15th Triennial World Congress, Barcelona, Spain

A PARAMETRIC OPTIMISATION APPROACH FOR ROBUST MPC

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Abstract: In this paper an algorithm is presented for deriving the explicit robust model based optimal control law. The system is represented by linear, discrete-time, time-invariant model with constraints on control and state variables and a quadratic objective function. Using the fundamentals of flexibility analysis the algorithm proposed in this paper derives the robust optimal control law off-line as a function of the state of the process, thus eliminating the repetitive solution of on-line optimisation problems. Hence, the on-line implementation is reduced to a sequence of simple function evaluations. The key advantageous features of the algorithm are demonstrated via an illustrative example. *Copyright © 2002 IFAC*

Keywords: Robustness, Model based control, process control, parametric control, quadratic programming.

1. INTRODUCTION

Model Predictive Control (MPC) refers to a class of computer control algorithms that compute a sequence of control variable adjustments to optimise the future behaviour of the plant over a specified time horizon. MPC has been gaining a lot of support in industrial applications, primarily due to its ability to explicitly incorporate operational restrictions, the so-called constraints, on *state* and *control* variables into the controller calculation. This makes it particularly attractive in the process industries, where the economical operating point typically lies at the intersection of constraints and the processes are sufficiently slow to allow its implementation (for reviews see Biegler and Rawlings, 1991; Mayne, *et al*., 2000). Classical MPC schemes determine the appropriate control law via on-line optimal control calculations based on measurements that represent the current process state. An approach for moving off-line the rigorous calculations, delimiting thus the applicability of MPC, has recently been reported (Pistikopoulos, *et al*., 2000). It is based on recently proposed parametric optimisation algorithms, developed in our research group at Imperial College, with which the explicit mapping of the optimal control actions in the space of the state measurements can be achieved.

The performance characteristics of MPC algorithms depend on their *feasibility, stability* and *robustness* properties. When a control system is said to be robust, it is meant that stability and feasibility are maintained for a specific uncertainty range over the considered time horizon. Inevitably, operation of manufacturing processes is subject to variations and uncertainties. These varying conditions demand development of *robust MPC algorithms* to ensure economical and safe operation. Min-max robust MPC was first introduced by Campo and Morari (1987) and further developed by Allwright and Papavasiliou (1992) and Zheng and Morari (1993). Kothare, *et al*. (1996) optimize robust performance for polytopic/multi-model and structured feedback

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uncertainty whereas Scokaert and Mayne (1998) for input disturbances. Bemporad and Garulli (1997) consider the worst input disturbance over the prediction horizon and enforce constraint fulfillment for all possible disturbance realizations. Badgwell (1997) proposed a robust MPC algorithm for stable, constrained, linear plants with multi-plant description. Using Lyapunov arguments, robust stability can be proved when a cost valued function based on stability constraint is imposed. Moreover, invariant ellipsoidal terminal sets can be extended to robust MPC. Invariant terminal ellipsoid sets lead to quadratically constrained quadratic programs, which can be solved through interior-point methods (Lobo, *et al.*, 1997). Alternatively, polyhedral robustly terminal invariant sets can be determined (Blanchini, 1999), that lead to linear constraints. However, all the above methodologies are generally based on optimization problems that have to be solved on-line.

In this paper, the implementation and development of a new generic *robustness synthesis* framework for MPC controllers based on the concepts of flexibility analysis (Pistikopoulos and Grossmann, 1988) and critical disturbance values for linear, time-invariant, discrete-time process models is presented. Using parametric optimization, the robust control law is derived as a function of the process states, thereby reducing the repetitive on-line optimizations to simple function evaluations. The rest of the paper is organized as follows. In the next section the mathematical description of the above problem is presented. In the following section the theoretical framework of this approach is presented and the description of the solution technique to obtain the robust control law as a function of the state variables. In section 4, an example is presented to illustrate the basic idea of this work and in section 5 the derived conclusions are discussed.

