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LINEARIZED CONTROLLER DESIGN FOR THE OUTPUT PDFS USING SQUARE ROOT BASED B-SPLINE MODELS

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Abstract: This paper presents a linearized approach for the controller design of the shape of the output probability density functions for general stochastic systems. A square root approximation to the output probability density function is realized by a set of B-spline functions. This generally produces a nonlinear state space model for the weights of the B-spline approximation. A linearized model is therefore obtained and embedded into a performance function that measures the tracking error of the output probability density function with respect to a given distribution. By using this performance function as a Lyapunov function for the closed loop system, a feedback control input has been obtained which guarantees the closed loop stability and realizes the perfect tracking. The algorithm described in this paper has been tested on a simulated example and desired results have been achieved. *Copyright IFAC2002*

Keywords: Dynamic stochastic systems; probability density function; B-splines neural networks; Lyapunov stability theory.

1. INTRODUCTION

Apart from the well developed minimum variance control, LQG and mean value control (Astrom, 1970), recently two new methods are developed: i) closed loop probability density function control; and ii)output probability density function control. The first approach was developed by Karny in 1996. In this approach, the closed loop probability density function is formed as a joint probability density function between the system and a *randomized* controller, which is characterized by a probability density function. A recursive algorithm has been established for the generation of the probability density function of the controller. However, this randomized controller is difficult to realize as an 'optimal' crisp control input cannot be generally produced from non-symmetric probability density functions of the controllers obtained from this algorithm.

In the second groups of approaches (Wang,1999; 2000), only the shape control of the output probability density function (not the closed loop probability density function) is addressed. The purpose of control design is to select a crisp control input so that the shape of the output probability density function of the stochastic system is made as close as possible to a given distribution function. Different from the work of Karny (1996), in these approaches (Wang,1999; 2000), the inputs of stochastic systems are taken as deterministic variables, and the outputs are taken as the measured probability density functions of the system output.

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The linear B-spline approximations have been used to approximate the probability density function of the system output directly. If the basis functions are fixed, then the weight of the approximation can be regarded as only being related to the control input. However, the main problem of linear B-spline approximation of probability density function is that the weight training trajectories can sometimes be partly negative. To overcome this difficulty, instead of approximating the probability density function directly, in Wang (2000) it has been proposed that the square root of the output probability density function should be approximated by the B-spline functions. However, to guarantee the stability of the closed loop system, a strict constraint has to be included for the dynamic part of the system. To improve this, a global Lyapunov based design has been made (Wang, 2001), where a nonlinear controller is formulated. However, the obtained control algorithm is very complicated and can cause non-smooth responses for the closed loop system. Improvement is therefore necessary by developing alternative design methods.

In this paper, we consider the use of a linearized model, where Lyapunov based design is still used. In fact, the Lyapunov function selected in this paper is the performance function which measures the difference between the output probability density function and the given distribution.

2. PRELIMINARIES ON SQUARE ROOT PDF MODEL

In this section, the formulation of the square root model by Wang et, al (1999, 2001) will be described for the completeness of this paper. For this purpose, let us denote $v(t) \in [a, b]$ as a uniformly bounded random process variable representing the output of a dynamic stochastic system, and $u(t) \in R^1$ as the control input vector which controls the distribution of $v(t)$, then $v(t)$ can be characterised by its probability density function $\gamma(y, u(t))$ which is defined by

$$P(a \le v(t) < \xi, u(t)) = \int_a^{\xi} \gamma(y, u(t))dy \qquad (1)$$

where $P(a \le v(t) < \xi, u(t))$ represents the probability of output $v(t)$ lying inside the interval $[a, \xi)$ when $u(t)$ is applied to the system. This means that the shape of probability density function $\gamma(y, u(t))$ of $v(t)$ is controlled by $u(t)$.

