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HIERARCHICAL MULTI-RESOLUTIONAL OPTIMAL CONTROL SYSTEMS

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Abstract: Complex systems with a very large number of state and control variables are difficult to analyse. multi level hierarchical control structure is proposed in this document. A method for goal propagation from the upper levels to the lower levels in the hierarchy is suggested. Copyright © 2002 IFAC

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1. INTRODUCTION

The traditional control systems essentially solve the control problem at a single level. The goal or cost function formulation and the control problem solving take place at a single level. Such single level control architectures can be useful for small-scale systems which are described by a fewer numbers of states and control variables. But as the number of state and control variables increases, the system becomes more and more complex which makes the cost function formulation and the optimal control computations at a single level, difficult.

Some examples of highly complex systems can be the Air Traffic Control (ATC) system (Pappas, *et al*, 1997). and the Intelligent vehicle / Highway systems (IVHS) (Varaiya, 1993). In (Pappas, *et al*, 1997), a hierarchical Air Traffic Management (ATM) structure, consisting of four layers viz. strategic planner, tactical planner, trajectory planner and the regulation layer that reduces the complexity and assists the pilots to perform their tasks better, has been proposed.

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In (Varaiya, 1993)., the IVHS whose control module is a four layered hierarchical architecture, reduces highway congestion.

A large-scale system is characterized by a large amount of complexity both in terms of the state variables and the control functions. For optimal control applications, setting the goals for each variable independently is a tedious task. In the hierarchical systems, the upper level dynamics are obtained as a result of the aggregation of the subsequent lower level (Pappas, *et al*, 2000).

During aggregation, the aggregated system captures the complete system behaviour of the original system (Pappas and Sastry, 1999). The amount of complexity is considerably reduced as a result of the aggregation. The goal propagates from the upper levels in the hierarchy to the lower levels. Thus the lower levels with more number of state and control variables have to simply track the goals propagated from the top. Thus, an extensive computations at lower levels is minimized. Another reason for aggregation is that not every state and control variable may be accessible, in case of the original system. During aggregation, these states and control variables of the original system are combined in a way so that the states and control variables of the aggregated system are now accessible.

In this paper, the hierarchical control systems are investigated from a multi-resolutional perspective. Both, the states and control variables are aggregated. Once these aggregated variables are evaluated at the top level in the hierarchy, the goals propagate to the lower levels. The same problem is solved at different levels but with different resolutions (Meystel, Maximov, 1993) Computational complexity is reduced (Albus, *et at*, 1997) as a result of solving the problem first at a lower resolution and then simply tracking the goals at the higher resolution levels. The dynamics at each level is comprised of two parts. One part of the dynamics is due to the aggregation from the bottom level and the other part is as a result of the levels own dynamics (non aggregated dynamics). In the next section we describe the architecture of a two level multi-resolutional hierarchical system.

2. ARCHITECTURE OF MULTI-RESOLUTIONAL HIERARCHICAL SYSTEMS

The relationship between the levels of a multi-resolutional hierarchical system is presented in Fig-1. The ith level denotes the original system. The aggregated system is denoted by the *(i-1)^st^* level. The original system has *n* number of state variables and *p* number of control variables. It is assumed that *n* and *p* are large to make the system under consideration a complex one. The aggregated system has *m* number of state variables, of which m_1 are the aggregated state variables and m_2 are the non aggregated state variables ($(i-1)^{st}$ level's own states). Thus, $m = m_1 + m_2$. Similarly there are *q* number of control variables with q_1 number of aggregated control variables and q_2 number of non aggregated control variables ($(i-1)^{st}$ level's own control) and $q = q_1 + q_2$. The original system can then be aggregated using some aggregation maps. The aggregation maps are non-square matrices that act on the state and control vectors of the original system.

Each level dynamics is composed of two parts. In case of the upper level, a part of the total dynamics is due to the aggregation from lower level. This dynamics at upper level is obtained using the aggregation maps. The other part of the dynamics is the non aggregated dynamics of the upper level that is unknown to the aggregated dynamics. Similar argument can be extended to the total dynamics of the lower level .