2. PROBLEM FORMULATION

The mathematical representation of a process system described by linear, time-invariant, discrete-time process model and subject to constraints as well as logical rules is considered:

$$\left.\begin{aligned} &\boldsymbol{x}_{t+k+1|t} = \boldsymbol{A}\boldsymbol{x}_{t+k|t} + \boldsymbol{B}\boldsymbol{u}_{t+k|t} + \boldsymbol{H}\boldsymbol{w}_{t+k|t} \\ &\boldsymbol{y}_{t+k|t} = \boldsymbol{C}\boldsymbol{x}_{t+k|t} \\ &\boldsymbol{h}(\boldsymbol{x}_{t+k|t}, \boldsymbol{u}_{t+k|t}, \boldsymbol{w}_{t+k|t}) \le 0 \\ &\boldsymbol{x}_{t|t} = \boldsymbol{x}_o \end{aligned}\right\} k \ge 0 \quad (1)$$

where t refers to time and k to the time instants ahead, $\boldsymbol{x}_t \in \mathrm{X} \subset \Re^n$ is the vector of process states, $\boldsymbol{y}_t \in \Re^r$ the vector of measurable outputs, $\boldsymbol{u}_t \in \mathrm{U} \subset \Re^s$ the vector of control inputs, $\boldsymbol{w}_t \in \mathrm{W} \subset \Re^q$ a vector to describe persistent, time varying disturbance inputs and the matrices $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}$ and $\boldsymbol{H}$ are of dimensions $n \times n$, $n \times s$, $r \times n$ and $n \times q$ respectively. The sets X, U and W are defined by constraints of the following form:

$$\left.\begin{aligned} &\boldsymbol{x}^L \le \boldsymbol{x}_{t+k|t} \le \boldsymbol{x}^U \\ &\boldsymbol{u}^L \le \boldsymbol{u}_{t+k|t} \le \boldsymbol{u}^U \\ &\boldsymbol{w}^L \le \boldsymbol{w}_{t+k|t} \le \boldsymbol{w}^U \end{aligned}\right\} k=0,1,\ldots,N_c \quad (2)$$

which are included in h along with other performance specification constraints; N_c is the control horizon and superscripts L and U refer to lower and upper bounds respectively.

From now on, a disturbance sequence satisfying $\boldsymbol{w}_t \in \mathrm{W} \subset \Re^q$ will be called *admissible*. Throughout this paper, the following assumptions hold:

- The pair $(\boldsymbol{A}, \boldsymbol{B})$ is stabilizable.
- The sets X, U and W contain the origin as an interior point.
- U and W are compact sets.

Based on this information the initial MPC formulation is set as follows:

$$\min_{U=\{u_t,\ldots,u_{t+Ny-1}\}} J\left(\boldsymbol{U}, \boldsymbol{x}(\boldsymbol{t})\right)$$

$$\begin{aligned} s.t. \quad &\boldsymbol{x}_{t+k+1|t} = \boldsymbol{A}\boldsymbol{x}_{t+k|t} + \boldsymbol{B}\boldsymbol{u}_{t+k|t} + \boldsymbol{H}\boldsymbol{w}_{t+k|t} \quad (3) \\ &\boldsymbol{y}_{t+k|t} = \boldsymbol{C}\boldsymbol{x}_{t+k|t}, \\ &\boldsymbol{h}(\boldsymbol{x}_{t+k|t}, \boldsymbol{u}_{t+k|t}, \boldsymbol{w}_{t+k|t}) \le 0, \quad k=0,1,\ldots,N_c \\ &\boldsymbol{x}_{t|t} = \boldsymbol{x}_o \end{aligned}$$

where $J\left(\boldsymbol{U}, \boldsymbol{x}(\boldsymbol{t})\right)$, the objective function, is a valued cost function that depicts the process economics and N_y is the prediction output horizon. The control objective is to regulate the final state to the origin. However, the presence of persistent disturbance acting on the system has the direct meaning that it is not possible to guarantee asymptotic regulation, i.e. $\lim_{t \to \infty} \boldsymbol{x}_t = 0$. Therefore, the control objective effectively becomes to steer the final state to a neighborhood as close as possible to the origin. In accordance, problem (3) has to be solved repetitively on-line for each time interval whenever a new state estimate becomes available. In the next section an approach is proposed, which avoids the on-line computational burden by obtaining the control variables being robust to uncertainty as an explicit function of the state variables.

3. DERIVATION OF ROBUST CONTROL LAW

The inherent uncertainty embedded in mathematical models as well as the numerous uncertain conditions present in every plant, provide a challenge to look for robust algorithms within the framework of model predictive control. A robust solution is characterized by its ability to guarantee stable and feasible operation over a specific time horizon.

