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A HIERARCHICAL NETWORK GAME WITH A LARGE NUMBER OF PLAYERS

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Abstract: We consider a hierarchical network game with multiple links, a single service provider, and a large number of users with multiple classes, where different classes of users enter the network and exit it at different nodes. Each user is charged by the service provider a fixed price per unit of bandwidth used on each link in its route, and chooses the level of its flow by maximizing an objective function that shows a trade-off between the disutility of the payment to the service provider and congestion cost on the link the user uses, and the utility of its flow. The service provider, on the other hand, wishes to maximize the total revenue it collects. We formulate this problem as a leader-follower (Stackelberg) game, with a single leader (the service provider, who sets the price) and a large number of Nash followers (the users, who decide on their flow rates). We show that the game admits a unique equilibrium, and obtain the solution in analytic form. A detailed study of the limiting case where the number of followers is large reveals a number of interesting and intuitive properties of the equilibrium, and answers the question of whether and when the service provider has the incentive to add additional capacity to the network in response to an increase in the number of users on a particular link.

Keywords: Game theory, Stackelberg Games, Nash Games, Communication Networks, Equilibrium

1. INTRODUCTION

We consider a network where users are subject to flow control through the use of congestion indication signals and bandwidth-usage pricing. Recently, there has been much interest in the design of mechanisms to ensure that small queue sizes are maintained at the routers of such networks (for example, see (Kelly *et al.*, 1998; Gibbens and Kelly, 1999; Low and Lapsley, 1999; Kelly, 2000; Kunniyur and Srikant, 2000; Kelly, 2001; Kunniyur and Srikant, 2001*b*; Kunniyur and Srikant, 2001*a*)). A key assumption driving such a design is the following well-known large deviations result: when the number of users in the network is large and the capacity of the network is large, then the probability that the arrival rate will exceed the available capacity is small. Thus, the probability of queue build-up is small (Botvich and Duffield, 1995). In a recent paper, we examined the economics of providing large capacity from a service provider's point of view by considering a single link accessed by many users (Başar and Srikant, 2002). In this paper, we show that the single-link results extend in a natural manner to a multiple link setting.

We note that there is an extensive literature on game-theoretic models of routing and flow control in communication networks (for example, see (Orda *et al.*, 1993; Korilis *et al.*, 1995; La and Anantharam, 1997; Altman *et al.*, 2001*a*; Altman *et al.*, 2001*b*)). These papers have presented conditions for the existence and

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uniqueness of an equilibrium. This has allowed, in particular, the design of network management policies that induce efficient equilibria (Korilis *et al.*, 1995). This framework has also been extended to the context of repeated games in which cooperation can be enforced by using policies that penalize users who deviate from the equilibrium (La and Anantharam, 1997). Our paper differs from the above papers due to the fact that our goal is to devise a revenue-maximizing pricing scheme for the service provider. Thus, a flow control game is played by the users (followers) in the Stackelberg game, while the goal of the leader is to set a price to maximize revenue. Specifically, we consider a network consisting of N links with $(N+1)$ classes of users. One class of users traverses all N links and the other N classes of users traverse only one link, with each class distinguished by the link that it traverses (see Figure 1). Our main results in this paper

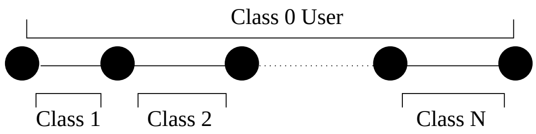

Fig. 1. An N-link network with $(N+1)$ classes of users. Class 0 users use all the links, users in other classes use only one link

can be extended to very general network topologies. However, due to page limitations, we present here only the simple N-link, $(N+1)$-class case which, however, illustrates the main ideas without complicated computations.

The rest of the paper is organized as follows. In Section 2, we formulate the service provider's pricing and users' flow control problems as a Stackelberg game, with the service provider as the leader and the users as followers playing a noncooperative game among themselves. In Section 3, we assume a fixed price and show the existence and uniqueness of a Nash equilibrium for the users' game and also derive the Nash equilibrium solution. In Section 4, we outline the steps in deriving the optimal price for the service provider and provide explicit expressions for the optimal price for certain choices of the problem parameters. In Section 5, we study the asymptotics of the solution to the Stackelberg game when the number of users is large. Concluding remarks are presented in Section 6.