The dynamics of the lower level is aggregated to generate the dynamics of the upper level. Due to this aggregation, there is a constraint on the lower levels, to track the trajectory of the upper levels. This constraint is the goal propagation from the upper levels to the lower levels

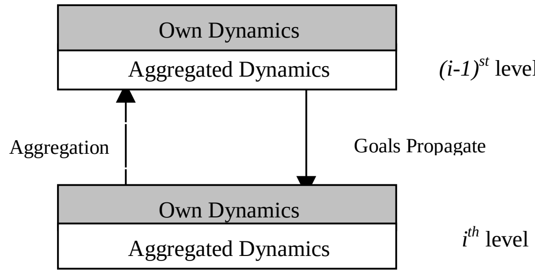

Fig-1. Schematic of a multi resolutional hierarchical system

Let the dynamics of the i^{th} level be given by

$$\dot{x}_i = A_i x_i + B_i u_i \qquad (1)$$

where x_i is the *n X 1* vector representing the states of the i^{th} level, and is of the form, $\begin{bmatrix} x_{1_i} & x_{2_i} & \cdots & x_{n_i} \end{bmatrix}^T$, where $x_i \in R^n$. u_i = *p X 1* vector representing the control variables of the i^{th} level, and is of the form, $\begin{bmatrix} u_{1_i} & u_{2_i} & \cdots & u_{3_p} \end{bmatrix}^T$, where $u_i \in R^p$. A_i and B_i are *n X n* and *n X p* matrices, of the i^{th} level. The i^{th} level is aggregated using the aggregation maps C_i^{i-1} and D_i^{i-1} such that

$$x_i^{i-1} = C_i^{i-1} x_i \qquad (2)$$

$$u_i^{i-1} = D_i^{i-1} u_i \qquad (3)$$

where C_i^{i-1} is *m X n*, matrix that aggregates the states of the i^{th} level x_i , to the states of the $(i\text{-}1)^{th}$ level and D_i^{i-1} is *q X p*, matrix that aggregates the control variables of the i^{th} level, u_i to the control variables of $(i\text{-}1)^{th}$ level.
The dynamics of the $(i\text{-}1)^{th}$ level can be formulated as,

$$\dot{x}_{i-1} = A_{i-1} x_{i-1} + B_{i-1} u_{i-1} \qquad (4)$$

where x_{i-1} is the state of the $(i\text{-}1)^{st}$ level and is of the form $x_{i-1} = \begin{bmatrix} x_i^{i-1} & x_{i-1}^{i-1} \end{bmatrix}^T$,where x_i^{i-1} is the state of the $(i\text{-}1)^{st}$ level, aggregated from the i^{th} level, such that $x_i^{i-1} = \begin{bmatrix} (x_i^{i-1})_1 & (x_i^{i-1})_2 & \cdots & (x_i^{i-1})_{m_1} \end{bmatrix}$. x_{i-1}^{i-1} is the non aggregated states of the $(i\text{-}1)^{st}$ level, such that $x_{i-1}^{i-1} = \begin{bmatrix} (x_{i-1}^{i-1})_1 & (x_{i-1}^{i-1})_2 & \cdots & (x_{i-1}^{i-1})_{m_2} \end{bmatrix}^T$. u_{i-1} is the control of the the $(i\text{-}1)^{th}$ level and is of the form $u_i = \begin{bmatrix} u_i^{i-1} & u_{i-1}^{i-1} \end{bmatrix}^T$ where u_i^{i-1} is the control of the $(i\text{-}1)^{th}$ level, aggregated from the i^{th} level, such that $u_i^{i-1} = \begin{bmatrix} (u_i^{i-1})_1 & (u_i^{i-1})_2 & \cdots & (u_i^{i-1})_{q_1} \end{bmatrix}$ and u_{i-1}^{i-1} is the non aggregated control of the $(i\text{-}1)^{th}$ level, such that $u_{i-1}^{i-1} = \begin{bmatrix} (u_{i-1}^{i-1})_1 & (u_{i-1}^{i-1})_2 & \cdots & (u_{i-1}^{i-1})_{q_2} \end{bmatrix}^T$
$A_{i-1} = f(A_i^{i-1}, A_{i-1}^{i-1})$, $B_{i-1} = f(B_i^{i-1}, B_{i-1}^{i-1})$ where A_i^{i-1}

and B_i^{i-1} are the system matrices due to aggregation and A_{i-1}^{i-1} and B_{i-1}^{i-1} are the system matrices due to the non aggregated dynamics of the *(i-1)*th level.

