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HIGH PERFORMANCE COMPUTING OF TIME FREQUENCY DISTRIBUTIONS FOR DOPPLER ULTRASOUND SIGNAL ANALYSIS

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Abstract: Typical methods for signal analysis utilize the Fourier Transform-based algorithms to estimate the spectral response of a signal. This current practice suffers from poor frequency resolution when estimating non-stationary signals. This paper describes some alternative methods based on time-frequency distributions from a Cohen´s class point of view. Four distribution cases are evaluated: Wigner Ville, Choi Williams, Bessel and Born Jordan. Continuous and discrete distributions are presented for each case. Simplified discretised expressions for the implementation of distributions are formulated, these leading to a reduction of the computations realized when comparing to original definitions. Also, two parallel approaches (intrinsic parallelism and data parallelism) for the computation of the distributions are proposed, implemented and assessed by using a parallel DSP-based system. Finally, a further simplification by truncating the simplified expressions is proposed; this truncation is restricted by the error in spectral estimations. Results are applied to the development of a real-time spectrum analyzer for Doppler blood flow instrumentation. *Copyright © 2002 IFAC.*

Keywords: Time-Frequency Distributions, Parallel DSP Architectures, Signal Analysis

1. INTRODUCTION

A classic method for spectral estimation is the so-called Fourier Transform. However, its use is limited to stationary signals giving as a result a poor frequency resolution when estimating non-stationary ones. Other types of spectral estimators, called time-frequency distributions, have been developed. Unlike conventional methods these distributions are not limited to the use of stationary signals. Despite of this important advantage, the number of calculations involved in obtaining the spectral estimation increases substantially compared to the traditional methods. Therefore, it is desirable to simplify the formulation of the distributions in such a way that the computations involved can be reduced without any loss in the spectral resolution. Furthermore, in order to get a real time signal processing, it is also desirable to implement the algorithms that calculate such distributions on a high performance DSP-based computational system. This paper deals with these issues.

2. TIME-FREQUENCY DISTRIBUTIONS

This section formulates the so-called Cohen's class for the time-frequency distributions.

2.1. The Cohen's Class

The Cohen´s class in terms of time frequency distributions can be formulated as follows. Let the time-frequency distribution kernel be defined as $\phi(\theta,\tau)$. This kernel will define the particular characteristics of each time-frequency distribution. Then, the Cohen's class for the time-frequency distributions with kernel $\phi(\theta,\tau)$ can be defined as:

$$TFD(t,\omega)= \tag{1}$$
$$=\frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty}\phi(\theta,\tau)e^{-j\theta(t-\mu)}d\theta\right]x\left(\mu+\frac{\tau}{2}\right)x^{*}\left(\mu-\frac{\tau}{2}\right)d\mu\right]e^{-j\omega\tau}d\tau$$

2.2. The Wigner Ville Distribution

According to its definition (Cardoso, et. al., 1996), the Wigner Ville distribution for the continuous case is given by:

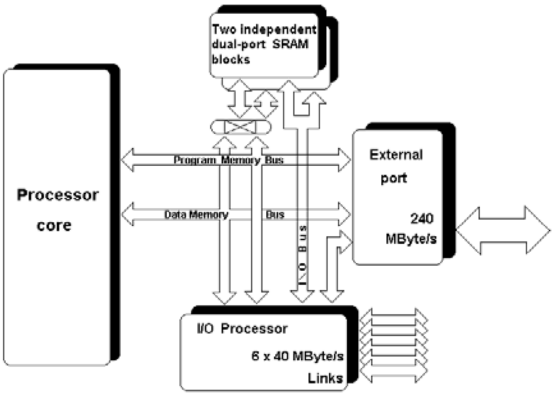


Fig. 1. ADSP-21060/62 SHARC block diagram.

$$WVD(t,\omega)=\int_{-\infty}^{\infty} x\left(t+\frac{\tau}{2}\right)x^*\left(t-\frac{\tau}{2}\right)e^{-j\omega\tau}d\tau \tag{2}$$

and, for the discrete case, by:

$$DWVD(n,k)= \tag{3}$$
$$=2\sum_{\tau=-N+1}^{N-1}W(\tau)W^*(-\tau)e^{-j\frac{2\pi k\tau}{N}}x(n+\tau)x^*(n-\tau)$$

where n represents the discrete time and k the discrete frequency; both variables are normalized.

