15th Triennial World Congress, Barcelona, Spain

CONTROL LIMITATIONS FOR UNSTABLE PLANTS

Sigurd Skogestad, [1] **Kjetil Havre, Truls Larsson**

Chemical Engineering, Norwegian University of Science and Technology (NTNU), N-7491 Trondheim, Norway

Abstract: This paper discusses for linear systems the performance limitations imposed when the plant to be controlled is unstable (with RHP-poles). The first limitation is that the plant needs to stabilized using feedback control, and this requires the active use of manipulated inputs. The instability imposes a lower bound on the $\mathcal{H}_2$- and $\mathcal{H}_\infty$-norms of the transfer function KS from outputs to inputs. Stabilization may thus be impossible if the input usage, due to measurement noise or disturbances, exceeds the saturation limits. These limitations are independent of the presence of RHP-zeros, but the combination of RHP-poles and RHP-zeros implies further performance deterioration. For a stabilized plant, the instability will manifest itself by the presence of a RHP-zero in the transfer function KS from the output to input used for control, which again imposes performance limitations in terms of the input movement.

Keywords: Linear systems, RHP-poles, RHP-zeros, performance limitations, input-output controllability multivariable system

1. INTRODUCTION

It is well understood that minimum-phase characteristics (RHP-zeros, time delays) impose fundamental performance limitations on the achievable control quality. It has also been clear that the presence of unstable (RHP) poles in combination with RHP-zeros implies further performance deterioration. Boyd and Desoer (1985) quantified this in terms of a lower bound on the $\mathcal{H}_\infty$-norm of the sensitivity S (a somewhat improved bound is presented in (4) below). Middleton (1991) reviewed the limitations for unstable plants, including limitations on the time response. His main conclusion is that the presence of RHP-poles imposes a lower limit on the system bandwith ω_B, which may be incompatible with the upper limit on ω_B imposed by RHP-zeros and time delays. Results leading to similar conclusions are given by Doyle (1986), Doyle *et al.* (1992), Kwakernaak (1995), Skogestad and Postlethwaite (1996), Seron *et al.* (1997) and Åström (1997).

However, the lower limit on the bandwidth imposed by instabiliy is not by itself a limitation, and led the first author of this paper to state the following (Skogestad and Postlethwaite, 1996, p. 140): *Recall that for "perfect control" we want $S \approx 0$ and $T \approx I$. We note that a RHP-zero imposes constraints which are incompatible with perfect control. On the other hand, the constraints imposed by the RHP-pole are consistent with what we would like for perfect control. Thus the presence of RHP-poles mainly impose problems when tight (high gain) control is not possible.*

The statement is misleading as it seems to indicate that RHP-poles do not by themselves impose performance limitations. However, this is not correct as shown more recently by (Havre and Skogestad, 2001). In short, whereas unstable (RHP) zeros impose limitations at the plant outputs, the unstable (RHP) poles impose limitations at the plant *inputs*, and in particular on the transfer

[1] e-mail: skoge@chembio.ntnu.no; phone: +47-7359-4154; fax: +47-7359-4080

function KS from outputs to inputs The main objective of this paper is to review and extend these results and put them into perspective.

Notation is fairly standard. The multivariable linear plant model is $G(s)$ and we have $y = Gu + G_d d$ where y is the output vector, u the input vector and d the disturbance vector. The negative feedback controller is $K(s)$, and we have $u = -K(y + n - r)$ where r is the reference for y and n is the measurement noise. For a system $z = M(s)w$ the $\mathcal{H}_\infty$-norm of M is

$$\|M(s)\|_\infty = \sup_\omega \bar{\sigma} M(j\omega) = \sup_{w(t)\neq 0} \frac{\|z(t)\|_2}{\|w(t)\|_2}$$

where here t is time and $\|z(t)\|_2$ is the usual Eucledian vector norm. The $\mathcal{H}_2$-norm of M is

$$\|M(s)\|_2 = \sqrt{\frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{tr}(M(j\omega)^H M(j\omega))d\omega}$$

2. POLES AND POLE VECTORS

Consider a multivariable plant with state-space matrices A, B, C and D, and transfer function matrix $G(s) = C(sI - A)^{-1}B + D$. The poles of the plant are the eigenvalues of A, and the plant is unstable if it has poles located in the RHP plane. By the right-half plane (RHP) we mean the closed right half of the complex plane, including the imaginary axis ($j\omega$-axis).

