \mathrm{trace}\,(\bar{\sigma}_i), \quad (9)$$

is the power of the monomial i-th of the series. In vector form

$$V_m(t, \theta) = \sum_{i=0}^{\infty} \frac{1}{2t\ (i)!} \left(\left(\theta^T \frac{\partial}{\partial \theta} \right)^{q_i} \int_0^t \varepsilon^m(\tau, \theta)\, d\tau \right) \Bigg|_{\theta = \theta^*} \quad (10)$$

where $\left(\theta^T \frac{\partial}{\partial \theta} \right) : \mathcal{C}^1 \rightarrow \Re$ is an operator with following properties

$$\text{a)}\ \left(\theta^T \frac{\partial}{\partial \theta} \right) V_m = \left(\theta^T \frac{\partial V_m}{\partial \theta} \right) = \theta_1 \frac{\partial V_m}{\partial \theta_1} + ... + \theta_{n_0} \frac{\partial V_m}{\partial \theta_{n_0}} \quad (11)$$

$$\text{b)}\ \left(\theta^T \frac{\partial}{\partial \theta} \right)^{q_i} V_m = \left(\theta^T \frac{\partial}{\partial \theta} \right) ... \left(\theta^T \frac{\partial V_m}{\partial \theta} \right) \quad (12)$$

$$\text{c)}\ \left(\theta^T \frac{\partial}{\partial \theta} \right)^0 V_m = V_m, \quad (13)$$

with $\mathcal{C}^1$ the set of infinitely differentiable functions on $\mathcal{D}_\theta$. The next theorem is the main contribution of the paper.

Theorem 1. (Persistent exciting regressor). Let V_m in (4) be the cost functional for the estimation of a nonlinear system of order n with unknown parameter vector $\theta^* \in \Re^{n_0}$ and model structure $\mathcal{M}(\theta)$. If

$$a_i > \sum_k \frac{1\text{- sign}(b_k)}{2} \frac{1}{2} |b_k| + \sum_k \frac{1}{2} |c_k| + \sum_k |d_k| \quad (14)$$

with $i = 1, ..., n_0$, where at θ^*

$$a_i(t) = \int_0^t \left(\frac{\partial \varepsilon}{\partial \theta_i} \right)^m d\tau \quad (15)$$

$$b_k(t) = \int_0^t \left(\frac{\partial \varepsilon}{\partial \theta_i} \right)^{\sigma_{b_i}} ... \left(\frac{\partial \varepsilon}{\partial \theta_j} \right)^{\sigma_{b_j}} d\tau \quad (16)$$

$$c_k(t) = \int_0^t \left(\frac{\partial \varepsilon}{\partial \theta_i} \right)^{\sigma_{c_i}} ... \left(\frac{\partial \varepsilon}{\partial \theta_j} \right)^{\sigma_{c_j}} d\tau \quad (17)$$

$$d_k(t) = \int_0^t \left(\frac{\partial \varepsilon}{\partial \theta_i} \right)^{\sigma_{d_i}} ... \left(\frac{\partial \varepsilon}{\partial \theta_j} \right)^{\sigma_{d_j}} d\tau, \quad (18)$$

$\sigma_{b_i}, ..., \sigma_{b_j} \in \Im_0^+$ are all even
$\sigma_{c_i}, ..., \sigma_{c_j} \in \Im_0^+$ are all odd, with $\sigma_{c_i} = ... = \sigma_{c_j}$
$\sigma_{d_i}, ..., \sigma_{d_j} \in \Im_0^+$, at least one of them is even,

with $\sigma_{b_i} + ... + \sigma_{b_j} = \sigma_{c_i} + ... + \sigma_{c_j} = \sigma_{d_i} + ... + \sigma_{d_j} = m$, then V_m is uniformly locally convex on a region centered at $\theta = \theta^*$.

