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ADAPTIVE EXTREMUM SEEKING CONTROL OF NONLINEAR DYNAMIC SYSTEMS WITH PARAMETRIC UNCERTAINTIES

M. Guay *,[1] **T. Zhang** *

* *Department of Chemical Engineering, Queen's University, Kingston, Ontario, Canada K7L 3N6*

Abstract: We pose and solve an extremum seeking control problem for a class of nonlinear systems with unknown parameters. Extremum seeking controllers are developed to drive system states to the desired set-points that optimize the value of an objective function. The proposed adaptive extremum seeking controller is "inverse optimal" in the sense that it minimizes a meaningful cost function that incorporates penalty on both the performance error and control action. Simulation studies are provided to verify the effectiveness of the proposed approach.

Keywords: Nonlinear systems, extremum seeking, inverse optimality, Lyapunov function, adaptive control

1. INTRODUCTION

Most adaptive control schemes in the literature ((Landau 1979), (Goodwin and Sin 1984), (Astrom and Wittenmark 1995), (Narendra and Annaswamy 1989), (Ioannou and Sun 1996), (Krstic *et al.* 1995)) are developed for regulation to known set-points or tracking known reference trajectories. In some applications, however, the control objective is the optimization of an objective function which may depend on unknown plant parameters, or the selection of the desired states to keep a performance function at its extremum value. Self-optimizing control and extremum seeking control are two methods to handle these kinds of optimization problems. The task of extremum seeking is to find the operating set-points that maximize or minimize an objective function. Since the early research work on extremum control in the 1920's (Leblanc 1922), many successful applications of extremum control approaches have been reported, (see (Vasu 1957), (Astrom and Wittenmark 1995), (Sternby 1980) and (Drkunov *et al.* 1995) for example). Although a large amount of research efforts has been done, a solid theoretical foundation has not yet been established for the stability and performance of extremum seeking control.

Recently, Krstic et. al ((Krstic and Deng 1998), (Krstic 2000), (Krstic and Wang 2000)) presented several extremum control schemes and stability analysis for extremum-seeking of linear unknown systems and a class of general nonlinear systems. Applications of these approaches have been reported for the maximization of pressure rise in an axial-flow compressor (Wang *et al.* 1998) and the maximization of biomass production rate in a continuous stirred tank bioreactor ((Wang *et al.* 1999),(Nguang and Chen 2000)).

In this paper, we investigate a different class of extremum seeking problems for nonlinear systems with parametric uncertainties. Unlike conventional extremum seeking schemes, the objective function to be maximized is not directly measurable for feedback. In contrast to the completely unknown objective function considered in (Krstic 2000), we require an explicit structure information for the objective function that depends on system states and unknown plant param-

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eters. The inverse optimal design technique is used to develop the extremum seeking controller. The paper is organized as follows. Section 2 presents some notations and the problem formulation. In Section 3, an inverse optimal extremum seeking controller is developed under the assumption that all plant parameters are known. Section 4 includes an optimal design of adaptive extremum seeking controller when unknown parameters exist in both plant model and the performance function. Numerical simulation results are shown in Section 5. A brief conclusion is given in Section 6.

2. PROBLEM

Consider an objective function of the form

$$y = p(x_p, \theta_p) \tag{1}$$

where $\theta_p \in R^p$ denotes a parameter vector satisfying

$$\theta_p \in \Omega_\theta = \left\{ \theta_p \in R^p \;\middle|\; \frac{\partial^2 p(x_p, \theta_p)}{\partial x_p \partial x_p} \leq c_0 I < 0 \right. \\ \left. ,\;\; x_p \in R^m \right\}. \tag{2}$$

The state $x_p \in R^m$ is driven by

$$\dot{x}_p = f(x) + F_p(x)\theta_p + F_q(x)\theta_q + G(x)u \tag{3}$$

where $x = [x_p^T \; x_s^T]^T \in R^n$ and $u \in R^m$ are system states and control input, respectively, $\theta_q \in R^q$ is a parameter vector, $f(x) : R^n \to R^m$ is a smooth vector-valued function, $F_p(x) : R^n \to R^{m\times p}$, $F_q(x) : R^n \to R^{m\times q}$ and $G(x) : R^n \to R^{m\times m}$ are smooth matrix-valued functions. We see from (1) and (3) that x_p and θ_p represent the system states and the parameters involved in the objective function. $x_s \in R^{n-m}$ denotes the remaining states that do not contribute to the objective function directly. In this paper, we assume that x_s is within a compact subset (or has been stabilized through feedback control).