2.3. The Choi Williams Distribution

According to its definition (Cardoso, *et. al*., 1996), the Choi Williams distribution for the continuous case is given by:

$$CWD(t,\omega)=$$
$$=\int_{-\infty}^{\infty}\sqrt{\frac{1}{4\pi\tau^2/\sigma}}\int_{-\infty}^{\infty}e^{-\frac{(t-\mu)^2}{4\tau^2/\sigma}}x\left(\mu+\frac{\tau}{2}\right)x^*\left(\mu-\frac{\tau}{2}\right)d\mu e^{-j\omega\tau}d\tau \tag{4}$$

where $\sigma>0$ is a scaling factor. And, for the discrete case, by:

$$DCWD(n,k)= \tag{5}$$
$$=2\sum_{\tau=-N+1}^{N-1}W(\tau)W^*(-\tau)e^{-j\frac{2\pi k\tau}{N}}\sum_{\mu=-M}^{M}\sqrt{\frac{1}{4\pi\tau^2/\sigma}}e^{-\frac{\mu^2}{4\tau^2/\sigma}}x(\mu+n+\tau)x^*(\mu+n-\tau)$$

2.4. The Bessel Distribution

According to its definition (Cardoso, *et. al*., 1996), the Bessel distribution for the continuous case is given by:

$$BD(t,\omega)= \tag{6}$$
$$=\int_{-\infty}^{\infty}\frac{2}{\pi\alpha|\tau|}\int_{-\infty}^{\infty}\sqrt{1-\left(\frac{t-\mu}{\alpha\tau}\right)^2}U_0\left(\frac{t-\mu}{\alpha\tau}\right)x\left(\mu+\frac{\tau}{2}\right)x^*\left(\mu-\frac{\tau}{2}\right)d\mu e^{-j\omega\tau}d\tau$$

where $\alpha>0$ is a scaling factor. And, for the discrete case, by:

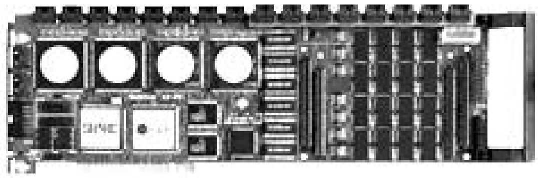


Fig. 2. ASP-P15 card, integrated by four ADSP-21060/62 SHARC DSP.

$$DBD(n,k)= \tag{7}$$
$$=2\sum_{\tau=-N+1}^{N-1}W(\tau)W^*(-\tau)e^{-j\frac{2\pi k\tau}{N}}\sum_{\mu=-2\alpha|\tau|}^{2\alpha|\tau|}\frac{1}{\pi\alpha|\tau|}\sqrt{1-\left(\frac{\mu}{2\alpha\tau}\right)^2}x(\mu+n+\tau)x^*(\mu+n-\tau)$$

2.5. The Born Jordan Distribution

According to its definition (Cohen, 1989), the Born Jordan distribution for the continuous case is given by:

$$BJD(t,\omega)=\frac{1}{2\alpha}\int_{-\infty}^{\infty}\frac{1}{\tau}\int_{t-\alpha\tau}^{t+\alpha\tau}x\left(\mu+\frac{\tau}{2}\right)x^*\left(\mu-\frac{\tau}{2}\right)d\mu e^{-j\omega\tau}d\tau \tag{8}$$

where $\alpha>0$ is a scaling factor. And, for the discrete case, by:

$$DBJD(n,k)= \tag{9}$$
$$=2\sum_{\tau=-N+1}^{N-1}W(\tau)W^*(-\tau)e^{-j\frac{2\pi k\tau}{N}}\sum_{\mu=-2\alpha|\tau|}^{2\alpha|\tau|}\frac{1}{4\alpha|\tau|}x(\mu+n+\tau)x^*(\mu+n-\tau)$$

3. REDUCING THE COMPUTATIONAL COMPLEXITY OF THE DISCRETE DEFINITIONS

In order to evaluate the different distributions for spectral estimation, a discrete signal *x(n)* is considered. Such a signal contains *2N-1* elements, where *N* is a power of *2* and the element range is from *–N+1* to *N-1*, therefore *x(0)* is the central element. Based on these elements, this section presents a reduction in computational terms of the number of calculations involved in the evaluation of each of the distributions considered in this paper.