Assume interval $[a, b]$ is known and the probability density function $\gamma(y, u(t))$ is continuous and bounded, then using the well known B-spline neural network, the following square root approximation is obtained

$$\sqrt{\gamma(y, u(t))} = \sum_{i=1}^{n} w_i(u(t))B_i(y) + e_0 \qquad (2)$$

where $w_i(u(t))$ are the weights which depends on $u(t)$, $B_i(y)$ are the pre-specified basis functions and e_0 represents the approximation error. Indeed, the above approximation is realisable in practice. This is largely due to the development of sensor techniques, where in papermaking probability density functions can now be easily measured (Wang, 2000).

To simplify the formulation, it is assumed that $e_0 = 0$. This means that only the following equality

$$\sqrt{\gamma(y, u(t))} = \sum_{i=1}^{n} w_i(u(t))B_i(y) \qquad (3)$$

will be considered. Since equation (3) means that

$$\gamma(y, u(t)) = (\sum_{i=1}^{n} w_i(u(t))B_i(y))^2 \ge 0 \quad \forall y \in [a, b] \qquad (4)$$

it can be seen that the positiveness of $\gamma(y, u(t))$ can be automatically guaranteed. Denote:

$$C_0(y) = (B_1(y), B_2(y), ..., B_{n-1}(y)) \in R^{n-1}$$
$$V(t) = (w_1, w_2, ..., w_{n-1})^T \in R^{1\times(n-1)} \qquad (5)$$

then it can be shown that at time t, the square root of the output probability density function becomes

$$\sqrt{\gamma(y, u(t))} = \begin{bmatrix} C_0(y) & B_n(y) \end{bmatrix} \begin{bmatrix} V(t) \\ w_n(t) \end{bmatrix} \qquad (6)$$

However, since $\gamma(y, u(t))$ is a probability density function, the following equality

$$\int_a^b \gamma(y, u(t))dy = \int_a^b \sqrt{\gamma(y, u(t))}^2 dy = 1 \qquad (7)$$

should always be satisfied. Using equation (7), it can be seen that the following equality

$$\int_a^b (C_0(y)V(t) + w_n B_n(y))^2 dy = 1 \qquad (8)$$

should hold for any set of weights and basis functions. This leads to

$$V^T(t)\Sigma_0 V(t) + 2\Sigma_1 V(t) w_n(t) + \Sigma_2 w_n^2(t) = 1 \qquad (9)$$

where

$$\Sigma_0 = \int_a^b C_0^T(y)C_0(y)dy \qquad (10)$$

$$\Sigma_1 = \int_a^b C_0(y)B_n(y)dy \qquad (11)$$

$$\Sigma_2 = \int_a^b B_n^2(y)dy \qquad (12)$$

Equation (6) gives an instantaneous expression of the considered probability density function at time t. However, in many systems, the actual probability density function of the system output is dynamically related to the control input $u(t)$. As such, the following dynamical relationship is considered:

$$\dot{V}(t) = AV(t) + Bu(t) \tag{13}$$

where A and B are known matrices of appropriate dimensions. It can be seen that only $n-1$ weights are independent and w_n is nonlinearly related to $V(t)$ to satisfy the main constraint in equation (7). Instead of treating this constraint independently, $w_n(t)$ can be arranged such that it changes dynamically to satisfy this constraint. For this purpose, it is necessary to calculate the first order derivative for both sides of equation (9), this leads to a dynamics of $w_n(t)$ as follows

$$\begin{aligned}&\dot{V}^T(t)\Sigma_0 V(t) + V^T(t)\Sigma_0\dot{V}(t) + 2\Sigma_1\dot{V}(t)w_n(t)\\&+2\Sigma_1 V\dot{w}_n(t) + 2\Sigma_2 w_n(t)\dot{w}_n(t) = 0\end{aligned} \tag{14}$$