The performance function $y = p(x_p, \theta_p)$, which may not be available for on-line feedback, is a smooth function to be optimized. The optimal extremum seeking problem is to design a controller u, which is optimal with respect to a meaningful cost function, such that the output y achieves its maximum value.

From the condition $\frac{\partial^2 p(x_p,\theta_p)}{\partial x_p \partial x_p} \leq c_0 I < 0$ given in (2), we see that the performance function $p(x_p, \theta_p)$ is strictly convex. According to the theorem of Global Solutions of Convex Programs in (Nash and Sofer 1996), there exists a unique constant vector x_p^* such that $\left.\frac{\partial p(x_p,\theta_p)}{\partial x_p}\right|_{x_p = x_p^*} = 0$. This means that the output $y = p(x_p, \theta_p)$ achieves its maximum at x_p^*. Hence the extremum seeking problem is solved if the system state x_p converges to x_p^*. Since $p(x_p, \theta_p)$ contains unknown parameter θ_p, the desired set-point x_p^* cannot be obtained directly. In some applications, even if the exact value of θ_p is known, an analytical expression of x_p^* may not be available due to the complexity of the nonlinear function $p(x_p, \theta_p)$.

Assumption 1: $G(x)G^T(x) \geq g_0 I, \forall x_p \in R^m$ with constant $g_0 > 0$.

Assumption 2: The Hessian matrix $\frac{\partial^2 p(x_p,\theta_p)}{\partial x_p \partial x_p}$ is a convex function with respect to θ_p.

Assumption 2 ensures that Ω_θ defined in (2) is a convex set. As will be shown later, the convexity of Ω_θ plays a key role in constructing a projection algorithm for the estimation of the unknown parameter θ_p.

3. ADAPTIVE EXTREMUM SEEKING

Let $\hat{\theta}_p$ and $\hat{\theta}_q$ denote estimates of the true parameters θ_p and θ_q, respectively, and $\hat{x}_p$ be a prediction of x_p generated by

$$\dot{\hat{x}}_p = f(x) + F_p(x)\hat{\theta}_p + F_q(x)\hat{\theta}_q + G(x)u \\ +Ke \tag{4}$$

with $K = K^T > 0$ and the prediction error $e = x_p - \hat{x}_p$. It follows from (3) and (4) that

$$\dot{e} = F_p(x)\tilde{\theta}_p + F_q(x)\tilde{\theta}_q - Ke \tag{5}$$

where $\tilde{\theta}_p = \theta_p - \hat{\theta}_p$ and $\tilde{\theta}_q = \theta_q - \hat{\theta}_q$. Consider a Lyapunov function candidate

$$V_a = \frac{1}{2}\left\| \frac{\partial p(x_p, \hat{\theta}_p)}{\partial x_p} - d(t) \right\|^2 + \frac{1}{2}\tilde{\theta}_p^T \Gamma_p^{-1}\tilde{\theta}_p \\ + \frac{1}{2}\tilde{\theta}_q^T \Gamma_q^{-1}\tilde{\theta}_q + \frac{1}{2}||e||^2 \tag{6}$$

where $\Gamma_p = \Gamma_p^T > 0$, $\Gamma_q = \Gamma_q^T > 0$, and $d(t) \in C^1$ is a dither signal vector that will be assigned later. Taking the time derivative of V_a, and substituting $\theta_p = \hat{\theta}_p + \tilde{\theta}_p$ and $\theta_q = \hat{\theta}_q + \tilde{\theta}_q$, we have

$$\dot{V}_a = a(x, \hat{\theta}) + b(x, \hat{\theta})u - e^T K e \\ -(\dot{\hat{\theta}}_p^T \Gamma_p^{-1} - \psi F_p(x))\tilde{\theta}_p \\ -(\dot{\hat{\theta}}_q^T \Gamma_q^{-1} - \psi F_q(x))\tilde{\theta}_q \tag{7}$$