3.1 Stability

Although recently the concept of *robust stability* (e.g. Campo and Morari, 1987; Allwright and Papavasiliou, 1992; Scokaert and Mayne, 1998) is adopted for that kind of problems, in this paper, to guarantee asymptotic stability in the design of the robust MPC algorithm, Lyapunov theory arguments are used. Most approaches for proving stability follow the ideas of Keerthi and Gilbert (1988), who establish that under specific conditions, the valued cost function $J(\overset{*}{U}(t),t)$ attained from the optimizer is a Lyapunov function for the system. Hence, the objective function of the MPC minimization algorithm is chosen to be the next, quadratic valued cost function as follows:

$$J(\boldsymbol{U},\boldsymbol{x}(t)) \overset{\Delta}{=} \sum_{k=1}^{N_y-1}\left\|\boldsymbol{Q}\cdot\boldsymbol{x}_{t+k|t}\right\|_q + \sum_{k=0}^{N_y-1}\left\|\boldsymbol{R}\cdot\boldsymbol{u}_{t+k}\right\|_q + \left\|\boldsymbol{P}\cdot\boldsymbol{x}_{t+N_y|t}\right\|_q \quad (4)$$

where $\|\boldsymbol{Q}\cdot\boldsymbol{x}\|_q = \boldsymbol{x}^T\cdot\boldsymbol{Q}\cdot\boldsymbol{x}$ and it is assumed that $\boldsymbol{Q}=\boldsymbol{Q}^T\succeq 0$, $\boldsymbol{R}=\boldsymbol{R}^T\succ 0$ and $(\boldsymbol{Q}^{0.5},\boldsymbol{A})$ detectable. The matrix $\boldsymbol{P}$ is such that $\boldsymbol{P}\succeq 0$ and it is chosen to be the solution of the Lyapunov equation, $\boldsymbol{P}=\boldsymbol{A}^T\cdot\boldsymbol{P}\cdot\boldsymbol{A}+\boldsymbol{Q}$.

3.2 Flexibility and Feasibility Analysis

Previously proposed techniques for designing a robust model-based controller rely on either minimizing on-line the worst-case cost (e.g. Campo and Morari, 1987) or incorporating a set of robustness constraints (e.g. Badgwell, 1997). In this work however, the issue of robustness is tackled using elements of *feasibility analysis theory*. The requirement to ensure feasibility of the MPC solution over the whole horizon considered and for the whole set of uncertain parameters can be cast mathematically as:

$$\exists \boldsymbol{u_t} \in \mathrm{U}:$$
$$\max_{\boldsymbol{w_t}}\min_{\boldsymbol{u_t}}\max_{j\in J}\left\{\boldsymbol{h}_j(\boldsymbol{x_t},\boldsymbol{u_t},\boldsymbol{w_t})\right\}\le 0,\ \forall j\in J, t\ge 0 \quad (5)$$

where j is an index over the set J of the constraints. This mathematical statement would complicate the solution of an MPC algorithm due to its max-min-max form. In order to impose this feasibility constraint, a multiperiod-type (Grossmann and Sargent, 1978; Grossmann, *et al.,* 1983) formulation is adopted in this paper.

Here, using the ideas of Grossmann, *et al.* (1983) we rigorously ensure feasible operation for the MPC controller over the specified set of bounded uncertainty values and the considered horizon. A finite number of critical points, $\boldsymbol{w^c}$, in $\boldsymbol{w}$-space are selected, so that by ensuring feasibility of the controller for those points, one can guarantee that the feasibility constraint (5) will be satisfied. In accordance with Halemane and Grossmann (1981), if the constraint functions h_j are jointly convex in $\boldsymbol{u}$ and $\boldsymbol{w}$, the global and local solutions to the subproblem (5) that lead to critical points w^c must lie at the vertices of the polyhedron W that defines the parameter space. This then implies that if the MPC controller (4) subject to (1) can be guaranteed to be feasible at the vertices of W, it can also be guaranteed to be feasible for all $w\in$ W. This theoretical approach then, allows one to replace the max-min-max constraint with the next set of constraints:

$$\boldsymbol{h}_j\left(\boldsymbol{x}_t^i,\boldsymbol{u}_t,\boldsymbol{w}_t^i\right)\le 0,\quad \forall j\in J,\ i\in I,\ t\ge 0 \quad (6)$$

where i is the index over the set of the critical uncertainty values as well as the nominal ones, I.