Proof: For V_m it holds

$$\frac{\partial^q V_m}{\partial \theta_i^{\sigma_i} ... \partial \theta_j^{\sigma_j}}(t, \theta^*) = 0, \quad \text{for } q < m \text{ and } q = \alpha_i + ... + \alpha_j \quad (19)$$

$$\frac{\partial^m V_m}{\partial \theta_i^{\sigma_i} ... \partial \theta_j^{\sigma_j}}(t, \theta^*) = \frac{(m-1)!}{t} \int_0^t \left(\frac{\partial \varepsilon}{\partial \theta_i}(\tau, \theta^*) \right)^{\sigma_i} ... \left(\frac{\partial \varepsilon}{\partial \theta_j}(\tau, \theta^*) \right)^{\sigma_j} d\tau \quad \text{with } m = \alpha_i + ... + \alpha_j\ . \quad (20)$$

So the first $(m-1)$ terms in (8) are null and V_m has a lower convex bound with degree $q_0 = m$. Thus V_m is bounded from below for all $t > 0$ on a region centered about θ^* as

$$V_m(t, \theta) \geqq \sigma_0\, V_{m0}(t, \theta), \quad (21)$$

with σ_0 a nonzero positive real constant and

$$V_{m0}(t, \theta) = \frac{1}{t} \left(\left(\theta^T \frac{\partial}{\partial \theta} \right)^m \int_0^t \varepsilon^m(\tau, \theta) \right) \Bigg|_{\theta = \theta^*}. \quad (22)$$

After applying $\left(\theta^T \frac{\partial}{\partial \theta} \right)^m$ onto the integral in (22), one achieves

$$V_{m_0}(t, \theta) = \frac{(m-1)!}{t\,(\sigma_i)! ... (\sigma_j)!} (\theta_i - \theta_i^*)^{\sigma_i} ... \left(\theta_j - \theta_j^* \right)^{\sigma_j} \left(\int_0^t \left(\frac{\partial \varepsilon}{\partial \theta_i}(\tau, \theta^*) \right)^{\sigma_i} ... \left(\frac{\partial \varepsilon}{\partial \theta_j}(\tau, \theta^*) \right)^{\sigma_j} d\tau \right) \quad (23)$$

with $\alpha_i + ... + \alpha_j = m$. Denote $\tilde{\theta}_i = (\theta_i - \theta_i^*)$ for $i = 1, ..., n_0$, and define the vector $\bar{\theta} \in \mathcal{B}_r \subset \Re^{n_q}$, with $n_q = \binom{m+n_0-1}{m-1}$

$$\bar{\theta} = \left(\tilde{\theta}^{\bar{\sigma}_1}, ..., \tilde{\theta}^{\bar{\sigma}_k}, ..., \tilde{\theta}^{\bar{\sigma}_{n_q}}\right)^T \tag{24}$$

$$\tilde{\theta}^{\bar{\sigma}_k} = \tilde{\theta}_1^{\sigma_{k1}} \tilde{\theta}_2^{\sigma_{k2}} ... \tilde{\theta}_{n_0}^{\sigma_{kn_0}} \tag{25}$$

$$\bar{\sigma}_k = \left(\sigma_{k_1}, \sigma_{k_2}, ..., \sigma_{k_{n_0}}\right)^T \tag{26}$$

$$\mathrm{trace}(\bar{\sigma}_k) = \frac{m}{2}\,. \tag{27}$$

Similarly let $\varphi : (0,t] \to \Re^{n_q}$ be

$$\varphi^T(t,\theta^*) = \prod_{i=1}^{n_0} \left(\frac{\partial \varepsilon}{\partial \theta_i}(\tau,\theta^*)\right)^{\sigma_{1_i}}, ..., \tag{28}$$
$$\prod_{i=1}^{n_0} \left(\frac{\partial \varepsilon}{\partial \theta_i}(\tau,\theta^*)\right)^{\sigma_{k_i}}, ..., \prod_{i=1}^{n_0} \left(\frac{\partial \varepsilon}{\partial \theta_i}(\tau,\theta^*)\right)^{\sigma_{n_{qi}}}.$$

Thus with (24) and (28)

$$V_{m_0}(t,\theta) =$$
$$\bar{\theta}^T \left(\frac{(m\text{-}1)!}{t} \int_0^t \varphi(\tau,\theta^*)\, \varphi^T(\tau,\theta^*)\, d\tau\right) \bar{\theta} \tag{29}$$

and $\Phi_m : (0,t] \to \Re^{n_q \times n_q}$ be

$$\Phi_m(t,\theta^*) = \frac{(m\text{-}1)!}{t} \int_0^t \varphi(\tau,\theta^*)\, \varphi^T(\tau,\theta^*)\, d\tau, \tag{30}$$

$$V_{m_0}(t,\theta^*) = \bar{\theta}^T \Phi_m(t,\theta^*)\, \bar{\theta}. \tag{31}$$

With ϕ_{ij} being an element of Φ_m and φ_i an element of φ, clearly, $\phi_{ij} = \varphi_i \varphi_j$ and $\phi_{ij} = \phi_{ji}$.