By re-arranging (14), it can be further obtained that

$$\begin{aligned}\dot{w}_n(t) &= \frac{f_1(w_n(t), V(t), u(t))}{2\Sigma_1 V(t) + 2\Sigma_2 w_n(t)}\\ f_1 &= -V^T(t)(A^T\Sigma_0 + \Sigma_0 A)V(t) +\\ &\quad 2\Sigma_1 AV(t)w_n(t) +\\ &\quad 2(\Sigma_1 Bw_n(t) + B^T\Sigma_0 V)u(t)\\ &= f(w_n(t), V(t), u(t))\end{aligned} \tag{15}$$

$$\Sigma_1 V(t) + \Sigma_2 w_n(t) = \int_a^b B_n(y)\sqrt{\gamma(y, u(t))}dy \neq 0 \tag{16}$$

Therefore, the relationship between all the weights and the input can be given in a compact form as follows:

$$\begin{bmatrix}\dot{V}(t)\\ \dot{w}_n(t)\end{bmatrix} = \begin{bmatrix}AV(t) + Bu(t)\\ f(w_n(t), V(t), u(t))\end{bmatrix} \tag{17}$$

It can be seen that equation (17) is a nonlinear state space equation which, together with

$$\sqrt{\gamma(y, u(t))} = \begin{bmatrix}C_0(y) & B_n(y)\end{bmatrix}\begin{bmatrix}V(t)\\ w_n(t)\end{bmatrix} \tag{18}$$

forms the mathematical expression of the stochastic system to be considered in this paper. Of course, to satisfy (9), the initial values of $V(t)$ and $w_n(t)$ must satisfy the nonlinear algebraic constraint in (9). It is also important to notice that the algebraic constraint (9) is equivalent to (15) only if A is a stable matrix.

3. THE CONTROL ALGORITHM

As discussed in Section 2, the purpose of the control algorithm design is to choose control input $\{u(t)\}$ such that the actual probability density function of the system output is made as close as possible to a pre-specified continuous probability density function $g(y)$, which is defined on $[a, b]$ and is independent of $\{u(t)\}$. This is equivalent to choosing $\{u(t)\}$ such that $\sqrt{\gamma(y, u(t))}$ is made as close as possible to $\sqrt{g(y)}$. To formulate the required performance function, a linearised model needs to be used. For this purpose, the following new state vector and a new matrix $C(y)$ are defined:

$$X(t) = \begin{bmatrix}V(t)\\ w_n(t)\end{bmatrix}, C(y) = \begin{bmatrix}C_0(y) & B_n(y)\end{bmatrix} \tag{19}$$

As a result, the square root of the output probability density function and the pre-specified distribution function $g(y)$ can be expressed in terms of B-spline basis functions to give

$$\begin{aligned}\sqrt{\gamma(y, u(t))} &= C(y)X(t)\\ \sqrt{g(y)} &= C(y)X_{ref}\end{aligned} \tag{20}$$

where X_{ref} is a constant vector deduced from the pre-specified $\sqrt{g(y)}$. Using the two new vectors in (20) and denoting u^0 is the input which maintains $X(t)$ at X_{ref}, then the error vectors $\tilde{X}$ and $\tilde{u}$ can be defined as

$$\tilde{X}(t) = X(t) - X_{ref} \text{ and } \tilde{u}(t) = u(t) - u^0 \tag{21}$$

By taking X_{ref} and u^0 as the equilibrium point, then from (17) it can be seen that the following equations must be satisfied.

$$\bar{A}X_{ref} + Bu^0 = 0 \text{ and } f(X_{ref}, u^0) = 0 \tag{22}$$

where $\bar{A} = [A, 0]$. This is simply because the dynamic relationship between state vector X and u can be given as

$$\dot{X}(t) = \begin{bmatrix}\bar{A}X(t) + Bu(t)\\ f(X(t), u(t))\end{bmatrix} \tag{23}$$