3.3 Robust MPC

The analysis discussed above, results in the next formulation:

$$\min_{U=\{\boldsymbol{u}_t,\ldots\boldsymbol{u}_{t+Ny-1}\}} \boldsymbol{J}(U,\boldsymbol{x}(t)) = \sum_{k=1}^{N_y-1}\left\|\boldsymbol{Q}\cdot\boldsymbol{x}_{t+k|t}\right\|_q + \sum_{k=0}^{N_y-1}\left\|\boldsymbol{R}\cdot\boldsymbol{u}_{t+k}\right\|_q + \left\|\boldsymbol{P}\cdot\boldsymbol{x}_{t+N_y|t}\right\|_q,\ \boldsymbol{w}_{t+k|t}=\boldsymbol{w}^N$$

$$\begin{aligned} s.t.\quad & \boldsymbol{x}^i_{t+k+1|t} = \boldsymbol{A}\boldsymbol{x}^i_{t+k|t}+\boldsymbol{B}\boldsymbol{u}_{t+k|t}+\boldsymbol{H}\boldsymbol{w}^i_{t+k|t} \\ & \boldsymbol{y}^i_{t+k|t} = \boldsymbol{C}\boldsymbol{x}^i_{t+k|t} \\ & \boldsymbol{h}\left(\boldsymbol{x}^i_{t+k|t},\boldsymbol{u}_{t+k|t},\boldsymbol{w}^i_{t+k|t}\right)\le 0,\quad k=0,1,\ldots,N_c \\ & \boldsymbol{x}_{t|t}=\boldsymbol{x_o} \end{aligned} \quad (7)$$

that guarantees robust operation for the considered MPC problem, (3), where $\boldsymbol{w}^N$ is the nominal value of the uncertainty considered.

3.4 Multi-parametric quadratic optimisation

Parametric programming is used in operations research for addressing parameter variations in mathematical programs. The key feature of parametric programming, which distinguishes it from the classical sensitivity analysis, is that the optimal solution is characterized with respect to the full range of parameter variations. Programs that depend on a vector of parameters are called *multi-parametric programs*. Thus, by treating $\boldsymbol{x_o}$ as parameters, problem (7) is a multi-parametric Quadratic Program (mp-QP).

Performing algebraic manipulations on process model (1), the following recursive equation is derived:

$$x_{t+k} = A^k \cdot x_o + \sum_{j=0}^{k-1} A^j \cdot B \cdot u_{t+k-j-1} + \sum_{j=0}^{k-1} A^j \cdot H \cdot w_{t+k-j-1},\ \forall k = 1,2,...,N_y \quad (8)$$

Introducing (8) into the objective and the constraints in problem (7) and carrying out the appropriate substitutions and manipulations the optimisation problem (7) is transformed to an equivalent finite dimensional problem of the following form:

$$\min_U J\ (U, x_o) = \frac{1}{2}\left(U^T L_1 U + x_o{}^T L_2 U + x_o{}^T L_3 x_o\right)$$
$$s.t. \quad G_1^i \cdot U \le G_2^i + G_3^i \cdot x_o,\ \forall i \in I \quad (9)$$

where $L_1, L_2, L_3, G_1, G_2, G_3$ are constant matrices of appropriate dimensions derived from (8) and (7). The solution of this quadratic program for all x_o provides a complete map of the robust control actions as a function of the current state values at any given time instant. A way to perform that would be to a priori process a very large number of optimisations for different values of x_o. This method is prohibitive due to its computational intensity and its inability to guarantee accurate solutions. On the contrary, parametric programming can be used to derive the explicit solution of the problem avoiding these computational difficulties. Problem (9) is recast as a multi-parametric quadratic program (mp-QP) and can be solved with the method proposed by Dua, *et al*. (2002).

The solution method for mp-QPs provides a set of V piecewise affine profiles (linear expressions) $u^v(x_o)$ for the robust control law for all admissible initial states x_o. The solution is partitioned in the x_o- space into a set of regions of optimality CR^v that are characterized by certain solution properties (e.g. activity of inequality constraints, optimal solution profile). The final parametric solution provides the relation between the control and the current state of the system that ensures the robustly optimum system regulation. The derived control law is proved to be (Dua et al., 2002) piecewise affine with respect to the states and is of the form:

$$u^v(x_o) = a^v \cdot x_o + b^v, \quad v \in V \quad (10)$$

$$CR^v: \ c^v \cdot x_o + d^v \le 0, \quad v \in V \quad (11)$$

where V is the number of region in the state space, a^v, c^v are matrices and b^v, d^v vectors that are determined from the solution of the parametric programming problem.

3.5 Robust MPC Algorithm

The above theoretical analysis can be summarized in the following robust MPC algorithm (Kakalis, 2001) consisting of four steps:

Step I: Calculate the critical disturbance values, w^i, as described in section 3.2.

Step II: Formulate the initial MPC problem of the form (3).