3.1. The Wigner Ville Distribution

Considering eq. (3) for estimating the Wigner Ville distribution and evaluating it in *n=0* (Fan, and Evans, 1994), an equivalent simplified expression would be given by:

$$DWVD(0,k)=4\,\mathrm{Re}\left[\sum_{\tau=0}^{N-1}W(\tau)W^*(-\tau)e^{-j\frac{2\pi k\tau}{N}}x(\tau)x^*(-\tau)\right]-2|x(0)|^2 \tag{10}$$

Assuming that $W(\tau)W^*(-\tau)$ is a single factor then, for each value of k in eq. (3) evaluated in *n=0*, there are *6N-3* complex multiplications, *2N-2* complex additions and *1* scalar multiplication, whereas in eq.

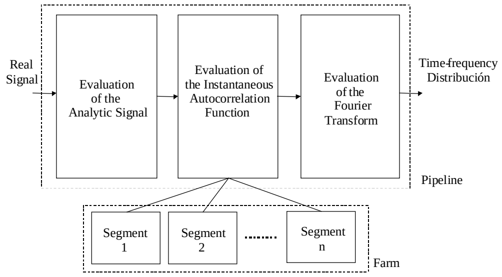


Fig. 3. Intrinsic parallel processing approach for execution of the time-frequency distributions.

(10) there are *3N+1* complex multiplications, *N* complex additions and *2* scalar multiplications.

3.2. The Choi Williams Distribution

Similarly, considering eq. (5) for estimating the Choi Williams distribution and evaluating it in *n=0*, an equivalent simplified expression would be given by:

$$DCWD(0,k) = -2|x(0)|^2 + \tag{11}$$
$$+4\,\mathrm{Re}\left[\sum_{\tau=0}^{N-1} W(\tau)W^*(-\tau)e^{-j\frac{2\pi k\tau}{N}} \sum_{\mu=-N+1+|\tau|}^{N-1-|\tau|} \sqrt{\frac{1}{4\pi\tau^2/\sigma}}\, e^{-\frac{\mu^2}{4\tau^2/\sigma}} x(\mu+\tau)x^*(\mu-\tau)\right]$$

where the summation respect to μ for $\tau=0$ is $x(0)x^*(0)$.

Assuming that $W(\tau)W^*(-\tau)$ and the square root multiplied by the exponential are single factors then, for each value of *k* in eq. (5) evaluated in *n=0*, there are $8N^2$-$4N$ complex multiplications, $4N^2$-$6N+2$ complex additions and *1* scalar multiplication, whereas in eq. (11) there are $2N^2$-$2N+1$ complex multiplications, N^2-$2N$ complex additions and *2* scalar multiplications.

3.3. The Bessel Distribution

Considering eq. (7) for estimating the Bessel distribution and evaluating it in *n=0*, an equivalent simplified expression would be given by:

$$DBD(0,k) = -2|x(0)|^2 + \tag{12}$$
$$+4\,\mathrm{Re}\left[\sum_{\tau=0}^{N-1} W(\tau)W^*(-\tau)e^{-j\frac{2\pi k\tau}{N}} \sum_{\mu=max\{-2\alpha|\tau|,-N+1+|\tau|\}}^{min\{2\alpha|\tau|,N-1-|\tau|\}} \frac{1}{\pi\alpha|\tau|}\sqrt{1-\left(\frac{\mu}{2\alpha\tau}\right)^2}\, x(\mu+\tau)x^*(\mu-\tau)\right]$$

where the summation respect to μ for $\tau=0$ is $x(0)x^*(0)$.

