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AN ON-LINE COLOR MANAGEMENT SYSTEM BASED ON A NON-LINEAR PREDICTIVE APPROACH

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Abstract: Color is an important variable in many processes since it defines the final appearance of the product. The color management system must not only consider the non-linearities of the process but also the operational constraints. This work proposes a Predictive Controller Approach, based on a sequential linear programming algorithm, for an on-line color recipe formulation and correction. Experiments with three colorants are presented, demonstrating the performance of the proposed algorithm. *Copyright* ©2002 IFAC

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1. INTRODUCTION

In many industries color control is of a paramount importance for the final product appearance. Colorants such as dyes, pigments and brightness are usually added at some part of the process to change the color appearance of the final product.

Computer supported color recipe management is based on the Kubelka-Munk theory. This theoretical framework assumes that the colorants affect the reflectance spectrum of the product. The reflectance of the colored product is a non linear combination of the reflectance of each individual colorant in it. In addition, the comparison between two colors requires the use of non-linear transformations and weighted integrals of the reflectance curves over a certain region in the visible spectra. All these aspects make manual management extremely difficult, and linear control algorithms not very useful, since they can only be used as regulators and requiring frequent tuning when the operating conditions change.

Model predictive control is a well established technique to deal with non-linear and constrained control problems, and therefore, it is an attractive tool to be applied to the color management problem.

The paper first introduces a model of the coloring process expressed by the Kubelka Munk theory and the dynamics associated with the colorant delivery system; the predictive control problem is then posed, and solved by means of a sequential Linear Programming algorithm. Finally, simulation examples are presented and some final remarks are given.

2. MODELING THE COLORING PROCESS

The description of the coloring process is based on the Kubelka-Munk theory, describing the process as the light traverses a thin layer of paint applied to a substrate. At any location in the paint, a certain fraction of light, K, travelling in each direction will be absorbed by the material and another portion, S, will be scattered. The spectrum of the reflected light at every wavelength is the base measurement for color and it is called reflectance R. For complete hiding, the reflectance can be expressed in terms of K and S as:

$$R = 1 + \frac{K}{S} - \sqrt{2\frac{K}{S} + \left(\frac{K}{S}\right)^2}. \qquad (1)$$

The properties of mixtures of pigmented solutions can be found by using the fact that combinations of absortion and scattering are linear:

$$K_M = \sum_{i=1}^{n} K_i c_i, \qquad (2)$$

$$S_M = \sum_{i=1}^{n} S_i c_i, \qquad (3)$$

and:

$$\left(\frac{K}{S}\right)_M = \frac{\sum_{i=1}^{n} K_i c_i}{\sum_{i=1}^{n} S_i c_i}, \qquad (4)$$

where K_M is the absorbtion of the mixture, S_M the scattering of the mixture, n number of pigments in the mixture, c_i concentration of the ith pigment in the mixture by weight of dry pigment, K_i absorbtion of the ith pigment,and S_i the scattering of the ith pigment .

Each equation must be calculated for each wavelength. The units of K and S are not important for the purposes of these equations, since in the Kubelka-Munk equations they are always used in conjunction with each other, the important factor is the ratio between them (Haase and Meyer, 1992). Taking into account this consideration and given a spectral reflectance curve for R, K can be calculated from equation (1) by setting the values of S equal to one at all wavelengths. Thus, equation (4) can be written as:

$$\left(\frac{K}{S}\right)_M = \sum_{i=1}^{n} \frac{K_i}{S_i} \frac{c_i}{\sum_{i=1}^{n} c_i}, \qquad (5)$$

this equation means that the concentrations enter the system as percentages.

Several simplifying assumptions were made by Kubelka and Munk. The pigmented solution is considered as a uniform material, assuming complete dispersion of pigments and homogeneous density of pigments particles over a planar surface. There is also no account for surface reflection, since they consider diffuse lighting and viewing conditions. There has been extensive research to overcome the limitation of this theory (Mitton, 1973).

