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SELECTION OF CONTROLLED VARIABLES AND ROBUST SETPOINTS

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Abstract: Self-optimizing control is achieved if a constant setpoint policy results in an acceptable (economic) loss L (without the need to reoptimize when disturbances occur). Skogestad (2000) presented a method for selecting controlled variables based on steady-state economics. The simplest is to select the setpoints of the controlled variables equal to their nominal optimum values. In this paper we extend the method by finding the robust optimal setpoints, or equivalently the optimal back-off from the nominal, for a given set of disturbances and implementation errors. As a case study we consider a reactor-separator-recycle process. For this process the control structures based on Luybens rule ("fix a flow in every recycle loop") give infeasibility if we use the nominal optimal setpoints, but it is feasible with acceptable loss with robust optimal setpoints.

Keywords: Self-optimizing control, back-off, control structure design

1. INTRODUCTION

This paper is concerned with the *implementation* of an optimal control policy. We consider a strategy where the optimization layer sends setpoints for the controlled variables to be implemented by the control layer, see figure 1. There are two classes of problems:

- *Constrained*: The optimal solution lies at active constraints for all expected disturbances
- *Unconstrained* (the focus of this paper): One or more of the optimization degrees of freedom are unconstrained for all or some expected disturbances.

Two important decisions are to be made:

- Decision 1: *Selection of controlled variables (c)*: This is a *structural* decision which is made off-line before implementing the control strategy.

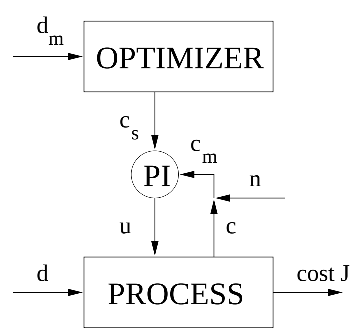

Fig. 1. A typical optimization system incorporating local feedback: The process is disturbed (d) and the control system tries to keep the controlled variables (c) at their setpoints (c_s). Steady-state optimization based on measured disturbances (d_m) is performed regularly to track the optimum by updating the setpoints.

- Decision 2: *Selection of setpoints* (c_s) for the controlled variables. This is a *parametric* decision which is usually done on-line.

For the constrained problem, we usually select the active constraints as controlled variables

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(Maarleveld and Rijnsdorp, 1970). To remain feasible it may be necessary to back off from the optimal value of the constraints, for example, when the constraints are difficult to measure or difficult to control due to poor dynamics. This is thoroughly discussed by Perkins and coworkers, e.g. Narraway *et al.* (1991). An exception to the rule of using active constraint control is when the optimal active constraint may move, and in order to avoid reconfiguration we choose to control unconstrained variables with good self-optimizing properties. For the unconstrained problem, the selection of what to control (Decision 1) is crucial. The controlled variables should yield feasible operation, that is, not violate any constraints for the expected disturbances and implementation errors. Otherwise, there may be both dynamic and steady-state problems such as instability, input saturation and operation outside constraints. To avoid such problems it may be necessary to back-off from the nominal optimum (Decision 2), for example , using robust optimization (Glemmestad *et al.*, 1999). Although the required back-off can be reduced by using logic, model predictive control and/or on-line optimization, a good choice of controlled variables may reduce the need for these remedies and give a simpler and cheaper system.

2. SOME DEFINITIONS

2.1 *Optimal operation*

From a steady-state point of view optimal operation for a given disturbance (d) can be found by solving the following problem:

$$\begin{aligned} &\min_{x,u} J(x,u,d) \\ &f(x,u,d) = 0 \\ &g(x,u,d) \leq 0 \end{aligned} \tag{1}$$

The scalar objective function J describes the quality (cost) of operation, f is the process model, g is the inequality constraints connected to operation, u is the independent variables (inputs) we can affect, d is the independent variables (disturbances) we cannot affect and x consists of internal variables, e.g. states. The inequality constraints are usually upper and lower bounds on the output and input variables.