where

$$\psi = \left[\left(\frac{\partial p(x_p, \hat{\theta}_p)}{\partial x_p} - d(t) \right) \frac{\partial^2 p(x_p, \hat{\theta}_p)}{\partial x_p \partial x_p} + e^T \right]$$

$$\phi = \left[\frac{\partial p(x_p, \hat{\theta}_p)}{\partial x_p} - d(t) \right]$$

$$a(x,\hat{\theta}) = \phi\left\{\frac{\partial^2 p(x_p,\hat{\theta}_p)}{\partial x_p \partial \hat{\theta}_p}\dot{\hat{\theta}}_p - \dot{d}(t) + \frac{\partial^2 p(x_p,\hat{\theta}_p)}{\partial x_p \partial x_p} \times \left[f(x) + F_p(x)\hat{\theta}_p + F_q(x)\hat{\theta}_q\right]\right\} \quad (8)$$

$$b(x,\hat{\theta}) = \phi\frac{\partial^2 p(x_p,\hat{\theta}_p)}{\partial x_p \partial x_p}G(x) \quad (9)$$

We propose the following parameter updating laws

$$\dot{\hat{\theta}}_q = \Gamma_q F_q^T(x)\psi^T \quad (10)$$

$$\dot{\hat{\theta}}_p = \text{Proj}\left\{\hat{\theta}_p, \Gamma_p F_p^T(x)\psi^T\right\} \quad (11)$$

where $\text{Proj}\{\cdot,\cdot\}$ denotes a projection algorithm chosen such that

$$\left\{\dot{\hat{\theta}}_p^T\Gamma_p^{-1} - \psi F_p(x)\right\}\tilde{\theta}_p \geq 0 \quad (12)$$

and $\hat{\theta}_p \in \Omega_\theta$ with Ω_θ defined in (2). Thus, $\frac{\partial^2 p(x_p,\hat{\theta}_p)}{\partial x_p \partial x_p} \leq c_0 I < 0$ is guaranteed during the parameter estimation. There exist several standard techniques to construct this projection algorithm. The reader is referred to (Goodwin and Mayne 1987)(Ioannou and Sun 1996) and the references therein for more details. By (7) and (10)-(12), the following inequality holds

$$\dot{V}_a \leq a(x,\hat{\theta}) + b(x,\hat{\theta})u - e^T K e. \quad (13)$$

Considering the control law $\tilde{u} = \alpha^*(x,\hat{\theta})$ with

$$\alpha^*(x,\hat{\theta}) = -b^T(x,\hat{\theta})\left[b(x,\hat{\theta})b^T(x,\hat{\theta})\right]^{-1} \times \left[a(x,\hat{\theta}) + |a(x,\hat{\theta})| + b(x,\hat{\theta})b^T(x,\hat{\theta})\right], \quad (14)$$

we have

$$\dot{V}_a \leq -|a(x,\hat{\theta})| - b(x,\hat{\theta})b^T(x,\hat{\theta}) - e^T K e. \quad (15)$$

By $K > 0$, it is concluded that

$$\lim_{t\to\infty} b(x,\hat{\theta})b^T(x,\hat{\theta}) = 0 \quad (16)$$

and $\lim_{t\to\infty} e = 0$. Since $\lim_{t\to\infty} b(x,\hat{\theta})b^T(x,\hat{\theta}) = 0$, it can be shown that $\lim_{t\to\infty}\left[\frac{\partial p(x_p,\hat{\theta}_p)}{\partial x_p} - d(t)\right] = 0$.

As $\lim_{t\to\infty} e = 0$, we have $\int_0^\infty \dot{e}dt = e(\infty) - e(0) = -e(0)$. This implies that $\dot{e}$ is integrable. It follows from the error equation (5) that $\dot{e}$ is a smooth function of x, e, $\hat{\theta}_p$ and $\hat{\theta}_q$. Since all these signals are bounded, we know that $\ddot{e} \in L_\infty$. This implies the uniform continuity of $\dot{e}$. By Barbalat's Lemma (Ioannou and Sun 1996), it is concluded that $\lim_{t\to\infty}\dot{e} = 0$. Let $F(x) = [F_p^T(x)\ F_q^T(x)]^T$ and $\theta = [\theta_p^T\ \ \theta_q^T]^T$. By (5), it can be seen that