Step III. Use the critical disturbance values, w^i, derived from Step I to reformulate (3) into the robust MPC problem of the form (7) and use (8) to reformulate (7) as an mp-QP (9).

Step IV. Solve program (9) as an mp-QP using the method of Dua, *et al.* (2002) to derive a set of V linear parametric solutions $u(x_o)$ and corresponding regions of optimality in the form of (10) and (11).

3.6 Remarks

- This approach (7) to feasibility of the MPC controller along with the stability assurance showed before leads to robust operation for every possible disturbance – uncertainty – realization without exploring the complete uncertainty space and corresponds to a less conservative control action since it minimizes the nominal and not the worst-case quadratic cost.
- The critical uncertainty values for systems like (1) can be derived by procedures similar to the one described in Kakalis (2001), where the "depth" of the MPC controller's feasible region is explored by studying the corresponding feasibility functions derived using parametric optimisation (Bansal, *et al.,* 2000).

4. ILLUSTRATIVE EXAMPLE

Consider the next simple SISO example which is a second order system with persisting additive, zero mean (W^N) disturbance considered and where discrete-time, state-space representation is given by the following model:

$$x_{t+1} = \begin{bmatrix} 0.7326 & -0.0861 \\ 0.1722 & 0.9909 \end{bmatrix} \cdot x_t + \begin{bmatrix} 0.0609 \\ 0.0064 \end{bmatrix} \cdot u_t + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \cdot w_t \quad (12)$$
$$y_t = \begin{bmatrix} 0 & 1.4142 \end{bmatrix} \cdot x_t$$

The task is to regulate this system to the origin while fulfilling the following constraints for output horizon $N_y = 2$:

$$-1 \le x_o \le 2.2 \quad (13)$$

$$x_t \ge -0.8, \quad t = 1,..., N_y \quad (14)$$

$$-5 \le u_t \le 5,\ t = 1,..., N_y - 1 \quad (15)$$

$$-0.2 \le w_t \le 0.2,\ t = 1,..., N_y - 1 \quad (16)$$

For illustrative purposes, first consider the nominal case ($w(t) = 0$). Solving this problem in the way described above, i.e. as an mp-QP, for

$R = 1$, $Q = 0.01$, $N_y = 2$, ten different control law expressions with the corresponding regions of optimality are derived, for instance:

$$\boldsymbol{u}_t^{10} = \begin{bmatrix} -20.4393 & -148.3393 \end{bmatrix} \cdot \boldsymbol{x}_o - 125 \qquad (17)$$

$$CR^{10} = \begin{cases} -\boldsymbol{x}_{o,2} \le 1 \\ -\boldsymbol{x}_{o,1} - 5.75436 \cdot \boldsymbol{x}_{o,2} \le 4.8316 \\ \boldsymbol{x}_{o,1} + 7.3092 \cdot \boldsymbol{x}_{o,2} \le -6.16615 \end{cases} \qquad (18)$$

Performing simulations on these results from the starting point $\begin{bmatrix} 1.6 & 1.6 \end{bmatrix}^T$, it can be seen in Figure 1, that the MPC algorithm indeed regulates the system to the origin, without any constraint violation (the state constraint (13) is shown in the graph).

Since in the control law derivation no uncertainty was taken into account, this algorithm cannot guarantee safe operation for all possible operations scenarios. Consider, for instance, that a sinusoidal type disturbance (with amplitude 0.2) enters the system. Then, as it can be seen in Figure 2, the state constraint is violated and no control law exists to recover the system leading to an undesired constraint violation. The remedy for such a case is to *design robust MPC algorithms* that can guarantee safe operation for all possible uncertainty –disturbance- realizations. To do that, the procedure described in section 3 is followed. First, following the procedure mentioned in Remark 2, the critical disturbance values $\boldsymbol{w}_t^c = -0.2$ and $\boldsymbol{w}_{t+1}^c = -0.2$ are identified. Then, using again the same values for the tuning parameters, the robust MPC problem (7) is formulated and solved as an mp-QP to obtain eighteen different robust control law expressions with the corresponding regions of optimality, for instance the fifth control law is given by:

$$\boldsymbol{u}_t^5 = \begin{bmatrix} -12.0296 & 1.4138 \end{bmatrix} \boldsymbol{x}_o - 9.85223 \qquad (19)$$

$$CR^5 : \begin{cases} -\boldsymbol{x}_{o,1} \le 1 \\ \boldsymbol{x}_{o,1} + 102.612 \boldsymbol{x}_{o,2} \le 70.9213 \\ \boldsymbol{x}_{o,1} - 1.49584 \boldsymbol{x}_{o,2} \le -1.82609 \end{cases} \qquad (20)$$

Performing again simulations on the robust, now control law, starting from the same starting point, it is shown in Figures 3, 4 and 5 that the robust MPC algorithm regulates the system to the origin, without any constraint violation despite the disturbance.