2. PROBLEM FORMULATION

Consider a network of N tandem links accessed by a total of M users of $N+1$ different classes. There are n_k users of class $k, k = 0, \ldots, N$, with the user of class 0 using all N links, while a user of class ℓ using only link $\ell, \ell = 1, \ldots, N$. Let c_ℓ be the capacity (bandwidth) of link $\ell, \ell = 1, \ldots, N$; p be the price per unit bandwidth charged by the network; and x_{kj} denote the transmission rate of the j'th user of class k. We will henceforth use the terminology "User kj" to refer to the j'th user of class k. Finally, let $\bar{x}_k$ denote the total flow of users of class k, that is

$$\bar{x}_k := \sum_{j=1}^{N} x_{kj}\,, \quad k = 0, \ldots, N\,;$$

x_{-k} denote the collection of flow rates of all users except those of class k, and x_{-jk} denote the collection of flow rates of all users of class k, except that of User kj.

The objective of User $0i$ is to maximize the following function with respect to x_{0i} over $[0, \min_\ell\{c_\ell - \bar{x}_0 + x_{0i} - \bar{x}_\ell\})$:

$$F_{0i}(x_{0i}, x_{-0i}, x_{-0}; p) = w_0 \log(1 + x_{0i}) - Npx_{0i} - \sum_{\ell=1}^{N} \frac{1}{c_\ell - \bar{x}_0 - \bar{x}_\ell}\,, \quad (1)$$

where $w_0 \log(1 + x_{0i})$ is the utility of the flow x_{0i} to User $0i$, with $w_0 > 0$ a preference parameter, and $1/(c_\ell - \bar{x}_0 - \bar{x}_\ell)$ represents the congestion cost on link ℓ. Likewise, User kj's objective, $k \geq 1$, is to maximize, with respect to $x_{kj} \in [0, c_k - \bar{x}_0 - \bar{x}_k + x_{kj})$, the function

$$F_{kj}(x_{kj}, x_{-kj}, x_{-k}; p) = w_k \log(1 + x_{kj}) - px_{kj} - \frac{1}{c_k - \bar{x}_0 - \bar{x}_k}\,. \quad (2)$$

Note that if we assume that the queueing process at the k'th link is $M/M/1$, then the congestion cost above is simply the delay on the link. For a given p, the objective functions above define a noncooperative game between the users of the network, where an appropriate solution concept is the Nash equilibrium (Başar and Olsder, 1999). For each fixed $p > 0$, a Nash equilibrium for this M-player game is an n-tuple $\{\{x^*_{kj}(p) \geq 0\}_{j=1}^{n_k}\}_{k=0}^{N}$ satisfying, for all $j, 1 \leq j \leq n_k$ and $k, 0 \leq k \leq N$,

$$\max_{x_{kj}} F_{kj}(x_{kj}, x^*_{-kj}, x^*_{-k}; p) = F_{kj}(x^*_{kj}, x^*_{-kj}, x^*_{-k}; p) \quad (3)$$

where the constraint interval is $[0, \min_\ell\{c_\ell - \bar{x}^*_0 + x^*_{0i} - \bar{x}^*_\ell\})$ for $k = 0$, and $[0, c_k - \bar{x}^*_0 - \bar{x}^*_k + x^*_{kj})$ for $k \geq 1$. Assuming that this M-player game admits a unique Nash equilibrium (which we will prove to be the case), we associate with the service provider a revenue maximization problem to determine the *optimum* price to charge, namely

$$\max_{p \geq 0} L(p; \bar{x}^*_k(p), 0 \leq k \leq N)\,,$$

where

$$L(p; \bar{x}_k(p), 0 \leq k \leq N) = Np\bar{x}_0 + p \sum_{k=1}^{N} \bar{x}_k\,.$$

What we have here is therefore a Stackelberg game (Başar and Olsder, 1999), with one leader (having objective function L) and $M := \sum_{k=0}^{N} n_k$ noncooperative Nash followers (with objective functions F_{kj}'s).

Remark 2.1. In our earlier recent work on pricing in a single link (Başar and Srikant, 2002), we assumed that the network charges a price proportional to the bandwidth consumed by a user. In the case of a single link, the bandwidth consumed by a user is also the same as the network resources consumed by the user. However, in a multiple link network where a user's route could consist of many links, the bandwidth consumed by the user is not necessarily the same as the amount of resources in the network that are allocated to that user. In fact, if a user traverses r links on its route and transmits data at rate x, then it consumes a total of rx units of network resources. Thus, it is logical to charge a user in proportion to the product of its bandwidth usage and the number of hops on its route. This is precisely the pricing scheme chosen in this paper.