Since X_{ref} is a constant vector once $g(y)$ is selected, in terms of $\tilde{X}$, the state space equation (17) of the system becomes

$$\dot{\tilde{X}} = \dot{X} = \begin{bmatrix}\bar{A}X + Bu\\ f(X, u)\end{bmatrix} \tag{24}$$

which can be further simplified to give

$$\begin{aligned}\dot{\tilde{X}} &= \begin{bmatrix}\bar{A}(X - X_{ref} + X_{ref}) + B(\tilde{u} + u^0)\\ f(X - X_{ref} + X_{ref}, \tilde{u} + u^0)\end{bmatrix}\\ &= \begin{bmatrix}\bar{A}\tilde{X} + B\tilde{u} + \bar{A}X_{ref} + Bu^0\\ f(\tilde{X} + X_{ref}, \tilde{u} + u^0)\end{bmatrix}\end{aligned} \tag{25}$$

where $\bar{A}X_{ref}+Bu^0=0$. Since $f(\tilde{X}+X_{ref},\tilde{u}+u^0)$ is differentiable (see (15)), it can be further expressed using the multivariable Cauchy's formula to give

$$f(\tilde{X}+X_{ref},\tilde{u}+u^0)=\frac{\partial f}{\partial X}\mid_{(X_{ref}+\eta_1\tilde{X},\tilde{u}+u^0)}\tilde{X}$$
$$+\frac{\partial f}{\partial u}\mid_{(X_{ref},u^0+\eta_2\tilde{u})}\tilde{u} \tag{26}$$

where $0<\eta_1<1$ and $0<\eta_2<1$ are the two numbers indicating that the first order partial derivatives in (26) are taken at $(X_{ref}+\eta_1\tilde{X},\tilde{u}+u^0)$ and $(X_{ref},u^0+\eta_2\tilde{u})$, respectively.

Using (23)-(26), the dynamics of $\tilde{X}$ can be finally expressed as

$$\dot{\tilde{X}}=\tilde{A}\tilde{X}+\tilde{B}\tilde{u} \tag{27}$$

where the linearized parameter matrices are given by

$$\tilde{A}=\left[\bar{A},\frac{\partial f}{\partial X}\mid_{(X_{ref}+\eta_1\tilde{X},\tilde{u}+u^0)}\right]$$
$$\tilde{B}=\left[B\frac{\partial f}{\partial u}\mid_{(X_{ref},u^0+\eta_2\tilde{u})}\right] \tag{28}$$

Since the purpose of the controller design is to realise a good tracking performance of the output probability density function with respect to the given distribution $g(y)$, the following performance function can be defined:

$$J=\int_a^b(\sqrt{\gamma(y,u(t))}-\sqrt{g(y)})^2dy+R(u(t)-u^0)^2 \tag{29}$$

where the first term characterises the difference between $\gamma(y,u(t))$ and $g(y)$, and the second term represents a constraint on the control input. R is a pre-specified weighting scaler. By substituting (27) - (28) into (29), it can be further formulated that

$$J=\int_a^b\tilde{X}(t)^TC(y)^TC(y)\tilde{X}(t)dy+R\tilde{u}(t)^2 \tag{30}$$

In this paper a new approach will be used by taking the performance function J in (29) as a Lyapunov candidate function for the closed loop system. As a result, the controller design can be formulated to give a stable and good tracking performance so long as it can guarantee that $\dot{J}\le 0$. Since the integral part of the equation (30) considers only $C^T(y)C(y)$, J can be re-written as

$$J=\tilde{X}(t)^TQ\tilde{X}(t)+R\tilde{u}(t)^2$$
$$Q=\int_a^bC(y)^TC(y)dy \tag{31}$$

It is assumed that the basis functions are chosen such that Q becomes positive definite. Indeed this can be regarded as a condition for choosing $C(y)$.