The color sensor contains a light source and a detector which measures the reflected light from the object to be measured. The spectrum of the reflected light compared to the spectrum of the source light at every wavelength is the reflectance, and it is expressed as a vector of values at each wavelength. The quality of the color adjustment is found by looking at the difference between the spectral distribution of the product and that of the target spectrum. The task of the color management system is to find the colorant concentrations so that, the best approximation of the target spectrum is reached. In general, it is imposible to obtain a perfect match, and finally the human eye has to decide on the quality of two colors. The human eye is evenly sensitive to each wavelength within the spectrum of visible light; this effect is represented by a set of weighted coefficients $\bar{\mathbf{x}}$, $\bar{\mathbf{y}}$, and $\bar{\mathbf{z}}$, called CIE standard Observer. The tristimulus values are defined by the relations:

$$X = \int \rho(\lambda)\bar{\mathbf{x}}S(\lambda)d\lambda, \qquad (6)$$

$$Y = \int \rho(\lambda)\bar{\mathbf{y}}S(\lambda)d\lambda, \qquad (7)$$

$$Z = \int \rho(\lambda)\bar{\mathbf{z}}S(\lambda)d\lambda, \qquad (8)$$

where $S(\lambda)$ is the relative spectral distribution function of the spectral power distribution of a standard illuminant, and $\rho(\lambda)$ is the spectral reflectance curve. Although, in many color control applications, the color of the material is determined by its reflectance spectrum, the more common representation of the color measurement is specified in terms of the triplet $[L, a, b]$, defined in terms of the tristimulus values as follows:

$$L = 116\left(\frac{X}{X_o}\right)^{\frac{1}{3}} - 16, \qquad (9)$$

$$a = 500\left((\frac{X}{X_o})^{\frac{1}{3}} - (\frac{Y}{Y_o})^{\frac{1}{3}}\right), \qquad (10)$$

$$b = 200\left((\frac{Y}{Y_o})^{\frac{1}{3}} - (\frac{Z}{Z_o})^{\frac{1}{3}}\right), \qquad (11)$$

where X_o, Y_o and Z_o are the tristimulus values of the light source.

The difference between two color positions is computed as the difference in the Lab space. Since the tristimulus considers the illumination, it is possible to have different samples, but with a similar look, and they are called metamers.

In on-line color monitoring and control system the colorants must be precisely added into the correct stages of the process. The colorants, in general, are delivered by positive displacement pumps and carried with a constant flow of water. As they are delivered to different points in the coloring process, the dynamic associated to each colorant can be different. In general a combination of a first order system plus a transport delay will be enough to describe the delivering process:

$$\frac{c_i(s)}{u_i(s)} = \frac{e^{-d_i s}}{s\tau_i + 1}. \qquad (12)$$

Summarizing, the final model is depicted in figure 1, the general equations can be represented as a multivariable Wiener model :

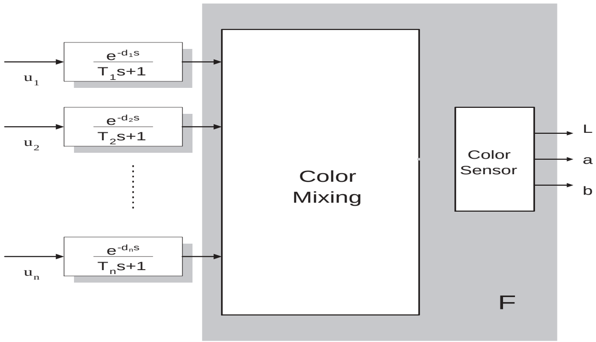

Fig. 1. Block Diagram of the Colorant Mixing Process

$$\mathbf{X}(t+1) = \mathbf{AX}(t) + \mathbf{BU}(t-\mathbf{d}) \tag{13}$$

$$\mathbf{Y}(t) = F(\mathbf{X}(t)) \tag{14}$$

where $\mathbf{X}$ is the vector of concentrations; i.e. $[c_1 c_2 \ldots c_n]'$, $\mathbf{U}(t-\mathbf{d})$ is the vector having the colorant rates $[u_1(t-d_1)u_2(t-d_2)\ldots u_n(t-d_n)]'$, and the output vector representing the color in the CIE space; i.e. $\mathbf{Y}(t) = [L\ a\ b]'$, $\mathbf{A} = diag(\tau_1 \ldots \tau_n)$. Function F represents the non-linear mapping between the colorant concentrations and the values of L, a and b.

3. THE NON-LINEAR PREDICTIVE CONTROLLER

The non-linear predictive approach offers the following advantages in dealing with the problem of an on-line color management system:

- In some cases, the color deviations are not always controllable; i.e. with a given set of active constraints it will not be possible to reach the desired color.
- It provides two alternatives to consider the degree of metamerism. The first one introduces a combination of different terms in the cost function, where each of them is calculated for a different light source or viewing conditions. In this way, the algorithm will try to keep the degree of metamerism as low as possible. The second alternative does not modify the cost function, but it defines a degree of metamerism tolerance,which is added as an extra constraint to the original problem.
- The relative cost of each colorant can be included in the cost function.