Assuming that $W(\tau)W^*(-\tau)$ and the square root divided by $\pi\alpha|\tau|$ are single factors then, for each value of *k* in eq. (7) evaluated in *n=0*, there are $8\alpha N^2$-$8\alpha N$ complex multiplications, $4\alpha N^2$-$4\alpha N$-$2N$ complex additions and *1* scalar multiplication, whereas in eq. (12) there are less than $4\alpha N^2$-$4\alpha N+1$

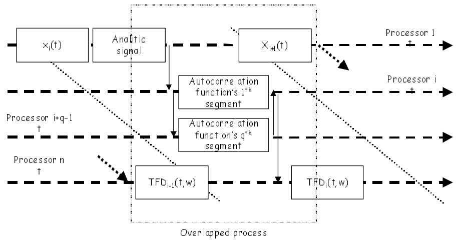


Fig 4. Time diagram of the intrinsic parallel processing approach.

complex multiplications, less than $2\alpha N^2$-$2\alpha N$-N and *2* scalar multiplications.

3.4. The Born Jordan Distribution

Considering eq. (9) for estimating the Born Jordan distribution and evaluating it in *n=0*, an equivalent simplified expression would be given by:

$$DBJD(0,k) = -2|x(0)|^2 + \tag{13}$$
$$+4\,\mathrm{Re}\left[\sum_{\tau=0}^{N-1} W(\tau)W^*(-\tau)e^{-j\frac{2\pi k\tau}{N}} \sum_{\mu=max\{-2\alpha|\tau|,-N+1+|\tau|\}}^{min\{2\alpha|\tau|,N-1-|\tau|\}} \frac{1}{4\alpha|\tau|}\, x(\mu+\tau)x^*(\mu-\tau)\right]$$

where the summation respect to μ for $\tau=0$ is $x(0)x^*(0)$. The analysis is similar to Bessel's.

4. PARALLEL PROCESSING OF THE TIME-FREQUENCY DISTRIBUTIONS

As stated previously, the use of time-frequency distributions for spectral estimation of signals opens the possibility of analyzing non-stationary signals. However, the associated computational cost is high. Section 3 has described strategies in order to achieve a reduction in the amount of calculations involved in evaluating the original definitions for each distribution. In this section, the use of parallel processing techniques in order to reduce the time required to perform the evaluations is proposed. Previous work has been addressed to investigate parallel processing techniques in signal analysis (Madeira, et. al., 1999; Solano, et. al., 1999).

In this work two approaches of parallelism are presented: intrinsic parallelism and data parallelism. The first approach considers a single data-time segment being processed by a number of processors, where each processor executes a different part of the signal analysis algorithm. In the second approach a sequence of N data-time segments is processed in parallel by using N processors, each processor executing the whole algorithm over each data-time segment. Both approaches are implemented and assessed by using a parallel DSP-based system.

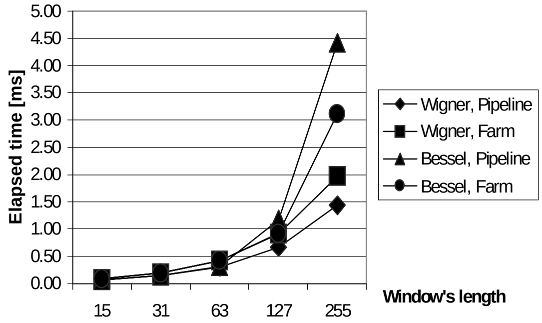


Fig. 5. Execution time vs. signal's length of Wigner Ville and Bessel distributions for pipeline and farm+pipeline parallel architectures using 4 SHARC processors.

4.1. Parallel Architecture

A parallel architecture based on the ADSP-21062 SHARC has been used for implementing the time-frequency distribution algorithms.

The ADSP-21062 SHARC is a high-performance signal processor using a Super Harvard Architecture (i.e. four independent buses for dual-data, instructions and I/O). It integrates three 32-bit IEEE floating-point computation units (multiplier, ALU and shifter), a 2 Mbits dual port on-chip SRAM and multiprocessing features. It performs 40 MIPS, 120 MFLOPS peak and 80 MFLOPS sustained and 6 DMA communication links with a maximum bandwidth of 240 MB/sec. Figure 1 depicts the ADSP-21060/62 SHARC block diagram.