$$\lim_{t\to\infty} \tilde{\theta}^T F^T(x)F(x)\tilde{\theta} = 0 \quad (17)$$

If $F^T(x)F(x)$ is positive definite, then $\tilde{\theta} = 0$ is guaranteed. However, it is impossible to satisfy this condition because $F^T(x)F(x)$ is singular at any given time. We consider the integral of $F^T(x)F(x)$ for $t \to \infty$. It follows from (17) that

$$\lim_{t\to\infty}\frac{1}{T_0}\int_t^{t+T_0}\left[\tilde{\theta}^T F^T(x)F(x)\tilde{\theta}\right]d\tau = 0 \quad (18)$$

with positive constant T_0. Since $\lim_{t\to\infty} e = 0$ and $\lim_{t\to\infty}\left[\frac{\partial p(x_p,\hat{\theta}_p)}{\partial x_p} - d(t)\right] = 0$, the adaptive laws (10)-(11) imply that $\lim_{t\to\infty}\dot{\hat{\theta}} = 0$. This means that $\tilde{\theta}$ is constant when $t \to \infty$. Hence, we obtain the condition

$$\tilde{\theta}^T\left\{\lim_{t\to\infty}\frac{1}{T_0}\int_t^{t+T_0}F^T(x)F(x)d\tau\right\}\tilde{\theta} = 0. \quad (19)$$

We are now ready to present a persistence of excitation condition for parameter convergence. If the dither signal $d(t)$ is designed to satisfy the following condition

$$\lim_{t\to\infty}\frac{1}{T_0}\int_t^{t+T_0}F^T(x)F(x)d\tau \geq c_1 I \quad (20)$$

for some $c_1 > 0$ and $x_p \in \Omega$

$$\Omega_d = \left\{x_p \;\middle|\; \frac{\partial p(x_p,\hat{\theta}_p)}{\partial x_p} = d(t),\ \ \hat{\theta}_p \in \Omega_\theta\right\} \quad (21)$$

then, the parameter error $\tilde{\theta}$ converges to zero asymptotically. Combining $\lim_{t\to\infty}\tilde{\theta} = 0$ and $\lim_{t\to\infty} e = 0$, we see from (6) that $\lim_{t\to\infty} V_a(t) = 0$.

Theorem 3.1. Consider the objective function (1) and system dynamic (3) satisfying Assumptions 1-2. If the dither signal $d(t)$ satisfies the PE condition (20), then the controller (14) with adaptive laws (10)-(11), (i) solves the adaptive extremum seeking problem, and (ii) is optimal with respect to the cost function

$$J_2 = \int_0^\infty\left[l(x,\hat{\theta}) + r(x,\hat{\theta})u^T u\right]dt \quad (22)$$

where

$$l(x,\hat{\theta}) = b(x,\hat{\theta})b^T(x,\hat{\theta}) + |a(x,\hat{\theta})| - a(x,\hat{\theta}) + 2e^T K e \quad (23)$$

$$r(x,\hat{\theta}) = \frac{b(x,\hat{\theta})b^T(x,\hat{\theta})}{a(x,\hat{\theta}) + |a(x,\hat{\theta})| + b(x,\hat{\theta})b^T(x,\hat{\theta})} \quad (24)$$

Proof: i) Re-expressing $\frac{\partial p(x_p,\hat{\theta}_p)}{\partial x_p}$ as

$$\frac{\partial p(x_p,\hat{\theta}_p)}{\partial x_p} = \frac{\partial p(x_p,\hat{\theta}_p)}{\partial x_p}\Big|_{x_p=x_p^*} + (x_p - x_p^*)^T$$
$$\times \int_0^1 \frac{\partial^2 p(x_\lambda,\hat{\theta}_p)}{\partial x_\lambda \partial x_\lambda} d\lambda$$

where $x_\lambda = \lambda x_p + (1-\lambda)x_p^*$. By $\lim_{t\to\infty} \tilde{\theta} = 0$, we have

$$\lim_{t\to\infty} \frac{\partial p(x_p,\hat{\theta}_p)}{\partial x_p}\Big|_{x_p=x_p^*} = \frac{\partial p(x_p,\theta_p)}{\partial x_p}\Big|_{x_p=x_p^*} = 0$$