The rest of the proof consists of demonstrating under what conditions $V_{m_0}(t,\theta^*)$ is positive definite, since under the same conditions $V_m(t,\theta^*)$ will be positive definite too. Clearly (31) is a quadratic form for $m=2$ and so the condition $V_{m_0}(t,\theta^*)$ to be convex about θ^* is $\Phi_m(t,\theta^*) \geqq 0$. For $m \neq 2$ to be positive definite more complex relations are needed to be satisfied. Let the function elements ϕ_{ij} be classified according to the sign definiteness of the monomials in (29). Thus

$$\text{for } \theta_i^m \ \to a_k = \phi_{ii} = \int_0^t \left(\frac{\partial \varepsilon}{\partial \theta_i}^m\right) d\tau$$

$$\text{for } \theta_i^{\sigma_{b_i}} ... \theta_j^{\sigma_{b_j}},\ \left\{\sigma_{b_i}, ..., \sigma_{b_j}\right\} \text{ even } \to$$
$$b_k = \phi_{ij} = \int_0^t \left(\frac{\partial \varepsilon}{\partial \theta_i}^{\sigma_{d_i}} ... \frac{\partial \varepsilon}{\partial \theta_j}^{\sigma_{b_j}}\right) d\tau$$

$$\text{for } \theta_i^{\sigma c_i} ... \theta_j^{\sigma_{c_i}},\ \sigma_{c_i} \text{ odd}$$
$$c_k = \phi_{ij} = \int_0^t \left(\frac{\partial \varepsilon}{\partial \theta_i}^{\sigma_{c_i}} ... \frac{\partial \varepsilon}{\partial \theta_j}^{\sigma_{c_i}}\right) d\tau$$

$$\text{for } \theta_i^{\sigma_{d_i}} ... \theta_j^{\sigma_{d_j}},\ \left\{\sigma_{d_i}, ..., \sigma_{d_j}\right\} \text{ even/odd } \to$$
$$d_k = \phi_{ij} = \int_0^t \left(\frac{\partial \varepsilon}{\partial \theta_i}^{\sigma_{d_i}} ... \frac{\partial \varepsilon}{\partial \theta_j}^{\sigma_{d_j}}\right) d\tau \tag{32}$$

where $\sigma_{b_i} + ... + \sigma_{b_j} = \sigma_{c_i} + ... + \sigma_{c_j} = \sigma_{d_i} + ... + \sigma_{d_j} = m$. Hence the monomials $\frac{(m-1)!}{t} a_i(t)\, \theta_i^m$ are uniformly positive definite, the monomials $\frac{(m-1)!}{t} b_k(t)\, \theta_i^{\sigma_{b_i}} ... \theta_j^{\sigma_{b_j}}$ are positive or negative semidefinite depending on the sign of b_k, the monomials $\frac{(m-1)!}{t} c_k(t)\, \theta_i^{\sigma_{c_i}} ... \theta_j^{\sigma_{c_j}}$ are symmetric and sign undefined, and the monomials $\frac{(m-1)!}{t} d_k(t)\, \theta_i^{\sigma_{d_i}} ... \theta_j^{\sigma_{d_j}}$ are antisymmetric and sign undefined. Therefore

$$b_k(t)\, \theta^{\bar{\sigma}_{b_k}} \geqq -\sum_{i=1}^{n_0} \frac{1\text{- sign}(b_k)}{4} \left|b_k(t)\right|\, \theta_i^m \tag{33}$$

$$c_k(t)\, \theta^{\bar{\sigma}_{c_k}} \geqq -\sum_{i=1}^{n_0} \frac{1}{2} \left|c_k(t)\right|\, \theta_i^m. \tag{34}$$

Additionally all d_k's have the same sign, since the function $d_k(t) = \int_0^t \frac{\partial \varepsilon}{\partial \theta_i}^{\sigma_{d_i}} ... \frac{\partial \varepsilon}{\partial \theta_j}^{\sigma_{d_j}} d\tau$ is antisymmetric and has a dual function with the same structure but interchanged time variables, i.e., $\int_0^t \frac{\partial \varepsilon}{\partial \theta_j}^{\sigma_{d_i}} ... \frac{\partial \varepsilon}{\partial \theta_i}^{\sigma_{d_j}} d\tau$. Thus

$$c_k(t)\, \theta^{\bar{\sigma}_{d_i}} \geqq -\sum_{i=1}^{n_0} \frac{1}{2} \left|d_k(t)\right|\, \theta_i^m. \tag{35}$$