Using (1) and (4) we get

$$A_i^{i-1} = C_i^{i-1} A_i (C_i^{i-1})^{\#} \quad (5)$$

$$B_i^{i-1} = C_i^{i-1} B_i (D_i^{i-1})^{\#} \quad (6)$$

where $(C_i^{i-1})^{\#}$ and $(D_i^{i-1})^{\#}$ are the generalized inverses.

3. OPTIMAL CONTROL FORMULATION

In this section the optimal problem formulation is given for the multi-resolutional hierarchical system. Given the dynamics of the i^{th} level as in (1), our aim is to find x_i and u_i. A two level hierarchy is considered for simplicity. Since there is no further aggregation beyond the upper level, it is assumed that x_i^{i-1} and u_i^{i-1} has already been calculated. The goals propagate from the upper level to the lower level. For the sake of compatibility with the notations used so far, the upper level will be referred to as the *(i-1)*st level and the lower level as the i^{th} level. The cost function at the i^{th} level can be formulated as

$$J_i = \phi(t_f) + \int_0^{t_f} (\phi_i(t) + (x_i(t))^T Q_i(x_i(t)) + \varphi_i(t) + (u_i)^T R_i^{!}(u_i))dt \quad (7)$$

where

$$\phi_i(t) = \frac{1}{2}(C_i^{i-1} x_i(t) - x_i^{i-1}(t))^T S_i(t)(C_i^{i-1} x_i(t) - x_i^{i-1}(t)) \quad (8)$$

$$\varphi_i(t) = \frac{1}{2}(D_i^{i-1} u_i(t) - u_i^{i-1}(t))^T T_i(t)(D_i^{i-1} u_i(t) - u_i^{i-1}(t)) \quad (9)$$

The $\phi(t)$ term in the cost function signifies that x_i has to track the states x_i^{i-1} of the $(i-1)^{st}$ level. It is evident from *Section-2* that the term $\phi(t)$ is essential in the cost function to satisfy the aggregation relationship (2). Hence the states x_i^{i-1} of the *(i-1)*th level act as a goal setting values on the i^{th} level and it has to track these states. Similarly the $\varphi(t)$ term sets the goal values for the control variables of the i^{th} level. This term satisfies the control aggregation relationship (3). The matrices, $Q_i(t), R'_i(t), S_i(t), T_i(t)$ are the appropriate dimensional weighing functions in the cost function. It should be noted that certain states at bottom level track the trajectory of the upper level while the others can be independently made to go to zero. So there is a quadratic function of x_i in the cost function.

From the principles of optimal control (Lewis and Syrmos, 1995) the Hamiltonian for the i^{th} level can be formed as,

$$H_i(t) = J_i + \lambda_i(t)(A_i x_i(t) + B_i u_i(t)) \quad (10)$$

where $\lambda_i(t)$ is the Lagrange's multiplier and is assumed to be of the form (Lewis and Syrmos, 1995)

$$\lambda_i(t) = p_i(t) x_i(t) + h_i(t) \quad (11)$$

where $p_i(t)$ and $h_i(t)$ are *n X n* and is *n X 1* matrices respectively. To find the optimal control $u_i(t)$ at the i^{th} level, (10) is partially differentiated with respect to u_i and equated to zero. $u_i(t)$ is found to be of the form ,

$$u_i(t) = -R_i^{-1}(B_i^T \lambda_i(t) - (D_i^{i-1})^T T_i u_i^{i-1}(t)) \quad (12)$$

where,

$$R_i = R'_i + (D_i^{i-1})^T T_i D_i^{i-1} \quad (13)$$

For the rest of the optimal control formulation, the argument t, has been omitted with the variables *x, u, p and h* for simplicity.
From (11),

$$\dot{\lambda}_i(t) = \dot{p}_i x_i + p_i \dot{x}_i + \dot{h}_i \quad (14)$$

Substituting value of x_i from (1) in (14),

$$\dot{\lambda}_i(t) = \dot{p}_i x_i + p_i A_i x_i + p_i B_i u_i + \dot{h}_i \quad (15)$$