From a practical point of view, it is intuitive that the service charge is proportional to bandwidth usage. The fact that it is also proportional to the number of hops, is similar to traditional long-distance telephone pricing schemes which charge more for calls over longer distances. ∎

3. THE NASH EQUILIBRIUM

We first address the issue of existence and uniqueness of Nash equilibrium for each fixed $p > 0$. The following lemma settles this in the affirmative.

Lemma 3.1. For each fixed $p > 0$, the M-person noncooperative game, where the players' objective functions (to be maximized) are given by (1) and (2), admits a **unique** Nash equilibrium $\{x^*_{kj}(p) \geq 0;\ 1 \leq j \leq n_k,\ 0 \leq k \leq N\}$, with $\bar{x}^*_0(p) + \bar{x}^*_\ell(p) < c_\ell,\ 1 \leq \ell \leq N$.

Proof: Let us first note that adding the quantity

$$w_0 \sum_{j\neq i} \log(1 + x_{0j}) + \sum_{k=1}^{N} w_k \sum_{j=1}^{n_k} \log(1 + x_{kj})$$
$$-Np \sum_{j=1,\neq i}^{n_0} x_{0j} - p \sum_{k=1}^{N} \bar{x}_k$$

to F_{0i}, and treating the resulting function as the new objective function of User 0i will not affect the Nash equilibrium. Likewise, adding the quantity

$$w_m \sum_{j\neq i} \log(1 + x_{mj}) + \sum_{k=0,k\neq m}^{N} w_k \sum_{j=1}^{n_k} \log(1 + x_{kj})$$
$$- \sum_{\ell=1,\neq m}^{N} \frac{1}{c_\ell - \bar{x}_0 - \bar{x}_\ell} - NP\bar{x}_0 - p \sum_{\ell=1}^{N} \bar{x}_\ell + px_{mi}$$

to F_{mi}, for each $i, m,\ 1 \leq i \leq n_m,\ 1 \leq m \leq N$, will not change the Nash equilibrium, as the quantity added to F_{mi} does not depend on the decision variable of User mi. Now note that all these new (modified) objective functions are identical, and given by

$$F(x_0, \ldots, x_N; p) = \sum_{k=0}^{N} w_k \sum_{j=1}^{n_k} \log(1 + x_{kj})$$
$$- \sum_{\ell=1}^{N} \frac{1}{c_\ell - \bar{x}_0 - \bar{x}_\ell} - NP\bar{x}_0 - p \sum_{\ell=1}^{N} \bar{x}_\ell$$

Hence, every Nash equilibrium solution of the original game is also a Nash equilibrium solution of a game with the common objective function F for all players. F is strictly concave in the M-tuple $(x_{01}, \ldots, x_{Nn_N})$ which is restricted to the nonnegative orthant bounded by the hyperplanes $\bar{x}_0 + \bar{x}_\ell = c_\ell$ on which F is unbounded from below. Then, from standard results in finite-dimensional optimization (Bertsekas, 1995), it follows that F has a unique maximum in this bounded region (where it is finite, except on the given hyperplane), and every *person-by-person optimal* solution is also globally optimal. Hence, the Nash equilibrium exists and is unique. Clearly, the maximizing solution cannot lie on the hyperplane, thus leading to the strict inequality on $\bar{x}^*_0(p) + \bar{x}^*_\ell(p)$, for all $\ell, 0 \leq \ell \leq N$. ∎

Depending on the value of p and values of other parameters, the unique solution alluded to in the lemma above could lie on the lower boundary of the constraint region (that is some of the x_{kj}'s could be *zero*). If this does not happen, we say that the Nash equilibrium is *inner* or *positive*. We now obtain necessary and sufficient conditions for the equilibrium to be positive, and obtain a characterization for it. Clearly, from first-order conditions (which are also sufficient), the Nash equilibrium will be positive if, and only if, the following set of equations (obtained by setting the partial derivative of F with respect to x_{kj} equal to *zero*, for all admissible j and k) admits a positive solution (for x_{kj}'s):

$$\frac{w_0}{1 + x_{0i}} - \sum_{\ell=1}^{N} \frac{1}{(c_\ell - \bar{x}_0 - \bar{x}_\ell)^2} - Np = 0,$$
$$\frac{w_k}{1 + x_{kj}} - \frac{1}{(c_k - \bar{x}_0 - \bar{x}_k)^2} - Np = 0\,,\ k \geq 1$$