Under the assumption that the linearised system (27) is stable, a controller should be designed which guarantees that Lyapunov function J for the closed loop system is always decreasing. For this purpose, the first order derivative of J with respect to time t is calculated to give:

$$\frac{dJ}{dt}=\dot{\tilde{X}}^TQ\tilde{X}+\tilde{X}^TQ\dot{\tilde{X}}+2R\tilde{u}\dot{\tilde{u}} \tag{32}$$

Using (27), the first order derivative of Lyapunov function J can be written in a new format as follows:

$$\frac{dJ}{dt}=(\tilde{X}^T\tilde{A}^T+\tilde{B}^T\tilde{u})Q\tilde{X}+\tilde{X}^TQ(\tilde{A}\tilde{X}+\tilde{B}\tilde{u})$$
$$+2R\tilde{u}\dot{\tilde{u}}=\tilde{X}^T(\tilde{A}^TQ+Q\tilde{A})\tilde{X}$$
$$+2\tilde{B}^TQ\tilde{X}\tilde{u}+2R\tilde{u}\dot{\tilde{u}} \tag{33}$$

Assuming that $\tilde{A}$ is stable, then it can be shown that $\tilde{A}^TQ+Q\tilde{A}<0$. As such, by defining another positive definite matrix $P=P^T>0$ such that

$$\tilde{A}^TQ+Q\tilde{A}=-P \tag{34}$$

it can be further obtained that

$$\frac{dJ}{dt}=-\tilde{X}^TP\tilde{X}+2\tilde{B}^TQ\tilde{X}\tilde{u}+2R\tilde{u}\dot{\tilde{u}} \tag{35}$$

To select $u(t)$ such that $\dot{J}\le 0$, it is sufficient to make

$$2\tilde{B}^TQ\tilde{X}\tilde{u}+2R\tilde{u}\dot{\tilde{u}}=0 \tag{36}$$

This leads to the required control input as

$$\dot{u}=\dot{\tilde{u}}=-\frac{1}{R}(\tilde{B}^TQ\tilde{X}) \tag{37}$$

From equation (37), it can be seen that the obtained control input is a linear feedback one directly related to $\tilde{X}$. To retrieve X from $\sqrt{\gamma(y,u(t))}$, we need to re-consider :

$$\sqrt{\gamma(y,u(t))}=C(y)X \tag{38}$$

By multiplying this equation with $C^T(y)$ from left and integrating both sides from a to b, it can be obtained that

$$\int_a^bC^T(y)\sqrt{\gamma(y,u(t))}dy=QX \tag{39}$$

Since Q is assumed non-singular, X can be solved from equation (39) to give

$$X=Q^{-1}\int_a^bC^T(y)\sqrt{\gamma(y,u(t))}dy \tag{40}$$

Using definition $\tilde{X} = X - X_{ref}$, it can be seen that $\tilde{X}$ can be written as

$$\tilde{X} = Q^{-1} \int_a^b C^T(y)[\sqrt{\gamma(y,u(t))} - \sqrt{g(y)}]dy \tag{41}$$

As a result, the final form of the control input, which stablises the closed loop system, can be expressed as:

$$\dot{\xi} = \frac{-1}{2R}(\tilde{B}^T \int_a^b C^T[\sqrt{\gamma(y,u)} - \sqrt{g(y)}]dy \tag{42}$$

$$u = \xi \tag{43}$$

This is a feedback control which receives the feedback signal directly from the measured output probability density function, where u is independent of u_0. To summarise, the following theorem can be obtained:

Theorem. (Main Result) *Suppose that $\tilde{A}$ in (27) is a stable matrix and Q in (31) is non-singular, then the control input given by (42)-(43) stablises the closed loop system and guarantees that*

$$\lim_{t\to+\infty} \gamma(y,u(t)) = g(y), \forall y \in [a,b] \tag{44}$$

Proof: Using the first order Lyapunov function given by (35), it can be seen that if

$$\gamma(y,u(t)) \neq g(y) \tag{45}$$

then the following inequality strictly holds

$$\frac{dJ}{dt} < 0 \tag{46}$$

This means that J will continue to decrease. Since J is always positive and has a lower bound (i.e., 0), there is a J_0 =constant such that

$$\lim_{t\to+\infty} J = J_0 \tag{47}$$

This means that in the end

$$\lim_{t\to+\infty} \tilde{X} = 0 \tag{48}$$

As a result, $\lim_{t\to+\infty} \gamma(y,u(t)) = g(y)$.