Substituting the value of u_i from (12) and $\lambda_i(t)$ from (11), in (15),

$$\dot{\lambda}_i(t) = p_i A_i x_i + p_i B_i(-R^{-1}{}_i B_i^T p_i^T x_i(t) - R_i^{-1} B_i^T h_i + R_i^{-1}(D_i^{i-1})^T T_i u_i^{i-1}) + \dot{p}_i x_i + \dot{h}_i \quad (16)$$

$\dot{\lambda}_i(t)$ can also be evaluated as, (Lewis and Syrmos, 1995)

$$\dot{\lambda}_i(t) = -\frac{\partial H_i}{\partial x_i} = -(A_i^T p_i x_i + A_i^T h_i + (C_i^{i-1})^T S_i C_i^{i-1} x_i - (C_i^{i-1})^T S_i x_i^{i-1} + Q_i x_i) \quad (17)$$

Equating (16) and (17) for the expressions of $\dot{\lambda}_i(t)$ and , rearranging the terms ,

$$(\dot{p}_i + p_i A_i + A_i^T p_i - p_i B_i R_i^{-1} B_i^T p_i + (C_i^{i-1})^T S_i C_i^{i-1} + Q_i) x_i + A_i^T h_i - p_i(t) B_i R_i^{-1} B_i^T h_i + p_i B_i R_i^{-1} (D_i^{i-1})^T T_i u_i^{i-1} - (C_i^{i-1})^T S_i x_i^{i-1} + \dot{h}_i = 0 \quad (18)$$

Hence from (18),

$$\dot{p}_i = -p_i A_i - A_i^T p_i + p_i B_i R_i^{-1} B_i^T p_i - (C_i^{i-1})^T S_i C_i^{i-1} - Q_i \quad (19)$$

$$\dot{h}_i = -A_i^T h_i + p_i(t) B_i R_i^{-1} B_i^T h_i - p_i B_i R_i^{-1} (D_i^{i-1})^T T_i u_i^{i-1} + (C_i^{i-1})^T Q_i x_i^{i-1} \quad (20)$$

Equations (19) and (20) can be solved to get $p_i(t)$ and $h_i(t)$. The following boundary conditions (Lewis and Syrmos, 1995) may be used for solving (19) and (20)

$$p_i(t_f) = \frac{\partial H_i}{\partial x_i(t_f)} = (C_i^{i-1})^T S_i(t_f) C_i^{i-1} \quad (21)$$

$$h_i(t_f) = -C_i^{i-1} S_i(t_f) x_i^{i-1} \quad (22)$$

Once $p_i(t)$ and $h_i(t)$ is obtained, it can be used to solve for $\lambda_i(t)$ from (11). $\lambda_i(t)$ can be used to solve for the optimal control $u_i(t)$ from (12). Once $u_i(t)$ has been calculated, (1) can be solved for $x_i(t)$.

