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SYNCHRONIZATION OF TWO HYPERCHAOTIC CHUA CIRCUITS: A GENERALIZED HAMILTONIAN SYSTEMS APPROACH [1]

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Abstract: In this paper, we use a Generalized Hamiltonian systems approach to synchronize two unidirectionally coupled hyperchaotic Chua circuits. Synchronization is obtained of transmitter and receiver dynamics in case the receiver is given via an observer. We apply this approach to transmit digital information signals in which the quality of the recovered signal is higher than in traditional observer techniques while the encoding remains potentially secure. *Copyrigth © IFAC 2002*

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1. INTRODUCTION

The application of chaotic synchronization to secure communication systems was suggested in an earlier article by (Pecora and Carroll 1990). Ever since the appearance of that paper, synchronization of chaotic systems has received a lot of attention and interest. Different approaches are being currently proposed and pursued (see, for instance, e.g., (Kocarev *et al.*, 1992; Ogorzalek, 1993; Wu and Chua, 1993; Ding and Ott, 1994; Feldmann, *et al.*, 1996; Nijmeijer and Mareels, 1997; Special Issue, 1997; Cruz and Nijmeijer, 2000; Sira-Ramírez and Cruz, 2001; Pikovsky *et al.*, 2001) and the references therein).

Data encryption using chaotic dynamics was reported in the early 1990s as a new approach for signal encoding which differs from the conventional methods using numerical algorithms as the encryption key. As a result, the synchronization of chaotic systems plays an important role in the area of chaotic communications. The issue of security, however, naturally arises in the consideration of chaotic communication and it constitutes an

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important motivation for chaotic communication research. In particular, several techniques, such as chaotic masking (Cuomo *et al.*, 1993), chaotic switching (Parlitz *et al.*, 1992; Cuomo *et al.*, 1993; Dedieu *et al.*, 1993) and chaotic parameter modulation (Yang and Chua, 1996) have been developed. However, subsequent works have shown that these techniques have a low degree of security (see (Short, 1996; Perez and Cerdeira, 1995; Yang, 1995)).

Two factors that are of primary importance in security considerations related to chaotic communication systems. These are; the dimensionality of the chaotic attractor and the effort required to obtain the necessary parameters for the matching of a receiver dynamics.

One way to enhance the level of security of the communication system can consist in applying proper cryptographic techniques to the information signal (Yang *et al.*, 1997). Another way to solve this security problem is to encode the message by using high dimensional chaotic attractors, or hyperchaotic atractrors, which take advantage of the increased randomness and unpredictability of the higher dimensional dynamics. In such option one generally encounters multiple positive Lyapunov exponents. However, the synchronization of hyperchaotic systems is a much more difficult problem (Brucoli *et al.*, 1996; Peng *et al.*, 1996).

The objective of this paper is to extend the approach developed in (Sira-Ramírez and Cruz, 2001) to the synchronization of hyperchaotic circuits through a Generalized Hamiltonian systems approach. In this work we show that the synchronization of two hyperchaotic Chua circuits is possible from this viewpoint and, moreover, we proceed to apply this approach to transmit digital information signals using chaotic parameter switching. We can enumerate several advantages over the existing methods:

- it enables synchronization in a systematic way;
- it can be successfully applied to several well-known hyperchaotic oscillators (e.g., Rössler's hyperchaotic system);
- it does not require the computation of any Lyapunov exponent;
- it does not require initial conditions belonging to the same basin of attraction.

2. SYNCHRONIZATION OF HYPERCHAOTIC CHUA'S CIRCUITS

Consider the following n-dimensional autonomous system

$$\dot{x} = f(x), \qquad x \in \mathbb{R}^n \tag{1}$$

which represents a circuit exhibiting a *hyperchaotic* behavior. Following the approach provided in (Sira-Ramírez and Cruz, 2001), many physical systems described by Eq. (1) can be written in the following "Generalized Hamiltonian" Canonical form,

$$\dot{x} = \mathcal{J}(x)\frac{\partial H}{\partial x} + \mathcal{S}(x)\frac{\partial H}{\partial x} + \mathcal{F}(x), \tag{2}$$