2.2 *Feasibility*

From a steady-state point of view operation is feasible when the following constraints are fulfilled:

$$\begin{aligned} &f(x,u,d) = 0 \\ &g(x,u,d) \leq 0 \end{aligned} \tag{2}$$

A constant setpoint policy is feasible if, with constant setpoints for the controlled variables ($c(x,u,d) = c_s + e$), none of the constraints are violated for expected variations in disturbances ($d \epsilon D$) and implementation errors ($e \epsilon E$).

The implementation errors e is the sum of the measurement errors ($c_m - c$) and the control errors ($c_s - c_m$), see figure 1. We distinguish between hard and soft constraints. Soft constraints may be violated in transients, but not at steady-state (average). Hard constraints must neither be violated in transients nor at steady-state. For controlled variables related to soft constraints we should only include the steady-state implementation error which with integral action equals the steady-state measurement error. For controlled variables related to hard constraints we must also include the worst-case dynamic control error.

2.3 *Self-optimizing control*

A set of controlled variables has good self-optimizing properties, when constant setpoints yield acceptable operation for expected variation in disturbances and implementation errors (Skogestad, 2000). More precisely, the loss L should be acceptable. The loss for a given disturbance (d) and implementation error (e) is the difference between the cost by keeping a set of controlled variables constant and the cost by re-optimizing

$$L(d,e) = J(c_s + e, d) - J_{opt}(d) \tag{3}$$

Figure 3 shows loss as function of disturbances for different sets of controlled variables. One problem is that in general it is not clear off hand whether such a self-optimizing controlled variable set exists.

2.4 *Back-off*

Back-off from nominal optimal setpoints is sometimes needed to achieve feasible operation, see Figure 2. *The "back-off" is the difference between the actual setpoints and the nominal optimal setpoints $b = c_s - c_{s,0}$.* The optimal back-off is

$$b_{opt} = c_{s,robust} - c_{s,0} \tag{4}$$

where $c_{s,0}$ is the nominal optimal setpoints found by solving equation (1) with respect to nominal disturbances (d_0), and the robust optimal setpoints $c_{s,robust}$ are found by minimizing the "mean" weighted cost (wJ) over all disturbances (d) and implementation error (e) (Glemmestad *et al.*, 1999):

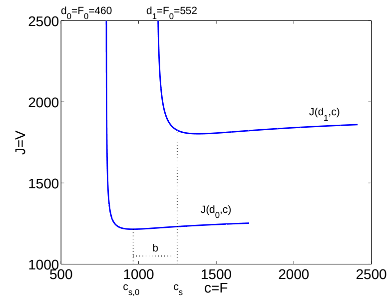

Fig. 2. Cost (J) as function of the controlled variable (c) at nominal point (d_0, lower curve) and with disturbance (d_1, upper curve). With the setpoint fixed at the nominal optimum $c_s = c_{s,0}$) we get infeasibility with the disturbances, but with back-off ($c_s = c_{s,0} + b$) we get close to optimal operation in both cases. Data are from the recycle case study with boilup as the cost ($J = V$), feedrate as the disturbance ($d = F_0$), and distillation feedrate as the controlled variable ($c = F$).

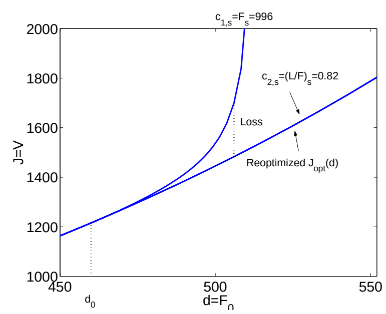

Fig. 3. Loss as a function the disturbance with reoptimized setpoint (lower curve) and constant setpoint. Data are from the recycle case study with boilup as the cost ($J = V$), feedrate as the disturbance ($d = F_0$), and distillation feedrate ($c_1 = F$) and reflux ratio as alternative controlled variable ($c_2 = L/F$). We see that the loss is negligible with c_2 as a controlled variable.