Some comments on selecting the form for $C_i^{i-1}, Q_i, R'_i, S_i$ and T_i are in order. From (8), it is evident that the term $\phi_i(t)$ in the cost function (7) makes $C_i^{i-1} x_i(t)$ follow $x_i^{i-1}(t)$, i.e. the states of the lower level try to track the aggregated trajectory of the upper level. So $\phi_i(t)$ in the cost function (7) tries to keep the cost of tracking to a minimum. However the other term in the cost function (7), $(x_i)^T Q_i (x_i)$ tries to drive $x_i(t)$ to zero. Thus there is a conflict between these two terms in the cost function (7). Similar conflicting case is also found with control as the term $\varphi_i(t)$ tries to keep the cost of tracking the aggregated control to minimum while the term $(u_i)^T R'_i u_i$, tries to drive u_i to zero. This conflicting situation results in an improper tracking of the aggregated states and aggregated control of the upper level. To avoid such conflict, it is essential to choose the range space of $(C_i^{i-1})^T S_i C_i^{i-1}$, i.e the weighing function for x_i after simplifying (8), different from the range space of Q_i. So those states of x_i that track the aggregated trajectory of the upper level will be different from those in the term $(x_i(t))^T Q_i (x_i(t))$. Thus the cost function (7) is minimized, as a result of some states following the aggregated trajectory of the upper level and thus minimizing $\phi_i(t)$, while the remaining states being driven to zero and thus minimizing the term $(x_i)^T Q_i (x_i)$. Any conflicts can thus be avoided and the tracking is proper.

4. SIMULATION RESULTS

In this section some simulation results on the hierarchical system are given. A two level subsystem is considered. The lower level is the i^{th} level. The upper level is the *(i-1)*st level. At the i^{th} level, there are 4 state variables *(n=4)* and 4 control variables *(p=4)*. At the *(i-1)*th level, there are 2 state variables *(m=2)* and 1 control variable *(q=1)*. It is assumed that the siystem is complex to solve at a single level. For the sake of simplicity it is assumed that there are no non-aggregated states at both the levels. Hence $A_{i-1} = A_i^{i-1}$ and $B_{i-1} = B_i^{i-1}$. The i^{th} level has the following dynamics

$$A_i = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 2 & 0 & 1 & 2 \end{bmatrix}, \; B_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (23)$$

and our aim is to find x_i and u_i using the principles of hierarchical control discussed in *Section 3.*

Next, this system is aggregated. Since the upper level has two aggregated state variables and one aggregated control variable the dimensions of C_i^{i-1} and D_i^{i-1} matrix are *2 X 4 and 1 X 4* respectively. Some caution should be followed in choosing these aggregation matrices, as they have to be of full rank, so that the system properties like controllability, observability and stability are preserved in the aggregated system (Anand Dasgupta, *et.al*, Submitted to ACC 2002). The aggregation matrices are,

$$C_i^{i-1} = \begin{bmatrix} 2 & 0.25 & 0 & 0 \\ 0 & 0 & 0.25 & 2 \end{bmatrix} \quad D_i^{i-1} = \begin{bmatrix} 0.25 & 0.25 & 0.25 & 0.25 \end{bmatrix} \quad (24)$$

Using the aggregation relation (2), $x_i^{i-1}(1) = 2x_i(1) + 0.25x_i(2)$ and $x_i^{i-1}(2) = 0.25x_i(3) + 2x_i(4)$. Similarly using (3), $u_i^{i-1} = 0.25u_i(1) + 0.25u_i(2) + 0.25u_i(3) + 0.25u_i(4)$. The term, $2x_i(1) + 0.25x_i(2)$ will be referred as '*Track1*' and $x_i^{i-1}(1)$ as 'X1'and $0.25x_i(3) + 2x_i(4)$ as '*Track2*' and $x_i^{i-1}(2)$ as 'X2'. Similarly we will refer $0.25u_i(1) + 0.25u_i(2) + 0.25u_i(3) + 0.25u_i(4)$ as '*Tracku*' and u_i^{i-1} as 'U1'.

The dynamics of the *(i-1)*st level is then obtained from (5) and (6) as,

$$A_{i-1} = \begin{bmatrix} 0.1231 & 0.0154 \\ 2 & 2.2154 \end{bmatrix}, B_{i-1} = \begin{bmatrix} 2.25 \\ 2.25 \end{bmatrix} \quad (25)$$

As there are only two levels, the *(i-1)*th level is the topmost level. This level formulates the overall goal for the system. The cost function at this level is given in the most general form as,

$$J_{i-1} = (x_i^{i-1}(t_f))^T S_{i-1}(t_f) x_i^{i-1}(t_f) + \int_0^{\infty} ((x_i^{i-1}(t))^T Q_{i-1} x_i^{i-1}(t) \tag{26}$$
$$+ u_i^{i-1}(t) R_{i-1} u_i^{i-1}(t))dt$$

where

$$Q_{i-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ R_{i-1} = [1] \text{ and } S_{i-1}(t_f) = [1] \tag{27}$$