where $H(x)$ denotes a smooth *energy function* which is globally positive definite in $\mathbb{R}^n$. The column *gradient vector* of H, denoted by $\partial H/\partial x$, is assumed to exist everywhere. We frequently use quadratic energy function $H(x) = 1/2\, x^T \mathcal{M} x$ with $\mathcal{M}$ being a, constant, symmetric positive definite matrix. In such case, $\partial H/\partial x = \mathcal{M}x$. The square matrices, $\mathcal{J}(x)$ and $\mathcal{S}(x)$ satisfy, for all $x \in \mathbb{R}^n$, the following properties, which clearly depict the *energy managing* structure of the system, $\mathcal{J}(x) + \mathcal{J}^T(x) = 0$ and $\mathcal{S}(x) = \mathcal{S}^T(x)$. The vector field $\mathcal{J}(x)\,\partial H/\partial x$ exhibits the *conservative* part of the system and it is also referred to as the *workless* part, or *work-less* forces of the system; and $\mathcal{S}(x)$ depicting the *working* or *nonconservative* part of the system. For certain systems, $\mathcal{S}(x)$ is *negative definite* or *negative semidefinite*. In such cases, the vector field is addressed to as the *dissipative* part of the system. If, on the other hand, $\mathcal{S}(x)$ is positive definite, positive semidefinite, or indefinite, it clearly represents, respectively, the global, semi-global and local *destabilizing* part of the system. In the last case, we can always (although nonuniquely) descompose such an indefinite symmetric matrix into the sum of a symmetric negative semidefinite matrix $\mathcal{R}(x)$ and a symmetric positive semidefinite matrix $\mathcal{N}(x)$. And where $\mathcal{F}(x)$ represents a *locally destabilizing* vector field.

We consider a special class of Generalized Hamiltonian systems with destabilizing vector field and linear output map , y, given by

$$\begin{aligned} \dot{x} &= \mathcal{J}(y)\frac{\partial H}{\partial x} + (\mathcal{I} + \mathcal{S})\frac{\partial H}{\partial x} + \mathcal{F}(y), \qquad x \in \mathbb{R}^n \\ y &= \mathcal{C}\frac{\partial H}{\partial x}, \qquad y \in \mathbb{R}^m \end{aligned} \tag{3}$$

where $\mathcal{S}$ is a constant symmetric matrix, not necessarily of definite sign. The matrix $\mathcal{I}$ is a constant skew symmetric matrix. The vector variable y is referred to as the system *output*. The matrix $\mathcal{C}$ is a constant matrix.

We denote the *estimate* of the state vector x by ξ, and consider the Hamiltonian energy function $H(\xi)$ to be the particularization of H in terms of ξ. Similarly, we denote by η the estimated output, computed in terms of the estimated state ξ. The gradient vector $\partial H(\xi)/\partial \xi$ is, naturally, of the form $\mathcal{M}\xi$.

A dynamic nonlinear state observer for the system (3) is readily obtained as

$$\begin{aligned}\dot{\xi} &= \mathcal{J}(y)\frac{\partial H}{\partial \xi} + (\mathcal{I}+\mathcal{S})\frac{\partial H}{\partial \xi} + \mathcal{F}(y) + K(y-\eta) \\ \eta &= \mathcal{C}\frac{\partial H}{\partial \xi} \end{aligned} \quad (4)$$

where K is a constant matrix, known as the *observer gain.*

The state estimation error, defined as $e = x - \xi$ and the output estimation error, defined as $e_y = y - \eta$, are governed by

$$\begin{aligned}\dot{e} &= \mathcal{J}(y)\frac{\partial H}{\partial e} + (\mathcal{I}+\mathcal{S}-K\mathcal{C})\frac{\partial H}{\partial e}, \qquad e \in \mathbb{R}^n \\ e_y &= \mathcal{C}\frac{\partial H}{\partial e}, \quad e_y \in \mathbb{R}^m \end{aligned} \quad (5)$$

where the vector, $\partial H/\partial e$ actually stands, with some abuse of notation, for the gradient vector of the *modified* energy function, $\partial H(e)/\partial e = \partial H/\partial x - \partial H/\partial \xi = \mathcal{M}(x-\xi) = \mathcal{M}e$. Below, we set, when needed, $\mathcal{I}+\mathcal{S} = \mathcal{W}$. We say that the receiver system (4) synchronizes with the transmitter system (3), if $e(t) \to 0$ as $t \to \infty$.