On the other hand, figure 2 shows the ASP-P15 card. This is a standard PCI full length card with 4 Analog Devices ADSP-2106x SHARC DSPs with up to 768Kbytes of zero wait state SRAM and 64Mbytes of fast page mode DRAM. A SHARCPAC module site provides for processor or I/O expansion. This development system programmed in C language and hosted in a personal computer has been used to implement the parallel approaches presented in this work.

4.2. Intrinsic Parallelism Approach

In this approach, the time-frequency distribution algorithms are carried out on single data-time segment of the signal. Parallelism is achieved by means of partitioning and allocating the composing processes of the algorithms on a number of processors. For the intrinsic parallelism approach, a 3-stages pipeline is proposed. The first stage calculates the analytic signal $x_a(t)$ of the real signal. The second stage calculates the generalized time-indexed auto-correlation function $R'_x(t,\omega)$ for $t=0$ of $x_a(t)$, this being the integral respect to μ in eq. (1).

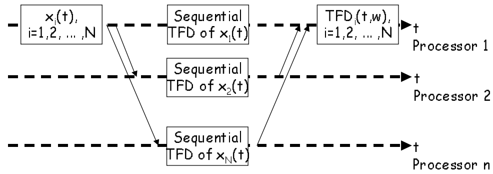


Fig. 6. Time diagram of the data parallel processing approach.

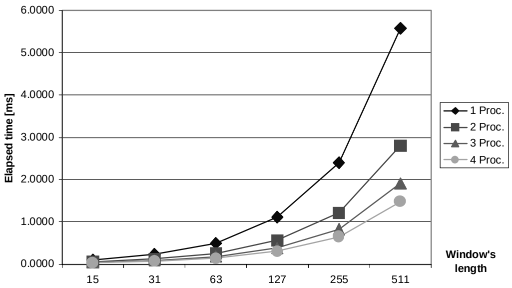
Finally, the third stage calculates the Fourier transform of $R'_x(0,\omega)$, this being the time-frequency distribution $TFD(t,\omega)$ for $t=0$ of the real signal. Figure 3 shows the pipeline structure of the process.

The calculations for first and third stages are relatively simple and a Fast Fourier Transform (FFT) algorithm is used. Second stage involves a more complex processing, then this stage is being paralleled by using a parallel farm computational model. Here, each node calculates a set of operations of the generalized time-indexed auto-correlation function. Although the expressions for the evaluation of each of the time-frequency distributions are different, the second stage can be adapted easily adding or subtracting processors according to the needs. Figure 4 shows the time diagram of the processes that are being executed on each pipeline stage.

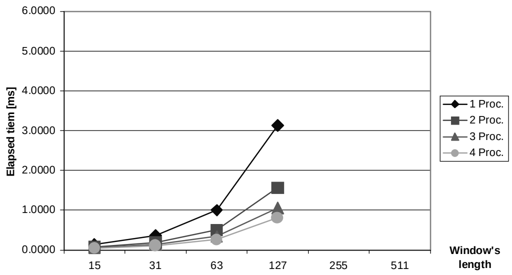
Figure 5 shows the time performance of the Wigner Ville and the Bessel distributions for two different parallel architectures (pipeline and farm+pipeline). Wigner Ville implemented on pipeline architecture performs better than the same implemented on farm+pipeline architecture; whereas Bessel implemented on farm+pipeline architecture performs better than the same implemented on a pipeline architecture. Observe that their generalized time-indexed auto-correlation functions are substantially different: Wigner Ville's one has a lower complexity than the Bessel's one. Finally, note that the Choi Williams and the Born Jordan distributions behave close to the Bessel distribution.