A non-linear predictive control problem is defined in terms of a cost function, a predictive model and a set of constraints. In this case, the cost function is defined as the absolute values of the color output error and the absolute values of the control increments;i.e.

$$\mathbf{J} = \sum_{i=1}^{3} |y_i^d(t+T) - y_i(t+T)| + \tag{15}$$
$$\lambda \sum_{j=1}^{N_u} \sum_{i=1}^{n} |u_i(t+j-1) - u_i(t+j-2)|,$$

where T and N_u define the predictive horizon and the control horizon, respectively. The motivation of using the $\mathbf{l}_1$ formulation stems from the existence of efficient numerical algorithms for solving $\mathbf{l}_1$ problems, and from its property that only few variables in the solution have nonzero values. As a result, a least-cost match can be obtained with a smaller set of colorant out of a bigger available set. The number of colorant to be used will also depend on a tolerated metamerism degree (Belanger, 1974).

The above cost function is subject to the following equality constraints:

$$\mathbf{X}(t+1) = \mathbf{AX}(t) + \mathbf{BU}(t-\mathbf{d}) \tag{16}$$
$$\mathbf{Y}(t) = F(\mathbf{X}(t))$$

$$\sum_{i=1}^{n} u_i(t) = 1 \tag{17}$$

$$0 \leq u_i(t) \leq 1 \tag{18}$$

Equation (17) represents the fact that the control signals are percentage of colorants. Equations (16), (15), (17), and (18) represent a non-linear optimization problem, which can be solved, using sequential linear programming. To this end, a series of variables μ_i and β_i are defined such that (Camacho and Bordons, 1998):

$$-\mu_i \leq y_i^d(t+T) - y_i(t+T) \leq \mu_i, \quad i = 1, 2, 3 \tag{19}$$

$$-\beta_i \leq u_i(t+j-1) - u_i(t+j-2) \leq \beta_i, \quad i = 1, \ldots, n, j = 1, \ldots, N_u \tag{20}$$

$$0 \leq \sum_{i=1}^{3} \mu_i + \lambda \sum_{i=1}^{N_u \times n} \beta_i \leq \gamma, \tag{21}$$

and, therefore, the cost function to be minimized is just:

$$\mathbf{I} = \gamma. \tag{22}$$

The vector of predictions $\mathbf{Y}(t+T)$ can be written as:

$$\mathbf{X}(t+T) = \mathbf{A}^T \mathbf{X}(t) + \sum_{j=0}^{T-1} \mathbf{A}^{T-j} \mathbf{BU}(t+j-\mathbf{d})$$
$$\mathbf{Y}(t+T) = F(\mathbf{X}(t+T)). \tag{23}$$

The error $\mathbf{Y}^d(t+T) - \mathbf{Y}(t+T)$ can be approximated along a given control trajectory; i.e. $u_i^k(t+j)$, $j = 1, \ldots, T-1$, $i = 1, \ldots, n$ as:

$$\mathbf{Y}^d(t+T) - \mathbf{Y}(t+T) \approx \mathbf{Y}^d(t+T) - (\mathbf{Y}^k(t+T) + \frac{\partial F}{\partial \mathbf{X}^k(t+T)}(\mathbf{X}^{k+1}(t+T) - \mathbf{X}^k(t+T)))$$

where:

$$\mathbf{X}^k(t+T) = \mathbf{A}^T\mathbf{X}(t) + \sum_{j=0}^{T-1} \mathbf{A}^{T-j}\mathbf{B}\mathbf{U}^k(t+j-\mathbf{d})$$
$$\mathbf{Y}^k(t+T) = F(\mathbf{X}^k(t+T)). \quad (24)$$

The partial derivatives $\frac{\partial F}{\partial \mathbf{X}^k(t+T)}$ can be computed from the equations (5),(1),(6),(7),(8),(9),(10) and (11).