$$
\begin{aligned}
\min_{x_i, u_i, c_s} \sum_i & w_i J(x_i, u_i, d_i) \\
f(x_i, u_i, d_i) &= 0 \\
g(x_i, u_i, d_i) &\leq 0 \\
c(x_i, u_i, d_i) &= c_s + e_i \\
d_i &\in D \\
e_i &\in E
\end{aligned}
\tag{5}
$$

The problem is infinite dimensional, but we have here simplified it by considering a discrete number of operation points ($i = 0, n$ where 0 denotes the nominal point and n is the number of “disturbed” operation points): The weights w_i in the objective (cost) function can be chosen in different ways. Using a nominal objective ($w_0 = 1$, $w_i = 0$ when $i \neq 0$) gives zero backoff if the nominal optimal setpoints are feasible. Preferably, the weights should be chosen equal to the probability for operation in the respective points. However, feasibility may be very important, and this can be handled by distinguishing between an economic and a feasible region, see figure 4, where we use $w_i = 0$ outside the economic region. The constraints must be fulfilled in the feasible region, whereas the cost is average in the economic region.

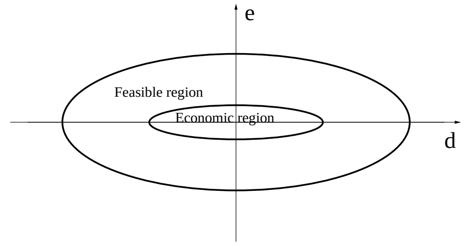

Fig. 4. Feasible and economic region

3. METHOD FOR SELECTING CONTROLLED VARIABLES AND ROBUST SETPOINTS

In the method presented by Skogestad (2000) the nominal optimal setpoints were used to identify promising sets of controlled variables. Here we focus on achieving feasible operation by implementing setpoints found by robust optimization (”optimal back-off”). We use a five step procedure:

(1) *Initial system analysis*:
Identify the number of degrees of freedom, define objective function and constraints, identify main disturbances and candidates for controlled variables, optimize at nominal and for expected disturbances (equation 1).
(2) *Identify sets of candidate controlled variables*:
Eliminate variables with no steady-state effect, use active constraint control, eliminate variables with large losses by using short-cut loss evaluation, eliminate variables based on process insight.
(3) *Evaluate loss and select setpoints for different sets of controlled variables, by using nominal optimization.*
(4) *Evaluate loss and select setpoints for different sets of controlled variables, by using robust optimization* (equation 5).
(5) *Final evaluation and selection of control structures*:
Stabilization, controllability analysis, selection of control configuration and simulation of proposed control structures.

The method is applied to a reactor, separator and recycle process in next section.

An alternative to initial screening (step 2) before evaluating the loss (step 3 and 4) is mathematical programming to find sets of controlled variables which imply small losses. The robust optimization is then the inner problem in a MINLP-problem. If including a controllability test (step 5) for different sets of controlled variables, the selection of controlled variables is done automatically.

4. EXAMPLE: REACTOR, SEPARATOR AND RECYCLE PROCESS

The process consists of reactor, distillation column and liquid recycle. There is no inert in the feed, and no purge is required. The model parameters and operation data are from Wu and Yu (1996). Larsson *et al.* (1999) identified promising sets of controlled variables for this process using nominal optimal setpoints.

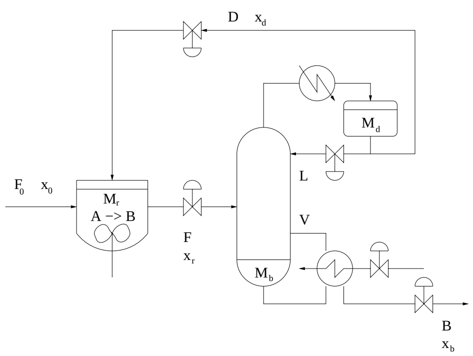

Fig. 5. Reactor/separator process with liquid recycle

4.1 *Initial system analysis*

The process has five manipulated variables (valves) which give five dynamic degrees of freedom.