4.3. Data Parallelism Approach

This approach regards the execution of a number N of data-time segments in parallel using an array of N processors. Each processor has a copy of the whole three-stage algorithm described previously and executes the analysis on a dedicated segment. As one dedicated processor is used for process each segment, the number of data-time segments to be processed depends on the available processors. A parallel star computational model has been used with N nodes, where N is the number of data-time segment to be analyzed simultaneously. Figure 6 shows the time diagram of the processes than are being executed on each star's node.



a)



b)

Fig. 7. a) Execution time vs. signal's length of Wigner Ville distribution for data parallel architecture using 1 to 4 SHARC processors. b) Similar explanation for Bessel distribution.

Figure 7 shows the time performance for Wigner Ville and Bessel time-frequency distributions using this data parallel approach.

5. TRUNCATING THE DISCRETE TIME FREQUENCY DISTRIBUTION.

It is possible to achieve a further reduction in the calculation time of the Choi Williams, Bessel and Born Jordan discrete distributions by truncating the summation respect μ in eq. (11), (12) and (13) respectively.

Although such a truncation is destructive, it can be experimentally characterized. This work considers a simulated non-stationary Doppler signal proposed in (Cardoso, et. al., 1996), that is, a band limited stochastic signal with Gaussian probability density function which models a signal sampled at the center of a normal carotid artery. This signal is shown in figure 8 and its time frequency distribution in figure 9.

Previous work has established the optimal parameters that minimize both the instantaneous frequency and the RMS bandwidth estimation errors. They are $\sigma=5$, $\alpha=2$ and $\alpha=1$ for the Choi Williams, Bessel and Born Jordan distributions, respectively. Note that they depend on the considered signal.

Considering a signal with a SNR of 20dB and a window's length of 127, the following results have been obtained. A truncation of $\mu=20$, $\mu=40$ and $\mu=20$ in eq. (11), (12) and (13) respectively, results in an

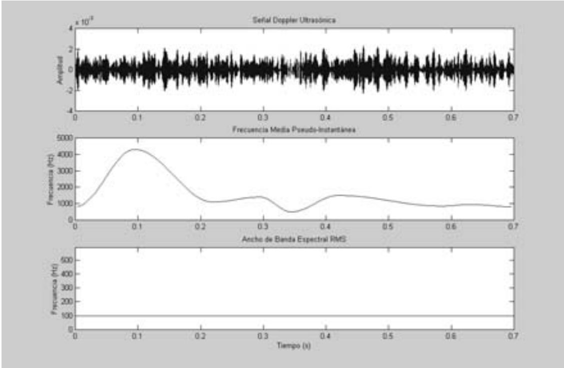


Fig. 8. Simulated non-stationary Doppler which models a signal sampled at the center of a normal carotid artery (up), its instantaneous frequency (center), and its RMS bandwidth (down).

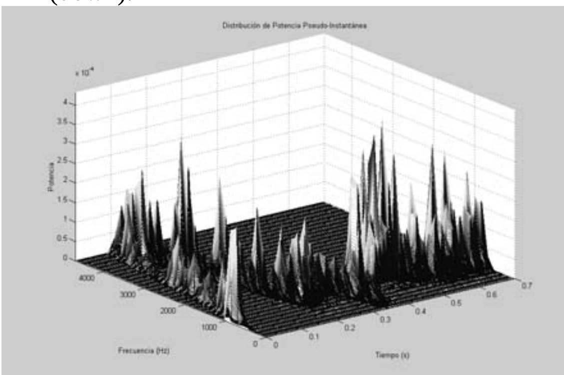


Fig. 9. Time frequency distribution of a simulated non-stationary Doppler which models a signal sampled at the center of a normal carotid artery.

increase of less than 5% in the estimation error of both the pseudo instantaneous mean frequency and the RMS bandwidth. The elapsed times involved in computation of the truncated distributions are only 86%, 98% and 97% of the elapsed time without truncation for eq. (11), (12) and (13) respectively. These results are shown in figures 10, 11 and 12. Similar results are valid for or noiseless signals.

6. CONCLUSIONS

Conventional methods for spectral estimation are limited to the analysis of stationary signals to produce a good estimate, however, these methods offer poor resolution when dealing with non-stationary signals. This paper has presented some alternative methods based on the so-called time frequency distributions for spectral analysis. Four methods based on the Cohen's class have been analyzed: Wigner Ville, Choi Williams, Bessel and Born Jordan distributions.