Given the approximation (24) and a starting control trajectory, the non-linear optimization problem can be solved by addressing the following LP problem:

$$-\mu \leq \mathbf{Y}^d(t+T) - (\mathbf{Y}^k(t+T) + \quad (25)$$
$$\frac{\partial F}{\partial \mathbf{X}^k(t+T)}(\mathbf{X}^{k+1}(t+T) - \mathbf{X}^k(t+T))) \leq \mu \quad (26)$$
$$-\beta_i \leq u_i^{k+1}(t+j-1) - u_i^{k+1}(t+j-2) \leq \beta_i, \quad (27)$$
$$i = 1, \ldots, n, j = 1, \ldots, N_u$$
$$-\delta \leq u_i^{k+1}(t+j) - u_i^k(t+j) \leq \delta \quad (28)$$
$$0 \leq \sum_{i=1}^{3} \mu_i + \lambda \sum_{i=1}^{N_u \times n} \beta_i \leq \gamma, \quad (29)$$

and:

$$\sum_{i=1}^{n} u_i^{k+1}(t+j) = 1. \quad (30)$$

The inequality (28) limits the change in the control trajectory to some small values, so that, the linearization will be valid.

This approach is different to the one presented in (Norquay *et al.*, 1998). Under this formulation, the constraints are explicitly considered and no inverse mapping of the output non-linearity is used. In general, when there are constraints and multivariable non-linearities with more inputs than outputs, it is not possible to use inverse mapping.

In order to reject offset errors the following differenced prediction model is considered:

$$\mathbf{X}(t+T) = \mathbf{A}^T\mathbf{X}(t) + \sum_{j=0}^{T-1} \mathbf{A}^{T-j}\mathbf{B}\mathbf{U}(t+j-\mathbf{d})$$
$$\mathbf{Y}(t+T) = F(\mathbf{X}(t+T)) + (\mathbf{Y}_m(t) - F((\mathbf{X})) \quad (31)$$

where $\mathbf{Y}_m(t)$ represents the color measurement vector. Thus, the final block diagram for the system is given in figure 2.

The general algorithm can be summarized as follows:

1 Initialize the control signal $u_i^0(t+j)$ with $i = 1, \ldots, n, j = 1, \ldots, T-1$.

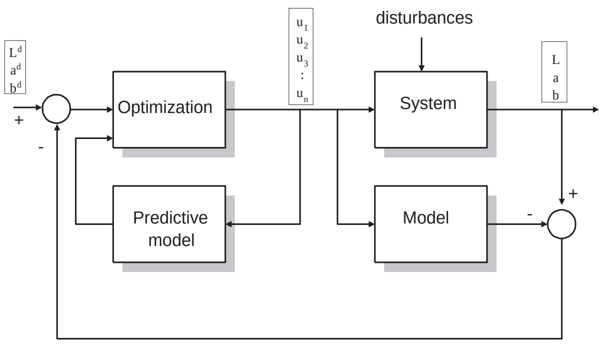

Fig. 2. Block diagram for on-line color management system.

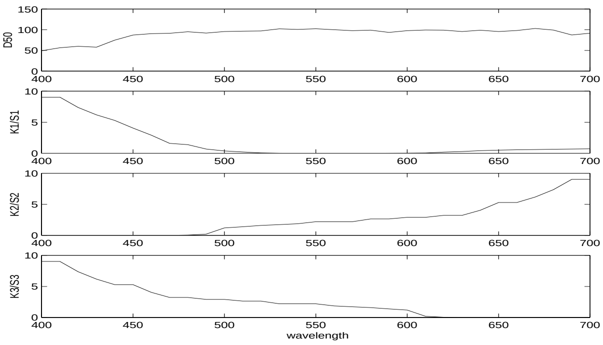

Fig. 3. Spectral reflectance for the light source and the colorants.

2 Measure the current color $\mathbf{Y}_m(t)$.
3 Solve sequentially the LP problem until $I(u_1^k(t)...u_n^k(t+T-1)) = I(u_1^{k+1}(t)...u_n^{k+1}(t+T-1))$.
4 Set the colorant rates as $u_i(t) = u_i^k(t)$, $i = 1, \ldots, n$.
5 Let $u_i^0(t+j) = u_i^k(t+j)$, $i = 1, \ldots, n$, $j = 1, \ldots, T-1$.
6 go to 2.

4. SIMULATION RESULTS

The example considers three colorants delivered at the same point in the process. Thus, the dynamic associated to each colorant is given by the following transfer function:

$$\frac{c_i(z)}{u_i(z)} = \frac{0.3z^{-3}}{z+0.7}. \quad (32)$$

In practice, the transfer functions associated to each color delivering system can be different. The lower time horizon is bounded by the dynamic characteristic of the system and the constrains (Camacho and Bordons, 1998); in this case, as the dynamic is given primarily by these transfer functions, a time horizon $T = 4$, bigger than the delay was chosen. The values for N_u and λ were 3 and 0.001, respectively.