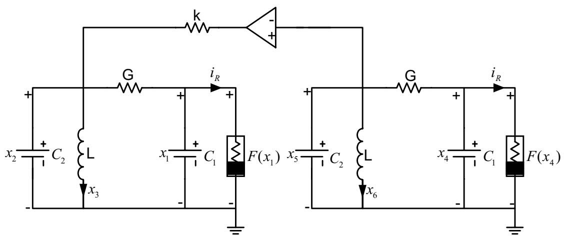

Fig. 1. Pair of identical unidirectionally coupled Chua circuits.

Two coupled Chua circuits (see, (Anishchenko *et al.*, 1994; Brucoli *et al.*, 1996)). The hyperchaotic circuit considered in this paper is formed by a pair of unidirectionally coupled identical Chua circuits as shown in Fig. 1. This circuit is described by

$$\begin{aligned} C_1\dot{x}_1 &= G(x_2 - x_1) - F(x_1), \\ C_2\dot{x}_2 &= G(x_1 - x_2) + x_3 + C_2 k (x_5 - x_2), \\ L\dot{x}_3 &= -x_2, \\ C_1\dot{x}_4 &= G(x_5 - x_4) - F(x_4), \\ C_2\dot{x}_5 &= G(x_4 - x_5) + x_6, \\ L\dot{x}_6 &= -x_5 \end{aligned} \quad (6)$$

where

$$F(x_1) = bx_1 + \frac{1}{2}(a-b)(|x_1+1| - |x_1-1|), a, b < 0,$$
$$F(x_4) = bx_4 + \frac{1}{2}(a-b)(|x_4+1| - |x_4-1|), a, b < 0.$$

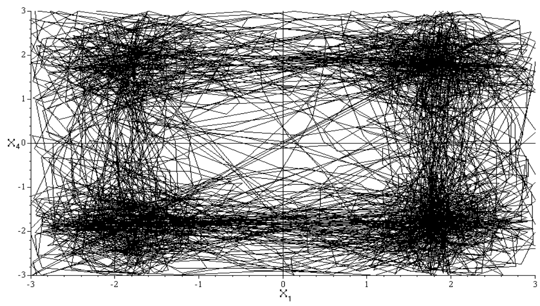

Fig. 2. Double-double scroll attractor in hyperchaotic Chua circuit with $k = 0.02$, $\alpha = 10$, $\beta = 14.87$, $a = -1.27$ and $b = -0.68$.

The parameter k individualizes the unidirectional coupling between the two Chua circuits.

The set of differential equations describing hyperchaotic Chua circuit in Hamiltonian Canonical form with a destabilizing field is given by

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\\ \dot{x}_5\\ \dot{x}_6\end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{C_2}k & \frac{1}{LC_2} & 0 & \frac{1}{2C_2}k & 0 \\ 0 & -\frac{1}{LC_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2C_2}k & 0 & 0 & 0 & \frac{1}{LC_2} \\ 0 & 0 & 0 & 0 & -\frac{1}{LC_2} & 0 \end{bmatrix}\frac{\partial H}{\partial x}$$
$$+ \begin{bmatrix} -\frac{G}{C_1^2} & \frac{G}{C_1C_2} & 0 & 0 & 0 & 0 \\ \frac{G}{C_1C_2} & -\frac{G}{C_2^2} & 0 & 0 & \frac{1}{2C_2}k & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{G}{C_1^2} & \frac{G}{C_1C_2} & 0 \\ 0 & \frac{1}{2C_2}k & 0 & \frac{G}{C_1C_2} & -\frac{G}{C_2^2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}\frac{\partial H}{\partial x}$$
$$+ \begin{bmatrix} -\frac{1}{C_1}F(x_1) \\ 0 \\ 0 \\ -\frac{1}{C_1}F(x_4) \\ 0 \\ 0 \end{bmatrix} \quad (7)$$

taking as the Hamiltonian energy function the scalar function

$$H(x) = \frac{1}{2}\left[C_1x_1^2 + C_2x_2^2 + Lx_3^2 + C_1x_4^2 + C_2x_5^2 + Lx_6^2\right] \quad (8)$$

and gradient vector as

$$\frac{\partial H}{\partial x} = \left[\, C_1x_1 \; C_2x_2 \; Lx_3 \; C_1x_4 \; C_2x_5 \; Lx_6 \,\right]^T$$