$$u^T = [L\ V\ B\ D\ F]$$

However, two of them (the reboiler holdup (M_b) and condenser holdup (M_d)) have no steady-state effect. There are then 3 degrees of freedom at steady-state. These may, for example, be selected as the reactor holdup (M_r), product composition (x_b) and recycle composition (x_d). The economic objective is to maximize the profit (the value of the products minus the cost of the utilities and raw materials). Since F_0 is given, B is given and L depends directly on V, the objective can be simplified to minimize the boilup flow rate:

$$J = V$$

The reactor volume and product purity are constrained:

$$0 \leq M_r \leq 2800$$
$$x_b \leq 0.015$$

The main disturbances are feed flow rate (F_0) and feed composition (x_0):

$$d^T = [F_0\ x_0] = [460 \pm 92 \text{ kmol/h } 0.90 \pm 0.05 \text{ molA/mol}]$$

We perform steady-state optimization, see (1), at the nominal operation point and for the four corner-points of the two disturbances. We find that the product composition (x_b) and the reactor holdup (M_r) are always at their constraints, leaving one unconstrained degree of freedom.

The 20 candidate controlled variables (9 manipulated variables and measurements and 11 flow ratios) are:

$$c^T = [u\ x_r\ x_b\ x_d\ L/F\ V/F\ B/F\ D/F\ V/L\ ...$$
$$B/L\ D/L\ B/V\ D/V\ B/D\ F/F_0]$$

The implementation error for the unconstrained variables are initially assumed as $\pm10\%$ for flow rates, $\pm0.5\%$ (absolute) for compositions and $\pm1\%$ for holdups. The average L is computed with $w_i = 1$ for the nominal point and the $n = 10$ perturbed points (two for each of the two disturbances and two for each of the three implementation errors).

4.2 *Identify sets of candidate controlled variables*

The 20 candidate controlled variables and three steady-state degrees of freedom give 1140 alternative sets of controlled variables, and we need to reduce the number of sets. We choose to control the active constraints, and then left with 18 candidate controlled variables and 1 steady-state degree of freedom, which give 18 possible sets.

Initial screening is performed by maximizing the steady-state gain ($|G(0)|$) where $G(0)$ is obtained with the active constraints $c_1 = x_b$ and $c_2 = M_r$ kept constant (Skogestad, 2000). The candidate controlled variables c_3 are scaled with respect to variation in optimal values and implementation errors. From Table 1 we see that x_d and L/F are the most promising controlled variables. However, this screening is based on a linear (local) analysis and need to be interpreted with care, although we will find that the ranking is surprisingly accurate.

At steady-state the product flow rate must be equal to the feed flow rate ($B = F_0$). Thus, keeping the product flow rate B constant when the feed flow rate changes, does not give feasible steady-state operation. Hence the product flow rate (B) is eliminated as a candidate controlled variable (agrees with the gain analysis in Table 1). Furthermore, the product flow rate (B) is given by the component balance of the product: $B = kM_rx_r/x_b$ Here M_r and x_b are controlled at their active constraints, whereas, as just noted, $B = F_0$. Thus the reactor composition (x_r) is fixed and can be eliminated as a candidate controlled variable (also agrees with the gain analysis in Table 1). 16 candidate controlled variables and 1 steady-state degree of freedom still remain, which give 16 possible sets.