The spectral reflectance curves of each colorant and of the light source are given in figure 3.

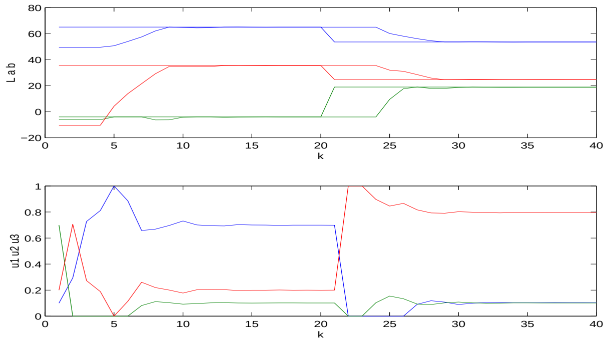

Fig. 4. Step changes in the reference color in Lab coordinates, and the control inputs.

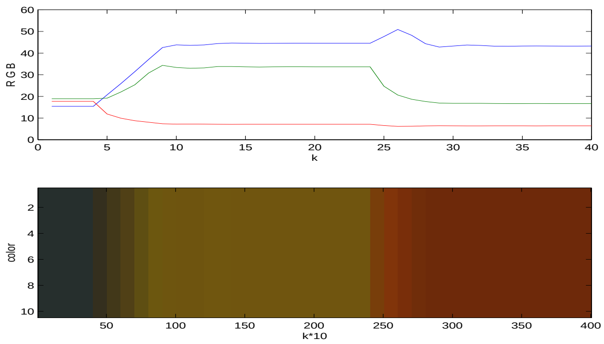

Fig. 5. The RGB coordinates and the final color.

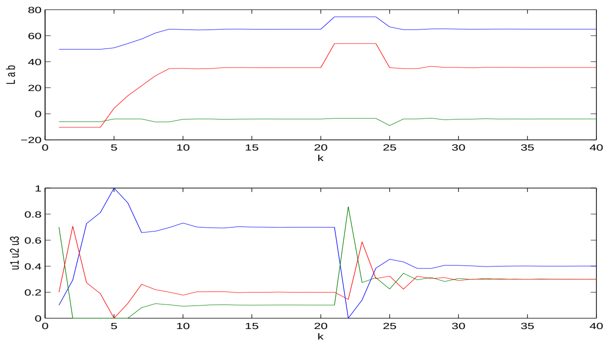

Fig. 6. Closed loop response under step disturbances

Two reference changes illustrate the response of the system to adapt the concentrations of each colorant to reach different output colors. As it can be seen in figure 4, the restrictions, concerning the inputs are satisfied all the times. The changes in color depicted in the RGB space are shown in figure 5.

The disturbance rejection capabilities are illustrated in figure 6. At time step 20, a sudden change in the concentrations occurs, and the control system is able to restore the desired color.

In order to illustrate the robustness of the approach, the same controller was applied to a system where the transfer functions associated to each colorant are:

$$\frac{c_i(z)}{u_i(z)} = \frac{0.45z^{-3}}{z + 0.77}. \quad (33)$$

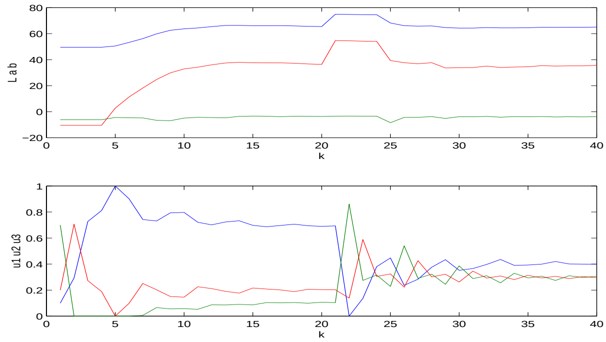

Fig. 7. Closed loop response under step disturbances

The results shown in figure 7 demonstrate that the algorithm can cope with these changes in the parameters, but the dynamic response is more oscillatory than the one obtained with a controller tuned with the right parameters .

5. FINAL REMARKS

The non-linear predictive control approach provides the right framework for dealing with the problems faced in on-line color management, since it can deal with non-linear systems and constraints. The non-linear color mixing process can be represented by a multivariable Wiener type of model, leading to a simple control algorithm. The simulation results have shown the effectiveness of the proposed approach. Further work is under way to consider experimental testing and some adaptive features for dealing with changing characteristics of both the colorants and the material.

6. ACKNOWLEDGEMENTS

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