The destabilizing vector field evidently calls for x_1 and x_4 to be used as the outputs, y_1 and y_2, of the transmitter (7). The matrices $\mathcal{C}$, $\mathcal{S}$ and $\mathcal{I}$, are given by

$$\mathcal{C}=\begin{bmatrix} \frac{1}{C_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{C_1} & 0 & 0 \end{bmatrix},$$

$$\mathcal{S}=\begin{bmatrix} -\frac{G}{C_1^2} & \frac{G}{C_1C_2} & 0 & 0 & 0 & 0 \\ \frac{G}{C_1C_2} & -\frac{G}{C_2^2} & 0 & 0 & \frac{1}{2C_2}k & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{G}{C_1^2} & \frac{G}{C_1C_2} & 0 \\ 0 & \frac{1}{2C_2}k & 0 & \frac{G}{C_1C_2} & -\frac{G}{C_2^2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathcal{I}=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{C_2}k & \frac{1}{LC_2} & 0 & \frac{1}{2C_2}k & 0 \\ 0 & -\frac{1}{LC_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2C_2}k & 0 & 0 & 0 & \frac{1}{LC_2} \\ 0 & 0 & 0 & 0 & -\frac{1}{LC_2} & 0 \end{bmatrix}.$$

The pair $(\mathcal{C},\mathcal{S})$ is neither observable nor detectable. However, the pair $(\mathcal{C},\mathcal{W})$ is observable. The system lacks damping in the x_3 and x_6 variables, and either in the x_1, x_2, x_4 or the x_5 variable as inferred from the negative semi-definite nature of the dissipation structure matrix, $\mathcal{S}$. If x_1 and x_4 are used as outputs, then the outputs error injection terms can enhance the dissipation in the error state dynamics. The receiver is designed as

$$\begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \\ \dot{\xi}_3 \\ \dot{\xi}_4 \\ \dot{\xi}_5 \\ \dot{\xi}_6 \end{bmatrix}=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{C_2}k & \frac{1}{LC_2} & 0 & \frac{1}{2C_2}k & 0 \\ 0 & -\frac{1}{LC_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2C_2}k & 0 & 0 & 0 & \frac{1}{LC_2} \\ 0 & 0 & 0 & 0 & -\frac{1}{LC_2} & 0 \end{bmatrix}\frac{\partial H}{\partial \xi}$$
$$+\begin{bmatrix} -\frac{G}{C_1^2} & \frac{G}{C_1C_2} & 0 & 0 & 0 & 0 \\ \frac{G}{C_1C_2} & -\frac{G}{C_2^2} & 0 & 0 & \frac{1}{2C_2}k & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{G}{C_1^2} & \frac{G}{C_1C_2} & 0 \\ 0 & \frac{1}{2C_2}k & 0 & \frac{G}{C_1C_2} & -\frac{G}{C_2^2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}\frac{\partial H}{\partial \xi}$$
$$+\begin{bmatrix} -\frac{1}{C_1}F(y_1) \\ 0 \\ 0 \\ -\frac{1}{C_1}F(y_2) \\ 0 \\ 0 \end{bmatrix}+\begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \\ k_5 \\ k_6 \end{bmatrix}e_y \tag{9}$$

The choice of k_i, $i=1,2,...,6$ as arbitrary strictly positive constants suffices to guarantee the asymptotic exponential stability to zero of the synchronization error. The synchronization error dynamics is governed by

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \\ \dot{e}_4 \\ \dot{e}_5 \\ \dot{e}_6 \end{bmatrix}=\begin{bmatrix} 0 & \frac{1}{2C_1}k_2 & \frac{1}{2C_1}k_3 & 0 & 0 & 0 \\ -\frac{1}{2C_1}k_2 & -\frac{2}{C_2}k & \frac{2}{LC_2} & 0 & \frac{1}{C_2}k & 0 \\ -\frac{1}{2C_1}k_3 & -\frac{2}{LC_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2C_1}k_5 & \frac{1}{2C_1}k_6 \\ 0 & -\frac{1}{C_2}k & 0 & -\frac{1}{2C_1}k_5 & 0 & \frac{2}{LC_2} \\ 0 & 0 & 0 & -\frac{1}{2C_1}k_6 & -\frac{2}{LC_2} & 0 \end{bmatrix}\frac{\partial H}{\partial e}$$
$$+\begin{bmatrix} \varphi & \psi & \phi & 0 & 0 & 0 \\ \psi & -\frac{G}{C_2^2} & 0 & 0 & \frac{1}{2C_2}k & 0 \\ \phi & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & \vartheta & \sigma \\ 0 & \frac{1}{2C_2}k & 0 & \vartheta & -\frac{G}{C_2^2} & 0 \\ 0 & 0 & 0 & \sigma & 0 & 0 \end{bmatrix}\frac{\partial H}{\partial e} \tag{10}$$