The control algorithm in (42) and (43) directly relates to the weighted integration of the square root of the measured probability density function. It is realisable in real-time so long as Q is a non-singular matrix.

4. DISCUSSIONS

The system considered here is a known dynamical system, where the parameter matrices A and B in equation (28) are known and fixed. This limits the application of the proposed methods to unknown systems. Indeed, recently two new control methods have been developed (Wang, 2001, 2002) as

- adaptive control that combines the parameter estimation with the control schemes in this paper;
- the control of random parameter systems (Wang, 2002).

In the first case, since the weights of the B-spline expansion can be directly calculated from equation (40), the linear part of the weight system (13) can be regarded as an input-output model for an MIMO ARMA system. This means that one can directly use the well known recursive least squares method to estimate (A, B) parameters. Using the estimated parameters the nonlinear function in equation (15) can still be defined. As such, the parameters in the linearized equation (27) can also be calculated at each time instant. This means that the estimated parameters can be naturally combined into the control equation (37), leading to an adaptive control strategy.

Especially in the second approach (Wang, 2002), a random ARMAX model has been considered as

$$y_k = \sum_{i=1}^{n} a_i(k)y_{k-i} + \sum_{j=1}^{m} b_j(k)u_{k-j} + \omega_k \tag{49}$$

where $y_k \in R^1$ and $u_k \in R^1$ are one dimensional output and input of the system, respectively, and $a_i(k)$, $(i = 1, 2, \cdots, n)$, $b_j(k), (j = 1, 2, \cdots, m)$, and ω_k are all independent and uniformly bounded random parameters characterized by their known probability density functions given by

$$P\{a \le y_k < \xi\} = \int_a^\xi \gamma_y(x, u_k)dx \tag{50}$$

$$P\{a \le a_i < \xi\} = \int_a^\xi \gamma_{ai}(x)dx \tag{51}$$

$$P\{a \le b_j < \xi\} = \int_a^\xi \gamma_{bj}(x)dx \tag{52}$$

$$P\{a \le \omega_k < \xi\} = \int_a^\xi \gamma_\omega(x)dx \tag{53}$$

Since at the current sample time k the system inputs and outputs in the past are measured, y_{k+1} is in fact a linear combination of independent random parameters $a_i(k)$ and $b_j(k)$, as such an (n+m+1)-folder convolution has to be used to calculate the probability density function of y_{k+1}. In Wang (2002), the well known Laplace transformation has been applied to all the probability density functions and thus transferred the (n+m) convolution into a simple algebraic equation.

Of course, one can use the square root model in this paper to express further the Laplace transferred probability density functions. This again leads to the design of Lyapunov based control algorithms that are similar to the one proposed in this paper. This belongs the area of future studies.

5. CONCLUSIONS

In this paper, a control algorithm has been developed for the shape control of the output probability density function for dynamic stochastic systems, where B-spline functions are used to approximate the square root of the measured output probability density function. A performance function has been defined and also used as a Lyapunov candidate function for the closed loop system. Under the conditions that the linearised system is stable and that the basis functions are selected such that matrix Q in (31) is non-singular, an output feedback control (42)-(43) is obtained which stablises the closed loop system, and makes the measured output probability density function tends to its target distribution asymptotically. Discussions are made on both adaptive control extension for unknownm, but fixed, parameter matrices A and B, and the random parameters composed ARMA systems (Wang, 2002).

The output probability density function control is a new research area that was originated through the examples seen in paper making systems, chemical process engineering (i.e., the particle size distribution control and polymerization systems), food processing (Campbell and Webb, 2001) and combustion flames distribution control in power generation. In the past, these systems are difficult to control due to the lack of sensors that can measure the output probability density functions. However, due to the fast development of sensing technology and image processing, these output probability density functions are now measurable. This provides a good opportunity to develop effective on-line control strategies that control the shape of the output probability density functions.

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