Then with (33)-(35)

$$V_{m0} \geqq \sum_{i=1}^{n_0} \left[a_i(t) \text{-} \sum_k \frac{1\text{-sign}(b_k)}{4} \left|b_k(t)\right| \right.$$
$$\left. -\sum_k \frac{1}{2} \left|c_k(t)\right| - \sum_k \left|d_k(t)\right| \right] \theta_i^m \tag{36}$$

and for V_{m0} to be positive definite it is sufficient that for $i = 1, ..., n_0$

$$a_i(t) > \sum_k \frac{1 - \mathrm{sign}(b_k)}{4} \left|b_k(t)\right| +$$
$$\sum_k \frac{1}{2} \left|c_k(t)\right| - \sum_k \left|d_k(t)\right|\,. \tag{37}$$

Consequently V_m is convex on a region centered about θ^*. ■

3.2 *Applications to local input design*

The sufficient conditions found in theorem (1) can be applied to solve the problem of optimal input design in the sense of achieving identifiability. The idea of taking advantage of simple algebraic conditions is the base of the following description.

Next one presents a methodology for input design could be settled in a simple form. This is the basis for more sophisticated algorithms.

To this goal consider the family of piecewise constant functions u with a set of amplitudes

$$\mathcal{S}_u = \{-s_p, -s_{p-1}, ..., -s_1, 0, s_1, ..., s_{p-1}, s_p\}$$

and a set of time intervals $\mathcal{S}_{t_i} = \{[t_{i-1}, t_i)\}$, with $t_i \in \Im_0^+$ and $t_i > t_{i-1}$. Besides consider a signal $u : \mathcal{S}_{t_i} \to \mathcal{S}_u$ such that

$$u(t) = u^*(t) = s_j \text{ for } t \in \mathcal{S}_{t_i} \text{ and } s_j \in \mathcal{S}_u. \quad (38)$$

The next step in the design consists in selecting the amplitude at every time t_i where an eventual change of amplitude is subject to the satisfaction of conditions (14). The additional task is to decide in which direction, i.e., upstairs or downstairs in the levels of u^* is changed. This will be accomplished by

$$\max_{s_j \in \mathcal{S}_u} (a_i - \textstyle\sum_k \frac{1-\operatorname{sign}(b_k)}{4} |b_k| - \sum_k \frac{1}{2} |c_k| - \sum_k |d_k|).$$

Here the $\theta(t)$ of the adaptive law trajectory is taking as the center point to evaluate a_i, b_i, c_i, and d_i.As the n_0 algebraic conditions are easy to compute numerically, the test can be verified online without too much time consuming.

If one condition in (14) is violated during the input generation, this is not necessary a symptom of nonconvexity. However, it may mean the parameter trajectory is crossing a nonconvex zone. A tensor-based adaptive law can provide insight in neighboring zones in order to direct the trajectory to a convex region. Two algorithms with these features are presented in (Bambill and Jordán, 1999*a*), (Bambill and Jordán, 1999*b*).

3.3 *Example 1*

Let the nonlinear dynamic system and regressor be described respectively by

$$\ddot{y}(t) = -\sin^3(\theta_1^* \dot{y}) - 2e^{\theta_2^* y + \theta_3^* u - 1} \quad (39)$$

$$\ddot{y}_\theta(t,\theta) = -\sin^3(\theta_1 \dot{y}) - 2e^{\theta_2 y + \theta_3 u - 1}, \quad (40)$$

which is Lipschitz continuous in the space of finite parameters and bounded signals. Using $m = 2$ in V_m it results $q_0 = 2$ and

$$\frac{\partial^2 V_2}{\partial \theta^2}(t,\theta^*) = \Phi(t,\theta^*) \quad (41)$$

$$a_1 = \frac{9}{t}\int_0^t \dot{y}^2 \cos^2(\theta_1^* \dot{y}) \sin^4(\theta_1^* \dot{y}) d\tau$$