Therefore,

$$\lim_{t\to\infty} \frac{\partial p(x_p,\hat{\theta}_p)}{\partial x_p} = \tag{25}$$
$$\lim_{t\to\infty}\left[(x_p - x_p^*)^T \int_0^1 \frac{\partial^2 p(x_\lambda,\theta_p)}{\partial x_\lambda \partial x_\lambda} d\lambda\right] = \lim_{t\to\infty} d(t)$$

Using the fact that $\frac{\partial^2 p(x_p,\theta_p)}{\partial x_p \partial x_p} \leq c_0 I < 0,\ x_p \in R^m$, we see that

$$\lim_{t\to\infty} \|x_p - x_p^*\| \leq$$
$$\lim_{t\to\infty}\left\| d(t)\left[\int_0^1 \frac{\partial^2 p(x_\lambda,\theta_p)}{\partial x_\lambda \partial x_\lambda} d\lambda\right]^{-1}\right\|$$
$$\leq \frac{1}{|c_0|}\lim_{t\to\infty}\|d(t)\|. \tag{26}$$

This implies that the state x_p converges to a neighborhood of the desired set-point x_p^* whose size depends on the amplitude of the injected dither signal. Subsequently, the performance function $p(x_p,\hat{\theta}_p)$ converges to a neighborhood of the maximum $p(x_p^*,\theta_p)$. It can be shown that under the assumptions stated hat the controller (14) is free from singularity. Hence, the control law (14) solves the adaptive extremum seeking problem.

ii) To prove the adaptive controller is optimal with respect to the performance index J_2, we suppose that the following controller

$$u = \alpha^*(x,\hat{\theta}) + v(t) \tag{27}$$

can also solve the adaptive extremum seeking problem. Using this controller in (13) follows that

$$\dot{V}_a \leq -|a(x,\hat{\theta})| - b(x,\hat{\theta})b^T(x,\hat{\theta}) + b(x,\hat{\theta})v(t)$$
$$-e^T K e. \tag{28}$$

Hence,

$$J_2 \leq V_a(0) - V_a(\infty) = V_a(0) \tag{29}$$

Substituting (23)-(24) and (27) into (22) and using the fact that

$$r(x,\hat{\theta})\alpha^{*T}(x,\hat{\theta})\alpha^*(x,\hat{\theta}) = a(x,\hat{\theta}) + |a(x,\hat{\theta})| +$$
$$b(x,\hat{\theta})b^T(x,\hat{\theta})$$
$$r(x,\hat{\theta})\alpha^{*T}(x,\hat{\theta}) = -b^T(x,\hat{\theta}),$$

we have

$$J_2 = \int_0^\infty \Big\{2|a(x,\hat{\theta})| + 2b(x,\hat{\theta})b^T(x,\hat{\theta}) -$$
$$2b^T(x,\hat{\theta})v(t) + 2e^T K e\Big\}dt$$
$$+ \int_0^\infty r(x,\hat{\theta})v^T(t)v(t)dt$$
$$\leq 2V(0) + \int_0^\infty r(x,\hat{\theta})v^T(t)v(t)dt. \tag{30}$$

Hence, $J_2 \leq 2V(0)$ can be guaranteed only if $v(t) = 0$. This proves that the controller (14) is optimal with respect to the cost function (22). **Q.E.D.**

4. EXAMPLE

Consider the plant

$$\dot{x}_1 = \theta_1 x_1^2 + \theta_2 x_2 + u \tag{31}$$
$$\dot{x}_2 = -x_2 + \theta_2 x_1^2 \tag{32}$$
$$y = p(x_1,\theta_1) = 1 + x_1 - \theta_1 x_1^2 \tag{33}$$

where θ_1 and θ_2 are constant parameters, and $\theta_1 > 0$. It is shown that the above system can be expressed in (3) by choosing $f(x) = 0$, $\theta_p = \theta_1$, $\theta_q = \theta_2$, $F_p(x_1) = x_1^2$, $F_q(x) = x_2$, and $G(x) = 1$. The objective of the extremum seeking design is to find an optimal controller u such that the objective function $1 + x_1 - \theta_1 x_1^2$ reaches its maximum. By (31), we see that

$$\frac{\partial p(x_1,\theta_1)}{\partial x_1} = 1 - 2\theta_1 x_1,$$
$$\frac{\partial^2 p(x_1,\theta_1)}{\partial x_1^2} = -2\theta_1 < 0.$$

Hence, the performance function $p(x_1,\theta_1)$ reaches its maximum at $x_1 = x_1^* = 1/2\theta_1$.