where

$$\varphi=-\frac{G+k_1C_1}{C_1^2},\ \psi=\frac{2G-k_2C_2}{2C_1C_2},\ \phi=-\frac{1}{2C_1}k_3$$
$$\mu=-\frac{G+k_4C_1}{C_1^2},\ \vartheta=\frac{2G-k_5C_2}{2C_1C_2},\ \sigma=-\frac{1}{2C_1}k_6$$

To ease the simulations we resorted the following normalized version of the hyperchaotic circuit (see (Anishchenko *et al.*, 1994))

$$\begin{aligned} \dot{x}_1 &= \alpha\left(x_2-x_1-f\left(x_1\right)\right), \\ \dot{x}_2 &= x_1-x_2+x_3+k\left(x_5-x_2\right), \\ \dot{x}_3 &= -\beta x_2, \\ \dot{x}_4 &= \alpha\left(x_5-x_4-f\left(x_4\right)\right), \\ \dot{x}_5 &= x_4-x_5+x_6, \\ \dot{x}_6 &= -\beta x_5 \end{aligned} \tag{11}$$

where

$$f\left(x_1\right)=bx_1+\frac{1}{2}\left(a-b\right)\left(|x_1+1|-|x_1-1|\right),$$
$$f\left(x_4\right)=bx_4+\frac{1}{2}\left(a-b\right)\left(|x_4+1|-|x_4-1|\right).$$

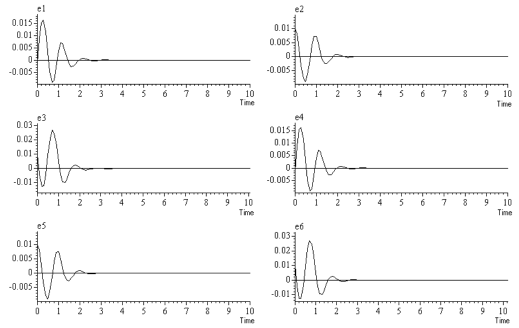

Fig. 3. Synchronization error evolution $e_i = x_i - \xi_i$, $i = 1, 2, ..., 6$ between (9) and (11).

The transmitter and the receiver are both characterized by the following parameter values: $\alpha = 10$, $\beta = 14.87$, $a = -1.27$, $b = -0.68$ and $k = 0.02$; and by the initial conditions:

$$x(0) = (0.01, 0.01, 0.01, 0.011, 0.01, 0.01),$$
$$\xi(0) = (0.011, 0, 0, 0.012, 0, 0).$$

These values assure the existence of the system hyperchaotic behavior. Figure 2 shows the double-double scroll attractor in hyperchaotic Chua circuit (11). While Figure 3 shows the synchronization error evolution ($e_i = x_i - \xi_i$, $i = 1, 2, ..., 6$) between transmitter system (7) and receiver system (9) for the observer gain $k_1 = 2$, $k_2 = 3$, $k_3 = 0$, $k_4 = 2$, $k_5 = 3$, and $k_6 = 0$. We can see, after some transient behavior, that all the hyperchaotic states of transmitter (7) synchronize with the corresponding states of receiver (9).

3. SYNCHRONIZATION STABILITY ANALYSIS

In this section, we examine the stability of the synchronization error (10) between hyperchaotic Chua circuit (7) and observer (9). A necessary and sufficient condition for global asymptotic stability to zero of the estimation error is given by the following theorem.

Theorem 1. (Sira-Ramírez and Cruz, 2001). The state x of the nonlinear system (7) can be globally, exponentially, asymptotically estimated, by the state ξ of the observer (9) if and only if there exists a constant matrix K such that the symmetric matrix

$$[\mathcal{W} - K\mathcal{C}] + [\mathcal{W} - K\mathcal{C}]^T = [\mathcal{S} - K\mathcal{C}] + [\mathcal{S} - K\mathcal{C}]^T = 2\left[\mathcal{S} - \frac{1}{2}\left(K\mathcal{C} + \mathcal{C}^T K^T\right)\right]$$

is negative definite.