This simple illustrative example shows clearly how the proposed method can guarantee the robust performance of an MPC controller providing thus a plant with stable and safer operation.

5. CONCLUSIONS

In this paper a framework is presented for deriving explicitly the robust MPC control law for process systems represented by linear, discrete - time, time-invariant process models. The robust control policy

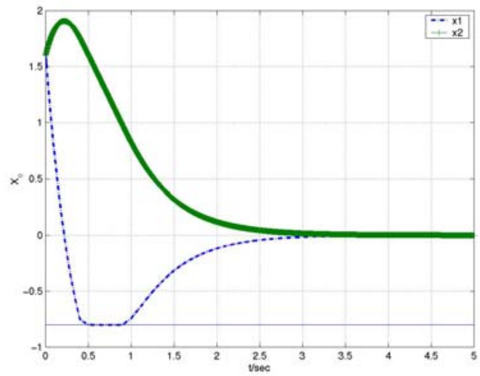

Fig. 1. Trajectory profile for the nominal MPC.

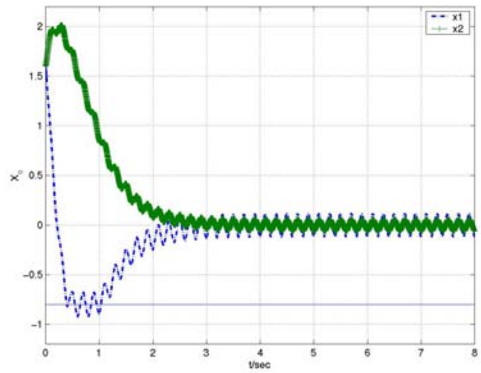

Fig. 2. Trajectory profile with disturbance for the nominal MPC.

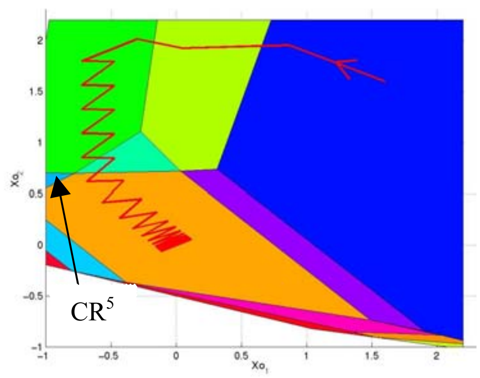

Fig. 3. Region partition of the robust optimal control policy

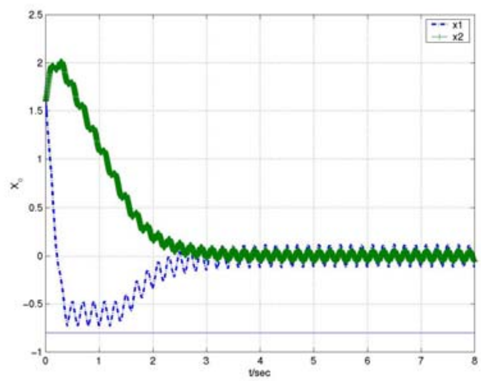

Fig. 4. Trajectory profile for the robust MPC .

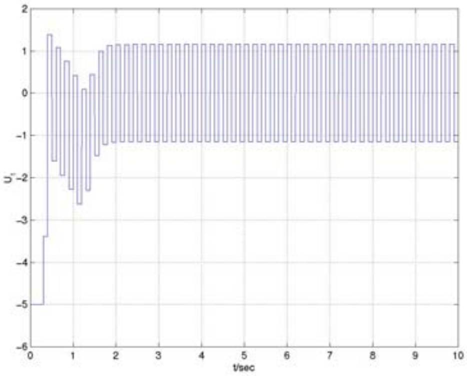

Fig. 5. Control profile for the robust MPC problem.

is given by a piecewise affine form as a function of the current state variables and it is shown that it guarantees robust and safe operation. The on-line implementation of the control action is achieved by simple function evaluations for the measurements that specify the system state. Recently, this work has been extended to proportional integral (PI) controllers (Sakizlis, *et al.,* 2002) and our current work focuses on extending it to general nonlinear systems.

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