A LQR model is used to solve for $x_i^{i-1}(t)$ and $u_i^{i-1}(t)$. Next, the goal propagates from the *(i-1)*th level to the i^{th} level. The i^{th} level has to satisfy the aggregation relation (2) and (3). The cost function of the i^{th} level is similar to (7). The various weighing functions, chosen are as follows

$$Q_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1000 & 0 & 0 \\ 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{28}$$

$$R'_i = \begin{bmatrix} 1.9 & 0 & 0 & 0 \\ 0 & 1.9 & 0 & 0 \\ 0 & 0 & 1.9 & 0 \\ 0 & 0 & 0 & 1.9 \end{bmatrix} \tag{29}$$

$$S_i = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix} \tag{30}$$

$$T_i = [1] \tag{31}$$

The i^{th} level, with the dynamics as in (23) is then solved to minimize the cost function (7). The Matrix-Riccatti equations (19) and (20) are solved backwards, with appropriate boundary conditions for t_f =10 sec . *Fig-2a* and *Fig-2b* shows the plots of the states of *(i-1)*st level as tracked by those of the i^{th} level, to satisfy the aggregation relation (2) . *Fig-2a* shows how '*Track1*' follows '*X1*'

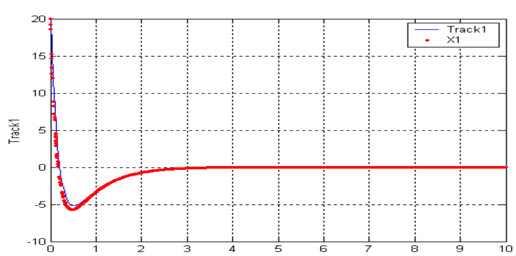

Fig- 2a.' Track1' following 'X1'

Fig-2b shows how '*Track2*' follows '*X2*'. '*X1*' and '*X2*', are obtained by solving the state equations.

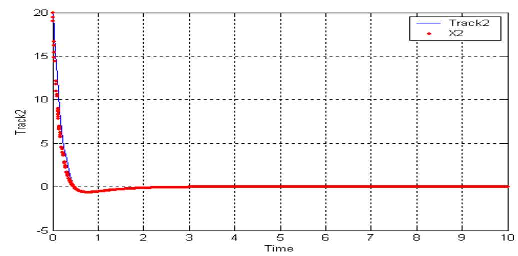

Fig- 2b.' Track2' following 'X2'

Fig-2c shows how *'Tracku'* follows *'U1'*. *Fig-2d* shows the trajectory of the individual control variables. The control variables are obtained by solving (12). Next (1) is solved to get the individual trajectories of the states of the i^{th} level *(Fig-2e).* These are obtained by solving (1), with the optimal control formulation as discussed in *Section-3*. We solved the complex problem using the principles of multi-resolutional hierarchical control. Thus extensive calculations with a large number of states can be avoided using the principles of aggregation and multi-resolution.

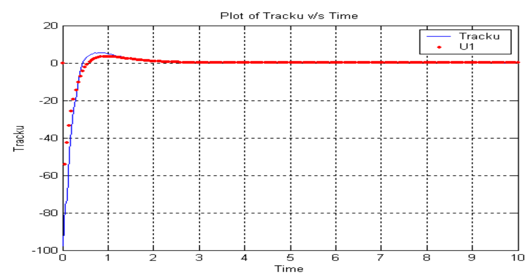

Fig- 2c. 'Tracku' following 'U1'