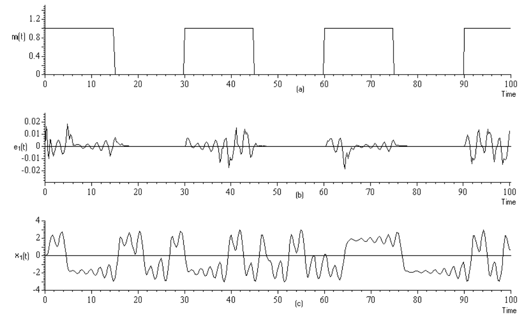

Fig. 4. (a) Digital information signal $m(t)$, (b) synchronization error detection $e_1(t)$, (c) transmitted hyperchaotic signal $x_1(t)$.

4. APPLICATION TO CHAOTIC COMMUNICATIONS

Here, we apply the Hamiltonian synchronization of hyperchaotic Chua circuits to chaotic switching. In this technique, the message $m(t)$ is a binary signal, and is used to modulate one or more parameters of the transmitter, i.e. $m(t)$ controls a switch whose action changes the parameter values of the transmitter. Thus, according to the value of $m(t)$ at any given time t, the transmitter has either the parameter set value p or the parameter set value $\bar{p}$. At the receiver, $m(t)$ is decoded by using the synchronization error to decide whether the received signal corresponds to one parameter value, or the other (it can be interpreted as an one or zero). In our case, to transmit $m(t)$ via chaotic switching, let β be the parameter to be modulated in the hyperchaotic Chua transmitter (11), the parameter α was fixed. We use a "modulation rule" to modulate $m(t)$ as follow $\beta(t) = \beta + r \cdot m(t)$, where $r = 0.2$ while the message is defined as

$$m(t) = \begin{cases} 1, & 0 \le t < 15, \\ 0, & 15 \le t < 30, \\ 1, & 30 \le t < 45, \\ 0, & 45 \le t < 60, \\ 1, & 60 \le t < 75, \\ 0, & 75 \le t < 90. \end{cases}$$

The following figures illustrate the binary transmission of $m(t)$ when β is switched between $\beta(1) = 14.87$ and $\beta(0) = 15.07$. Figure 4 depicts: (a) the message to be transmitted, (b) the synchronization error detection $e_1(t)$ and (c) the transmitted hyperchaotic signal $x_1(t)$; whereas, Figure 5 shows: (a) the message to be transmitted, (b) the synchronization error detection $e_4(t)$ and (c) the transmitted hyperchaotic signal $x_4(t)$. Figures 4(b) and 5(b) clearly show that the original message is recovered at the receiver.

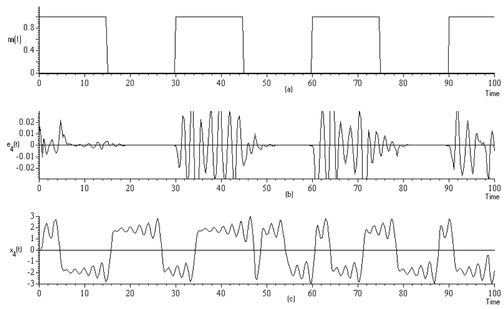

Fig. 5. (a) Digital information signal $m(t)$, (b) synchronization error detection $e_4(t)$, (c) transmitted hyperchaotic signal $x_4(t)$.

5. CONCLUDING REMARKS

In this paper, we have approached the problem of synchronization of two hyperchaotic Chua circuits from the perspective of Generalized Hamiltonian systems including dissipation and destabilizing terms. The approach allows one to give a simple design procedure for the receiver system and clarifies the issue of deciding on the nature of the output signal to be transmitted. Our scheme can be successfully applied to several well-known hyperchaotic osillators (e.g., Rössler's hyperchaotic system with a scalar transmitted signal). The approach is easily implemented on experimental setups, and shows great potential for actual communication systems in which the encoding is required to be secure.

In a forthcomming article we will be concerned with a physical implementation of the method with a specific quantization of the degree of safety of the proposal in actual communication systems.

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