$$a_2 = \frac{1}{t}\int_0^t y^2 \exp(2\theta_2^* y + 2\theta_3^* u) d\tau$$

$$a_3 = \frac{1}{t}\int_0^t u^2 \exp(2\theta_2^* y + 2\theta_3^* u) d\tau$$

$$c_{1,2} = \frac{3}{t}\int_0^t \dot{y} y \exp(2\theta_2^* y + 2\theta_3^* u) \cos(\theta_1^* \dot{y}) \sin^2(\theta_1^* \dot{y}) d\tau$$

$$c_{3,4} = \frac{3}{t}\int_0^t \dot{y} u \exp(2\theta_2^* y + 2\theta_3^* u) \cos(\theta_1^* \dot{y}) \sin^2(\theta_1^* \dot{y}) d\tau$$

$$c_{5,6} = \frac{1}{t}\int_0^t y \exp(2\theta_2^* y + 2\theta_3^* u) d\tau \ .$$

The sufficient conditions (14) require

$$\operatorname{sign}(c_3(u)) = \operatorname{sign}(c_5(u)) \quad (42)$$

$$\begin{cases} a_1(u) > \frac{1}{2}|c_1(u)| + \frac{1}{2}|c_3(u)| + \frac{1}{2}|c_5(u)| \\ a_2(u) > \frac{1}{2}|c_1(u)| + \frac{1}{2}|c_3(u)| + \frac{1}{2}|c_5(u)| \\ a_3(u) > \frac{1}{2}|c_1(u)| + \frac{1}{2}|c_3(u)| + \frac{1}{2}|c_5(u)| \end{cases} .$$

The (42) implies

$$\operatorname{sign}\int_0^t \dot{y} u \cos(\theta_1^* \dot{y}) d\tau = \operatorname{sign}\int_0^t y d\tau. \quad (43)$$

It could appear that classic conditions ($\det(\Phi(u)) \neq 0$) are more easily to check, but it is apparently. First, by complex relations in $\mathcal{M}_\theta$, testing condition $\Phi(u) > 0$ at every time may be cumbersome. The sufficient conditions (14), on the contrary, can be handled on-line much more easily. Second, $\Phi(u) > 0$ is valid only for $m = 2$, while conditions (14) are general. The next example illustrates this feature.

3.4 *Example 2*

Let the nonlinear dynamic system and regressor be described respectively by

$$\dot{y}(t) = \sin^{1/3}(\theta_1^* y + \theta_2^* u) \quad (44)$$

$$\dot{y}_\theta(t) = \sin^{1/3}(\theta_1 y + \theta_2 u). \quad (45)$$

Since often one is interested in using gradient-based algorithms for parameter estimation, in which the gradient be simply proportional to θ, the cost functional is designed for having locally quadratic order of magnitude. Thus, one chooses $m = 6$. Consequently at θ^* it holds $V_6 = \frac{\partial V_6}{\partial \theta} = \frac{\partial^2 V_6}{\partial \theta^2} = \frac{\partial V_6^3}{\partial \theta^3} = \frac{\partial V_6^4}{\partial \theta^4} = \frac{\partial V_6^5}{\partial \theta^5} = 0$, and

$$\bar{\theta}(t) = \left(\tilde{\theta}_1^3, \tilde{\theta}_2^3, \tilde{\theta}_1^2 \tilde{\theta}_2, \tilde{\theta}_1 \tilde{\theta}_2^2\right)^T \quad (46)$$

$$\frac{\partial^6 V_6}{\partial \theta^6}(t,\theta^*) = \Phi_6(t,\theta^*) \in [0,t] \times \Re^{4\mathrm{x}4} \ . \quad (47)$$

The sufficient conditions (14) for $V_6 > 0$ require

$$\operatorname{sign}\int_0^t y^5 u \frac{\cos^5(\theta_1^* y)}{\sin^{10/3}(\theta_1^* y)} \frac{\cos(\theta_2^* u)}{\sin^{2/3}(\theta_2^* u)} d\tau = \operatorname{sign}\int_0^t y u^5 \frac{\cos(\theta_1^* y)}{\sin^{2/3}(\theta_1^* y)} \frac{\cos^5(\theta_2^* u)}{\sin^{10/3}(\theta_2^* u)} d\tau \tag{48}$$