4.1 Simulation of the Optimal Adaptive Extremum Seeking Design

We consider the extremum seeking control for the case of unknown parameters θ_1 and θ_2. From (31) and (4), the predicted state $\hat{x}_1$ is generated by

$$\dot{\hat{x}}_1 = \hat{\theta}_1 x_1^2 + \hat{\theta}_2 x_2 + u + ke \tag{34}$$

Since $\frac{\partial^2 p(x_1,\theta_1)}{\partial x_1 \partial x_1} = -2\theta_1 < 0$, we know that if the parameter θ_1 is within a convex set $\Omega_\theta = \{\theta_1 \mid \theta_1 > 0\}$, then the convexity of $1 + x_1 - \theta_1 x_1^2$ can be guaranteed.

By $\frac{\partial p(x_1,\hat{\theta}_1)}{\partial x_1} = 1 - 2\hat{\theta}_1 x_1$, $\frac{\partial^2 p(x_1,\hat{\theta}_1)}{\partial x_1 \partial x_1} = -2\hat{\theta}_1$ and (10)-(11), the projection adaptive laws are designed as

$$\psi = \left\{2\hat{\theta}_1[1 - 2\hat{\theta}_1 x_1 - d(t)] - e\right\}$$

$$\dot{\hat{\theta}}_1 = \begin{cases} -\gamma_1 \psi x_1^2, & \text{if } \hat{\theta}_1 > \varepsilon \\ & \text{or } \hat{\theta}_1 = \varepsilon \text{ and } \psi x_1^2 \le 0 \\ 0, & \text{otherwise} \end{cases}$$

$$\dot{\hat{\theta}}_2 = -\gamma_2 \psi x_2$$

with $\hat{\theta}_1(0) \ge \varepsilon$ and a sufficiently small $\varepsilon > 0$ satisfying $\varepsilon \le \theta_1$. This projection algorithm ensures that $\hat{\theta}_1 \in \Omega_\theta$ for all time. By $\frac{\partial^2 p(x_1,\hat{\theta}_1)}{\partial x_1 \partial \hat{\theta}_1} = -2x_1$, we know from (8)-(9) that

$$\begin{aligned} a(x,\hat{\theta}) = &-\left[1 - 2\hat{\theta}_1 x_1 - d(t)\right]\left[2x_1\dot{\hat{\theta}}_1 + \dot{d}(t) \right. \\ &\left. + 2\hat{\theta}_1(\hat{\theta}_1 x_1^2 + \hat{\theta}_2 x_2)\right] \\ b(x,\hat{\theta}) = &-2\hat{\theta}_1\left[1 - 2\hat{\theta}_1 x_1 - d(t)\right]. \end{aligned}$$

The following parameters are used in the simulation experiment:

$$k = 0.1, \quad \gamma_1 = \gamma_2 = 2.0, \quad \varepsilon = 0.1$$
$$\hat{x}_1(0) = x_1(0) = 0, \quad \hat{\theta}_1(0) = 0.3, \quad \hat{\theta}_2(0) = 0.3$$

The dither signal is $d(t) = 0.3\sin(3t)exp(-0.05t)$. The exponential term used in $d(t)$ ensures that the excitation signal $d(t)$ vanishes as t increases.

Figures 1-5 present the simulation result of the inverse optimal adaptive extremum seeking control. It is shown from Figure 1 that the performance function converges to its maximum value 1.25 after $t = 30$. It is interesting to note from Figure 1 that the performance function $p(x_1, \theta_1)$ reaches the maximum several times during the transient period. This behaviour is associated with the estimation procedure. Since the parameter estimates do not converge to their true values immediately, a state prediction error remains before $t = 30$ (see Figure 3, 4 and 5). The control action in Figure 2 keeps changing to provide the persistent excitation necessary to identify the extremum point and to ensure that the estimated parameters converge to the true parameters, as confirmed from the results. It may be desirable in practice to remove the excitation signal when no further improvement is achieved.