Note that the solution to the set of equations above depends only on the *class* of each user, and not on the individual user within each class. Hence, $x_{kj} = \bar{x}_k/n_k\,,\ \ k \geq 0$, in view of which, we have the following necessary and sufficient conditions for the positive Nash equilibrium to satisfy:

$$\frac{n_0 w_0}{n_0 + \bar{x}_0} = \sum_{k=1}^{N} \frac{n_k w_k}{n_k + \bar{x}_k} \tag{4}$$

and

$$\frac{n_k w_k}{n_k + \bar{x}_k} = p + \frac{1}{(c_k - \bar{x}_0 - \bar{x}_k)^2} \tag{5}$$

Considerable simplification is possible here if we assume that the users that use single links all have the same preference parameter in their utility functions, that is w_k is independent of k for $k \geq 1$, and also that n_k is independent of k for $k \geq 1$. Let us further assume that the capacity of each link is proportional to the total number of users using that link, with the proportionality constant being c. Hence, $c_\ell = (n_0 + n_1)c =: nc\,, \forall\, \ell$. Further let,

$$\bar{y}_0 := n_0 + \bar{x}_0\,, \quad \bar{y}_1 := n_1 + \bar{x}_1\,, \quad \bar{y} := \bar{y}_0 + \bar{y}_1$$
$$\bar{w} := n_0 w_0 + N n_1 w_1\,,\ w_{av} := (\bar{w}/n)$$

Then, we have a positive Nash equilibrium if, and only if, there exists a $\bar{y}(p)$ solving

$$g(\bar{y}) := \frac{\bar{w}}{\bar{y}} - \frac{N}{(nc+n-\bar{y})^2} - Np = 0\,, \qquad (6)$$

and satisfying

$$\min\big(w_0, N w_1\big)\,\frac{\bar{y}(p)}{\bar{w}} > 1 \qquad (7)$$

which is the positivity constraint. Hence, the Nash equilibrium is positive for a given p if, and only if, there exists a $\bar{y} > n$ solving (6) and further satisfying (7). Since $g(\bar{y} \uparrow n(c+1)) = -\infty$, and $g(\bar{y} = n) = w_{av} - N(nc)^{-2} - Np$, and g is strictly decreasing in $[n, (c+1)n)$, there will exist a unique solution to (6) in the open interval $(n, (c+1)n)$ if, and only if, $g(n) > 0$, that is

$$p < \frac{w_{av}}{N} - \frac{1}{(nc)^2} =: \hat{p}\,. \qquad (8)$$

Hence, there exists a range of values of p for which the unique Nash equilibrium is positive.

4. LEADER'S PROBLEM AND ITS SOLUTION

We now proceed on to the maximization problem faced by the service provider (leader), and restrict the discussion to the case of two classes of users as introduced above. Because of the one-to-one correspondence between $\bar{y}$ (or equivalently $\bar{x}$) and p through the constraint (6), an equivalent problem for the leader is the maximization of the following function with respect to $\bar{y} > n$ (obtained by substitution of p from (6) in terms of $\bar{y}$):

$$\tilde{L}(\bar{y}) = \bar{w}\big(1 - \frac{n}{\bar{y}}\big) - \frac{N(\bar{y}-n)}{(n(c+1)-\bar{y})^2}\,.$$

We seek a solution to this maximization problem in the open interval $(n, (c+1)n)$. $\tilde{L}$ is analytic over this interval, and

$$\tilde{L}_{\bar{y}} = \frac{n\bar{w}}{\bar{y}^2} - \frac{N[n(c-1)+\bar{y}]}{(n(c+1)-\bar{y})^3}\,, \quad \tilde{L}_{\bar{y}\bar{y}} < 0.$$

Hence, $\tilde{L}$ is strictly concave, and further since it becomes unbounded negative at the upper end of the interval, it follows that it has a unique maximum in the half-open interval $[n, (c+1)n)$. Moreover, the situation $\bar{y} = n$ can be avoided by requiring that $\tilde{L}_{\bar{y}}(n) > 0$, which translates into the simple condition

$$n^2 c^2 w_{av} > N\,. \qquad (9)$$

Under this condition, there exists a unique solution, $\bar{y}^* \in (n, (c+1)n)$, to $\tilde{L}_{\bar{y}}(\bar{y}) = 0$ which we rewrite here for future reference:

$$\frac{n\bar{w}}{\bar{y}^2} - \frac{N[n(c-1)+\bar{y}]}{(n(c+1)-\bar{y})^3} = 0\,. \qquad (10)$$