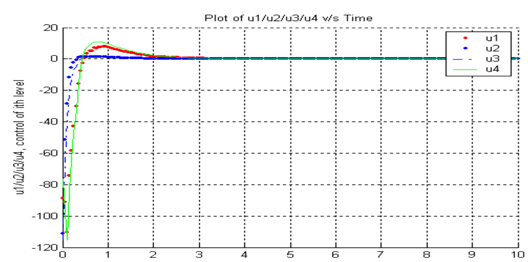

Fig- 2d. Control of the i^{th} level

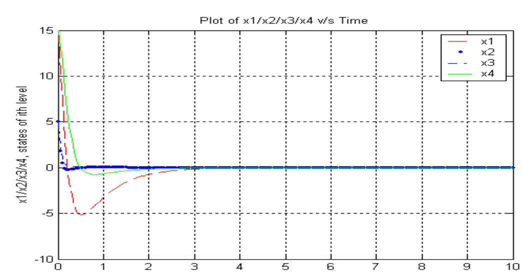

Fig-2e. States of the i^{th} level

Next the range space of the terms, $\phi_i(t)$ and $(x_i)^T Q_i (x_i)$ is found out. The subspace of $\phi_i(t)$ will be the range space of the weighing function $(C_i^{i-1})^T S_i C_i^{i-1}$.

$$(C_i^{i-1})^T S_i C_i^{i-1} = \begin{bmatrix} 1200 & 150 & 0 & 0 \\ 150 & 18.7 & 0 & 0 \\ 0 & 0 & 18.7 & 150 \\ 0 & 0 & 150 & 1200 \end{bmatrix} \qquad (32)$$

From (32) it is clear that this weighing function, penalizes the first and the last states more. Since $\phi_i(t)$, is the cost of tracking, the first and the last states, track the aggregated trajectory $x_i^{i-1}(t)$. The second and third state variables do not play a major role in tracking. The range space of $(x_i)^T Q_i (x_i)$ is the range space of Q_i. From (28), it follows that Q_i penalizes the second and third state variables much more then the first and the last state variables of x_i. Thus the term $(x_i)^T Q_i (x_i)$ tries to drive the second and third state variables to zero. Thus the cost function is minimized first by the first and last state variables that track the aggregated trajectory of the upper level and then by the second and third state variables being driven to zero.

Next lets choose Q_i of the form

$$Q_i = \begin{bmatrix} 1000 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1000 \end{bmatrix} \qquad (33)$$

Now the first and the last states are driven to zero. Thus there is a conflict with the $\phi_i(t)$ term in the cost function, as $\phi_i(t)$ tries to make the first and the last states track the aggregated trajectory of the upper level. The tracking in this case is not proper as evident from *Fig –2f* and *Fig-2g.*Thus to avoid conflicts of such form the range spaces of the weighing functions should be properly chosen.

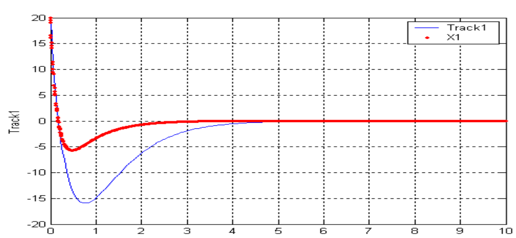

Fig-2f. Improper Tracking by 'Track1' of 'X1'

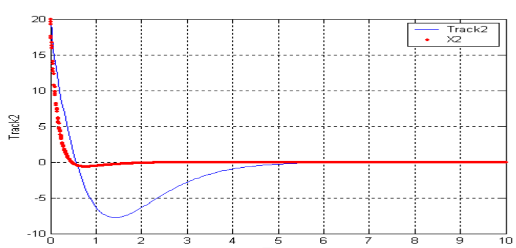

Fig-2g. Improper Tracking by 'Track2' of 'X2'

5. CONCLUSIONS

Some concepts about hierarchical multi-resolution systems were introduced in this paper. Aggregation of states and control are essential in the case of large scale systems in order to avoid computational complexity in solving the problem. The resolution of the hierarchical system is lower at the upper level and higher at the bottom level. Reduction in complexity is obtained by solving the system at the higher level at a lower resolution. The system goals then propagate from the higher levels to the lower levels in the hierarchy. The lower level (higher resolution) has to simply track the goals (trajectory) of the upper level. A two level system was simulated to demonstrate these principles.

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