5. CONCLUSION

We have solved a class of extremum seeking control problems for nonlinear systems with unknown parameters. The proposed extremum seeking controllers drive system states to unknown desired states that optimize the value of an objective function. In addition, the inverse optimality has been achieved in the sense that the controller minimizes a meaningful cost function. It has been shown that if the external dither signal is designed such that the persistent excitation condition is satisfied, then the proposed adaptive extremum seeking controller guarantees that the objective function converges to a neighborhood of its maximum.

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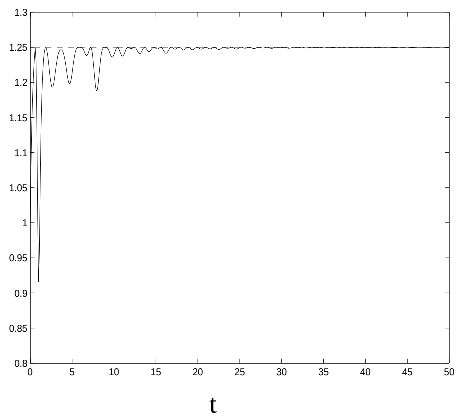

Fig. 1. Performance function $p(x_1, \theta_1)$ (“—") and its maximum $p(x_1^*, \theta_1)$ (“- -")

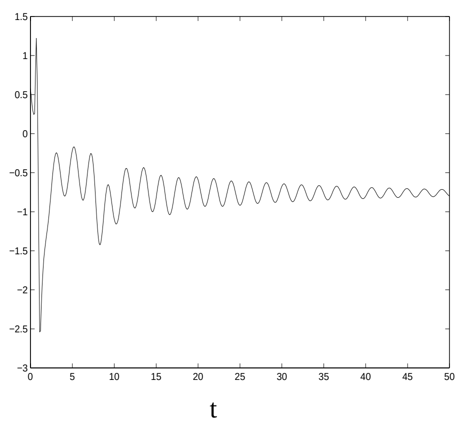

Fig. 2. Control input $u(t)$

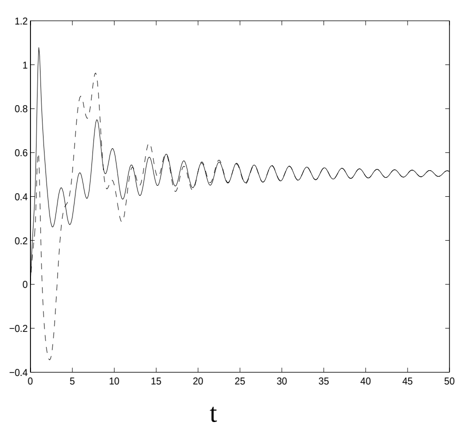

Fig. 3. State x_1 (“—") and $\hat{x}_1$ (“- -")

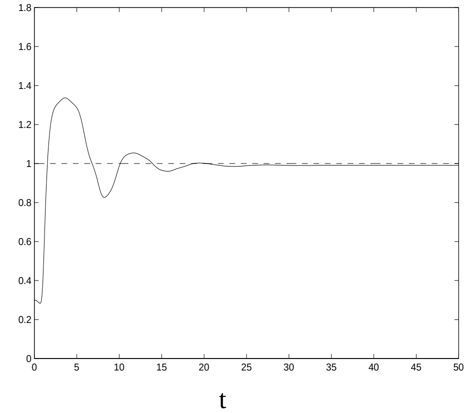

Fig. 4. Parameter θ_1 (“- -") and its estimate $\hat{\theta}_1$ (“—")

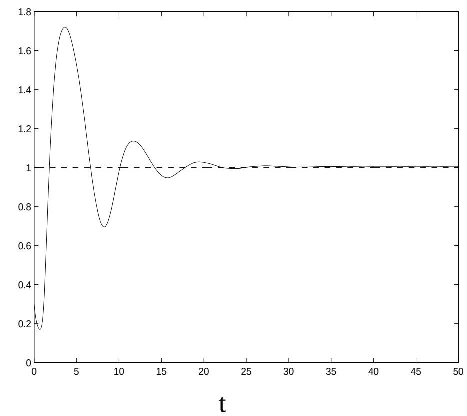

Fig. 5. Parameter θ_2 (“- -") and its estimate $\hat{\theta}_2$ (“—")