This work has proposed a simplification in the complexity of the expressions utilized for calculating the time-frequency distributions giving as a result a reduction of at least half the operations involved in the original definition (equations 10 to 13).

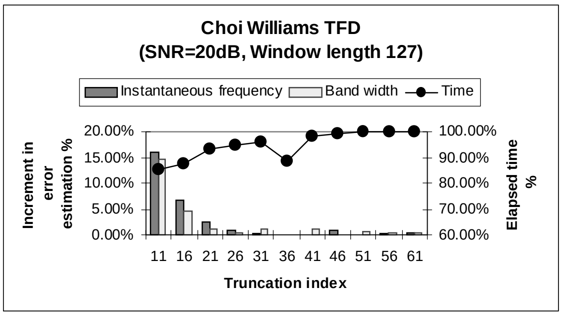


Fig. 10. Increment in error estimation (%) and elapsed time (%) vs. index truncation for Choi Williams TFD (1 SHARC processor).

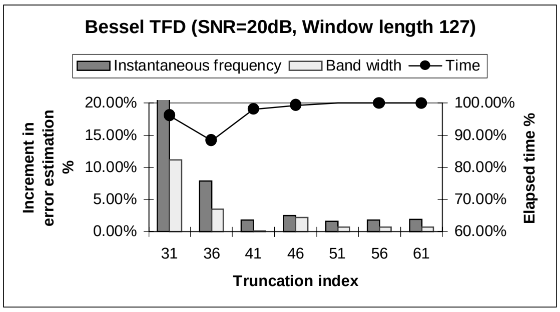


Fig. 11. Increment in error estimation (%) and elapsed time (%) vs. index truncation Bessel TFD (1 SHARC processor).

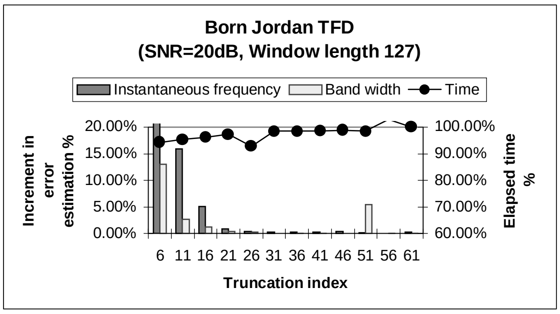


Fig. 12. Increment in error estimation (%) and elapsed time (%) vs. index truncation for Born Jordan TFD (1 SHARC processor).

Two approaches of parallelism have been presented: intrinsic parallelism and data parallelism (sections 4.2 and 4.3 respectively). In the first approach parallelism has been achieved by means of partitioning and allocating the composing processes of the time frequency distribution algorithms on a number of processors. A single data-time segment has being processed by a number of processors, each processor executing a dedicated part of the signal analysis algorithm. Here, a pipeline scheme with three stages is utilized, corresponding to the second stage to deal with the more expensive computational process (evaluation of the generalized time-indexed auto-correlation function). A generalized scheme has been implemented on a parallel DSP based system, which can be adapted easily according to the time-frequency distribution under consideration. As a result, the Wigner Ville distributions shows a better performance in a pipeline parallel architecture, while the Choi Williams, Bessel and Born Jordan distributions perform better in a farm+pipeline scheme.

The second parallel approach, data parallelism, deals with several data-time segments that are processed by a number of processors. Here, the whole signal analysis algorithm for each data-segment has been calculated by each processor. In contrast with the first approach, multiple data-time segments are processed in parallel. This approach has shown better flexibility and scalability than the intrinsic parallelism one. An advantage of this approach is that the communications between processors are reduced only to the distribution of the input data to each one of the processors, at the beginning of the loop, and at the end of the process when each processor sends back the results to the master processor, this leading to a higher performance than the intrinsic parallelism approach.

Also, this work has proposed a truncation in the simplified expressions in order to improve the time performance keeping a reasonable accurate in the spectral estimations (section 5). This approach is especially suitable for Choi Williams distribution.

Results are applied to the development of a real-time spectrum analyzer for Doppler blood flow instrumentation.

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