Table 2. Average costs (wJ) and loss (L) when using nominal optimal setpoints

Rank	c_1	c_2	c_3	$c_{1,s}$	$c_{2,s}$	$c_{3,s}$	wJ	$L[\%]$
Optimal	–	–	–	–	–	–	1231.23	0.0
1	x_b	M_r	x_d	0.9900	2772	0.8186	1322.35	7.4
2	x_b	M_r	L/F	0.9900	2772	0.8207	1324.20	7.6
3	x_b	M_r	D/L	0.9900	2772	0.6379	1324.17	7.6
4	x_b	M_r	D/V	0.9900	2772	0.3893	1325.87	7.8
5	x_b	M_r	V/F	0.9900	2772	1.3446	1331.10	8.2
6	x_b	M_r	L	0.9900	2772	793	1336.88	8.8
7	x_b	M_r	V/L	0.9900	2772	1.6585	1329.79	9.9
8	x_b	M_r	B/D	0.9900	2772	0.9098	1330.97	11.0
–	x_b	M_r	B/F	0.9900	2772	0.4764	infeas	infeas
–	x_b	M_r	B/L	0.9900	2772	0.5804	infeas	infeas
–	x_b	M_r	B/V	0.9900	2772	0.3544	infeas	infeas
–	x_b	M_r	D	0.9900	2772	506	infeas	infeas
–	x_b	M_r	D/F	0.9900	2772	0.5236	infeas	infeas
–	x_b	M_r	F	0.9900	2772	966	infeas	infeas
–	x_b	M_r	F/F_0	0.9900	2772	2.0990	infeas	infeas
–	x_b	M_r	V	0.9900	2772	1298	infeas	infeas
–	x_b	F	x_d	0.9900	966	0.8186	infeas	infeas
–	x_b	x_r	x_d	0.9900	0.4334	0.8186	infeas	infeas
–	x_b	F/F_0	x_d	0.9900	2.0990	0.8186	infeas	infeas
–	V/B	F/F_0	x_d	2.8219	2.0990	0.8186	infeas	infeas
–	x_b	F/F_0	V/B	0.9900	2.0990	2.8219	infeas	infeas
–	x_b	F/F_0	L/D	0.9900	2.0990	1.5678	infeas	infeas
–	x_b	M_r/F	L/D	0.9900	2.8710	1.5678	infeas	infeas

Table 1. Candidate controlled variables c_3 ranked by steady-state gain ($|G(0)|$)

Rank	c_3	$\lvert G(0)\rvert \cdot 10^3$
1	x_d	13.1
2	L/F	8.9
3	D/L	7.7
4	D/V	5.8
5	V/L	4.5
6	B/L	4.1
7	V/F	4.0
8	B/D	3.3
9	L	3.0
10	B/F	2.6
11	D	2.6
12	F/F_0	2.5
13	D/F	2.5
14	F	1.9
15	B/V	0
15	V	0
15	x_r	0
15	B	0

4.3 *Loss evaluation with nominal optimal setpoints*

For the remaining 16 alternative sets we evaluate the economic losses imposed by using constant setpoints instead of re-optimization. The average loss L (as defined above) when using the *nominal* optimal setpoint for each alternative controlled variables is shown in Table 2. The reason why the losses are much larger than the in Larsson *et al.* (1999) is because we have included simple back-off from the active constraints.

We rank the alternatives based on average loss. Control of x_d is best, closely followed by L/F(figure 6), D/V and D/L, gives the smallest average loss, see also Larsson *et al.* (1999). Control of F or D, which follows Luybens rule ("fix a flow in

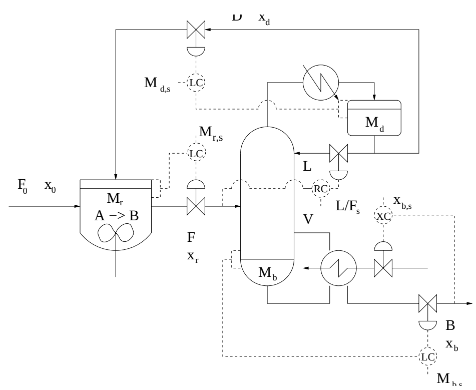

Fig. 6. Control structure with $c = L/F$ as the unconstrained controlled variable

every recycle loop") (Luyben *et al.*, 1997), and also control of D/F and F/F_0, give infeasibility. In addition we have evaluated (under the line) some alternatives from the literature that do not control reactor holdup. None of these yield feasible operation for all disturbances.