The corresponding value of p, which in fact maximizes $L(p; \bar{x}^*(p))$, is then obtained directly from (6):

$$p^* = \frac{\bar{w}}{N\bar{y}^*} - \frac{1}{(nc+n-\bar{y}^*)^2} \qquad (11)$$

which, by construction, satisfies (8) and the positivity constraint. To complete the solution to the problem, however, we still have to require that (7) holds with p replaced by p^*, or equivalently

$$\min\big(w_0, N w_1\big)\,\bar{y}^* > \bar{w} \qquad (12)$$

with $\bar{y}^*$ solving the third-order polynomial equation (10).

The solution to (10) cannot be obtained in closed form except for some special cases. One such case is $c = 1$, for which the unique solution is:

$$\bar{y}^* = \frac{2n(n\bar{w})^{\frac{1}{3}}}{N^{\frac{1}{3}} + (n\bar{w})^{\frac{1}{3}}} \qquad (13)$$

provided that

$$n\bar{w} > N$$

which ensures that $\bar{y}^* > n$, that is the total throughput is positive. Now, for the individual throughput levels to be positive, we need also the condition (12), which can be rewritten as

$$\left(\frac{2\min(w_0, Nw_1)}{w_{av}} - 1\right)\big(n^2 w_{av}\big)^{\frac{1}{3}} > 1\,. \qquad (14)$$

This condition is of course more restrictive than the earlier one, $n^2 w_{av} > N$, which can therefore be dropped.

The revenue-maximizing price (for the service provider) can now be obtained for this special case by setting $c = 1$ in (11), and using (13):

$$p^* = \frac{w_{av}}{2N}\big(1 + N^{\frac{1}{3}}\big(n^2 w_{av}\big)^{-\frac{1}{3}}\big)$$
$$-\frac{1}{4n^2}\big(1 + N^{\frac{1}{3}}(n^2 w_{av})^{\frac{1}{3}}\big)^2 \qquad (15)$$

which can easily be checked to be positive provided that $n^2 w_{av} > N$, a condition already assumed to hold. It is also easy to check that $p^* < \hat{p}$.

5. ASYMPTOTIC BEHAVIOR

We now study the behavior of the solution obtained in the previous section for large n. Studying this *many-followers game* will allow us to obtain an explicit expression for $\bar{y}^*$ (or $\bar{x}^*$), which was not possible for finite n, unless $c = 1$. The study will also enable us to ask (and answer) questions like whether it is possible for the service provider to admit new users to the network by increasing the capacity, and whether the existing users would benefit (measured in terms of their utilities) from a "crowding" of the network.

An underlying assumption (or rather convention) throughout this section is that as $n \to \infty$, the sequence $\{w_{av}(n)\}$ has a well-defined limit, $w_{av} > 0$. This would be the case, for example, when there exists an $\alpha \in (0,1)$, such that $n_0 = \alpha n$, which means that there will be infinitely many users of both classes as $n \to \infty$. In this case, of course, $w_{av} = \alpha_0 w_0 + (1-\alpha_0)Nw_1$. An immediate implication of this assumption is that now the condition (9) is readily satisfied.

Now, to study the asymptotic behavior, it is convenient to work with the arithmetic mean of the x_i's (or y_i's), rather than their sum, denoted for the former as

$$x_{av}(n) = \frac{1}{n}(\bar{x}_0 + \bar{x}_1)\,.$$

Then, we can rewrite (10) as

$$\frac{w_{av}(n)}{N(x_{av}(n)+1)^2} = \frac{c + x_{av}(n)}{n^2(c - x_{av}(n))^3}\,.$$

Under our assumption that $w_{av}(n) \to w_{av}$ as $n \to \infty$, a positive solution to $x_{av}(n)$ exists for large n if, and only if,

$$\lim_{n\to\infty} n^2(c - x_{av}(n))^3 = \alpha$$

for some $\alpha > 0$. Substituting this in (6), we obtain

$$p \sim \frac{w_{av}}{N(c+1)} + \frac{2c-1}{\alpha^{2/3}n^{2/3}}\,, \qquad (16)$$

where we have again used the notational convention that $f(n) \sim h(n)$ if $\lim_{n\to\infty}\big(f(n)/h(n)\big) = 1$. Using (16) in (10), and letting $n \to \infty$, yields