$$0 < \int_0^t y^6 \frac{\cos^4(\theta_1^* y)}{\sin^2(\theta_1^* y)} d\tau - 2\left|\int_0^t y^3 u^3 \frac{\cos^3(\theta_1^* y)}{\sin(\theta_1^* y)} \frac{\cos^3(\theta_2^* u)}{\sin(\theta_2^* u)} d\tau\right| - \left|\int_0^t \left(y^5 u \frac{\cos^5(\theta_1^* y)}{\sin^{10/3}(\theta_1^* y)} \frac{\cos(\theta_2^* u)}{\sin^{2/3}(\theta_2^* u)} + y u^5 \frac{\cos(\theta_1^* y)}{\sin^{2/3}(\theta_1^* y)} \frac{\cos^5(\theta_2^* u)}{\sin^{10/3}(\theta_2^* u)} d\tau \right)\right| \tag{49}$$

$$0 < \int_0^t u^6 \frac{\cos^4(\theta_2^* u)}{\sin^2(\theta_2^* u)} d\tau - 2\left|\int_0^t y^3 u^3 \frac{\cos^3(\theta_1^* y)}{\sin(\theta_1^* y)} \frac{\cos^3(\theta_2^* u)}{\sin(\theta_2^* u)} d\tau\right| - \left|\int_0^t \left(y^5 u \frac{\cos^5(\theta_1^* y)}{\sin^{10/3}(\theta_1^* y)} \frac{\cos(\theta_2^* u)}{\sin^{2/3}(\theta_2^* u)} + y u^5 \frac{\cos(\theta_1^* y)}{\sin^{2/3}(\theta_1^* y)} \frac{\cos^5(\theta_2^* u)}{\sin^{10/3}(\theta_2^* u)} \right)\right|. \tag{50}$$

Conditions (49)-(50) are now checked for a single-harmonic signal $u(t) = 1.8\sin(3t)$ (Fig. 1, bottom) and for an optimal signal $u^*(t)$ (Fig. 2, bottom) according to our presented approach. Condition (49) is violated permanently by $u(t)$. It is worth noticing that in the linear case this $u(t)$ would be rich of order two, i.e., it is sufficient for estimating two parameters. Finally an optimal multilevel excitation generated for fixed changing time points satisfies (49)-(50) permanently after an insignificant violation of (49) at the beginning.

4. CONCLUSIONS

In this work sufficient time-varying conditions were established for assuring local persistent excitation for identification of a large class of nonlinearly parametrized model structures under integral cost functionals of arbitrary degree. The conditions are algebraic in nature. They can be set up off-line in a symbolic form and evaluated on-line. The potential benefit of these conditions mainly reclines in the design of excitations for nonlinear system identification. A simple method for optimal input design is presented. Examples illustrate the features of our approach.

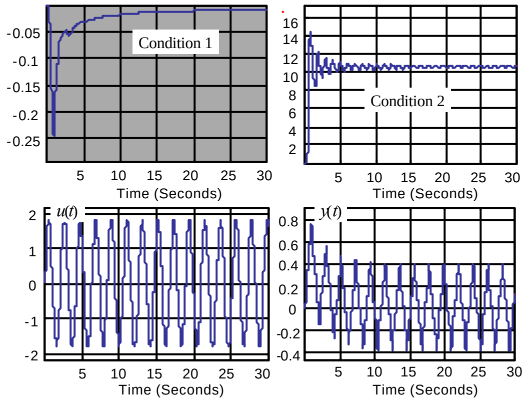

Fig. 1. PE conditions for a non-rich single-harmonic input $u(t)$ (Condition 1 is violated permanently)

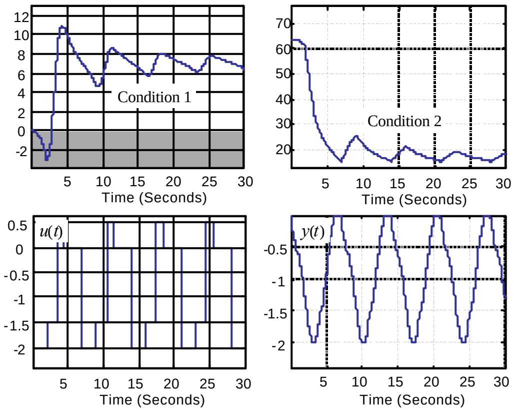

Fig. 2. PE conditions for an optimal multilevel input $u^*(t)$

5. REFERENCES

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