4.4 *Loss evaluation with robust optimal setpoints*

Use of nominal optimal setpoints may exclude controlled variables that are workable. In the worst case we may not find any feasible sets of controlled variables at all. We therefore consider computing the robust optimal setpoints, see equation (5), with equal weight on all eleven operating points ($i = 0, 10$). We rank the different sets of controlled variables based on their cost in optimum (average loss), see Table 3. Interestingly, there are only minor changes and improvements compared to Table 2 among the best alternatives. However, there are large improvements with

Table 3. Average costs (wJ) and loss (L) when using robust optimal setpoints

Rank	c_1	c_2	c_3	$c_{1,s}$	$c_{2,s}$	$c_{3,s}$	wJ	L[%]
Optimal	–	–	–	–	–	–	1231.23	0.0
1	x_b	M_r	x_d	0.9900	2772	0.8243	1322.35	7.4
2	x_b	M_r	L/F	0.9900	2772	0.8061	1324.20	7.6
3	x_b	M_r	D/L	0.9900	2772	0.6863	1324.17	7.6
4	x_b	M_r	D/V	0.9900	2772	0.4159	1325.87	7.7
5	x_b	M_r	V/L	0.9900	2772	1.8455	1329.79	8.0
6	x_b	M_r	B/D	0.9900	2772	0.7741	1330.97	8.1
7	x_b	M_r	V/F	0.9900	2772	1.3084	1331.10	8.1
8	x_b	M_r	B/L	0.9900	2772	0.6441	1332.65	8.2
9	x_b	M_r	F/F_0	0.9900	2772	2.3856	1333.47	8.3
10	x_b	M_r	B/F	0.9900	2772	0.4219	1334.96	8.4
11	x_b	M_r	D/F	0.9900	2772	0.5849	1336.65	8.6
12	x_b	M_r	L	0.9900	2772	716	1336.88	8.6
13	x_b	M_r	F	0.9900	2772	1250	1347.75	9.5
14	x_b	M_r	D	0.9900	2772	881	1350.97	9.7
20	x_b	M_r	B/V	0.9900	2772	0.2703	1735.32	40.9
–	x_b	M_r	V	–	–	–	infeas	infeas
15	x_b	F/F_0	V/B	0.9900	2.4963	3.4371	1581.08	28.4
16	x_b	F/F_0	x_d	0.9900	2.4963	0.8522	1585.98	28.8
17	x_b	M_r/F	L/D	0.9900	2.0231	1.2803	1586.64	28.9
18	x_b	x_r	x_d	0.9900	0.4852	0.8522	1588.36	29.0
19	x_b	F/F_0	L/D	0.9900	3.2497	0.5973	1652.97	34.3
21	x_b	F	x_d	0.9900	1955	0.7135	1794.04	45.7
22	V/B	F/F_0	x_d	3.9475	2.6167	0.8366	1815.83	47.5

control of F and D which follow Luybens rule. These are now feasible and give acceptable losses. Also alternatives which do not control the reactor holdup, are feasible, but give very large losses (28-48%). Anyway, the conclusion has not changed. The loss is smaller and control is simpler, if we keep x_d or L/F at nominal optimal setpoints rather than controlling D or F at robust optimal setpoints.

4.5 *Final evaluation and selection of control structures*

Finally, we check the control properties of the four alternatives with the smallest loss (x_d, L/F, D/L and D/V) and the two alternatives that follow Luybens rule (F and D). The reactor holdup, reboiler holdup and condenser holdup are first stabilized. The controllability analysis reveals no problems for the six alternatives.

5. CONCLUSION

An extended method for the selection of controlled variables with good self-optimizing control properties which include the choice of setpoints, is presented. We focus on achieving feasible operation by implementing setpoints found by robust optimization ("optimal back-off"). Preferably we would like to avoid using back-off because the complicates implementation. The method is applied to a reactor, separator and recycle process. x_d and L/F show best self-optimizing control properties and require no back-off. Alternatives which follow Luybens rule (F and D), give infeasibilty with nominal setpoints and thus require back-off and then give somewhat larger losses. Alternatives with variable reactor holdup also require back-off and give very large losses.

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