$$\alpha = \frac{2c(c+1)^2N}{w_{av}}\,. \qquad (17)$$

Then,

$$x_{av}(n) \sim c - n^{-2/3}\alpha^{1/3}\,, \qquad (18)$$

and the positivity condition is

$$x_{av}(n) > \max\left(\frac{w_{av}}{w_0} - 1, \frac{w_{av}}{Nw_1} - 1\right)$$

Letting, as before, $\alpha_0 := n_0/n$, and assuming that $\alpha_0 < 1$ for all n and as $n \to \infty$, the condition above can be shown to be equivalent to, as $n \to \infty$,

$$\frac{\alpha_0}{c+\alpha_0} < \frac{Nw_1}{w_0} < \frac{c}{1-\alpha_0} + 1 \qquad (19)$$

which is a necessary and sufficient condition for an inner solution to exist. Note that this places an upper bound on the number of links, namely,

$$N < \frac{w_0}{w_1}\left(\frac{c}{1-\alpha_0} + 1\right)$$

assuming that w_0 is not a function of N. If, however, we pick $w_0 = \tilde{w}_0 N$, for some constant $\tilde{w}_0 > 0$, then there is no upper bound on the number of links.

It is worth noting that the optimal price charged by the network is positive for sufficiently large n, but whether it is an increasing or decreasing function of n depends on the specific value of c. For $c > 1/2$, it decreases with n, whereas for $c < 1/2$, it increases with n. In spite of this c-dependent behavior of the optimum price, the revenue per unit bandwidth per link exhibits a c-independent trend–increasing with n in the many-users regime:

$$\text{Revenue/bw/link} = \frac{px_{av}}{c} \sim \frac{w_{av}}{(c+1)} - \alpha^{-\frac{2}{3}}n^{-\frac{2}{3}}\,.$$

Suppose that $w_{av}/(c+1)$ is larger than the cost of adding one unit of bandwidth. Then, as the number of users increases, the service provider's profit increases. Thus, the service has an incentive to increase the link capacity which drives the congestion cost to zero as shown next.

The congestion cost decreases with n in the many-users regime:

$$\text{Congestion cost} = \frac{1}{n(c - x_{av}(n))} \sim \alpha^{-\frac{1}{3}}n^{-\frac{1}{3}}\,,$$

and so does the net utility of each user. For users of class 0,

$$F^*_{0i} = w_0 \log\frac{N(c+1)w_0}{w_{av}} - Nw_0 + \frac{w_{av}}{c+1} - N\alpha^{-\frac{1}{3}}n^{-\frac{1}{3}}\,.$$

and for users of class $k, k \geq 1$

$$F^*_{kj} = w_1 \log\frac{N(c+1)w_1}{w_{av}} - w_1 + \frac{w_{av}}{c+1} - \alpha^{-\frac{1}{3}}n^{-\frac{1}{3}}\,.$$

6. CONCLUSIONS

In this paper, we have considered the important, emerging problem of choosing a pricing scheme for the Internet based on bandwidth usage for user. Assuming a pricing scheme whereby a user is charged in proportion to its total resource consumption (measured as the product of the bandwidth consumed by the user and the number of links on the user's route), we have presented a Stackelberg formulation of the pricing problem. The network (leader) sets the price and the users (followers) react to the price as well as the congestion caused by the overall traffic in the network, using a flow control algorithm. In this setting, we have derived expressions for the optimal price to maximize network revenue under a many-users regime. A significant observation is the fact that there is revenue-incentive for the network to increase the available capacity in the network in proportion to the number of users in the network. From a purely Quality-of-Service (QoS) point of view, increasing the capacity in the network decreases the delay seen by the users of the network. Thus, our Stackelberg game formulation and the solution show that it is profitable for the network to provide better QoS for the users of the network.

The results in the paper can be generalized to arbitrary network topologies with different call classes. For an arbitrary network topology, the Nash equilibrium solution of the followers' non-cooperative game acts as a constraint to the revenue maximization problem faced by the network. Under simple network topologies, this constrained network optimization problem can be explicitly solved. The case of arbitrary network topologies requires the use of Lagrange multiplier techniques and the computation of asymptotic expansions of the Lagrange multipliers. Due to space limitations, these results are not presented here. Instead, we remark that the fundamental observation regarding pricing continues to hold: it is profitable for the network to provide better QoS to the users by increasing the available bandwidth in proportion to the number of users at congested links.

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