In multivariable system poles have directions associated with them. To quantify this we define for a pole p_i the input and output *pole vectors* (Havre and Skogestad, 2002)

$$y_{p_i} = Ct_i, \quad u_{p_i} = B^H q_i \tag{1}$$

where t_i and q_i are the corresponding right and left eigenvectors of A, respectively. The closely related *pole directions* are the pole vectors scaled to have unit length, $\frac{1}{\|y_{p_i}\|_2} y_{p_i}$.

To motivate the introduction of pole vectors, consider for the case when A has n distinct eigenvalues, the following dyadic expansion of the transfer function,

$$G(s) = \sum_{i=1}^{n} \frac{Ct_i q_i^H B}{s - \lambda_i} + D = \sum_{i=1}^{n} \frac{y_{p_i} u_{p_i}}{s - \lambda_i} + D$$

From this we see that the i'the input pole vector u_{p_i} is an indication of how much the i'th mode is excited (and thus may be "controlled") by the inputs. Similarly, the i'the output pole vector y_{p_i} indicates how much the i'th mode is observed in the outputs. Thus, the pole vectors may be used for checking the state controllability and observability of a system. Indeed, if $u_{pi} = B^H q_i = 0$ then the corresponding pole is not state controllable, and if $y_{pi} = Ct_i = 0$ the corresponding pole is not state observable (Zhou *et al.*, 1996, p.52).

In (1) we defined the pole directions in terms of the state space matrices A, B and C. The pole directions may alternatively be defined in terms of the transfer matrix, by evaluating $G(s)$ at the pole p_i and considering the directions of the resulting complex matrix $G(p_i)$. The matrix is infinite in the direction of the pole, and we may somewhat crudely write

$$G(p_i)u_{p_i} = \infty \cdot y_{p_i} \tag{2}$$

where u_{p_i} is the input pole direction, and y_{p_i} is the output pole direction. (2) gives useful insight into the significance of the pole directions. The pole directions may then in principle be obtained from an SVD of $G(p_i) = U\Sigma V^H$. Then u_{p_i} is the first column in V (corresponding to the infinite singular value), and y_{p_i} the first column in U.

3. FUNDAMENTAL ALGEBRAIC LIMITATIONS

Let us recall some fundamental properties for an unstable plants. Consider a plant G with an unstable pole p which is stabilized with the feedback controller K. For internal stability, all closed-loop transfer functions such as $S = (I + GK)^{-1}$, $T = GKS$, $KS = K(I + GK)^{-1}$ etc. must be stable. But if $T = GKS$ (complementary sensitivity) is stable, and G has a RHP-pole located at p, then it follows for internal stability that the transfer function $KS = K(I + GK)^{-1}$ must have a RHP-zero located at p. This fundamental requirement is used repeatedly in the following.

Furthermore, since K cannot have a RHP-zero located at p (this would imply a RHP-pole cancellation between G and K resulting in internal instability) it follows that $S = (I+GK)^{-1}$ (sensitivity) must have a RHP-zero located at p. Thus, $S(p)$ is zero in the output direction of the pole, $S(p)y_p = 0$, and since $T = I - S$, the presence of an ustable pole p in the plant G requires for internal stability (Zames, 1981)

$$T(p)y_p = y_p \tag{3}$$

where y_p is the output pole direction. This interpolation constraints forms the basis for several of the results presented below.

4. RHP-POLES COMBINED WITH RHP-ZEROS

Let us start by presenting a result that confirms the well-known fact that stabilization is very difficult if there is a RHP-zero close to the RHP-pole.

Here "close" in a multivariable systems means both close in the complex plane as well as close with respect to its direction.

In theory, stabilization of a rational linear system is always possible, provided the unstable (RHP) poles are state controllable and observable. However, in practice the presence of RHP-zeros may limit possibility to stabilize the plant. First, stabilization may require an unstable controller. Second, and more importantly, performance will be poor if the plant has a RHP-pole located close and in the same directions as a RHP-zero. For example, for a MIMO plant with single RHP-zero z and single RHP-pole p we have that (Havre and Skogestad, 1998a)(Skogestad and Postlethwaite, 1996)

$$\|S\|_\infty \geq c; \quad \|T\|_\infty \geq c$$

$$c = \sqrt{\sin^2\phi + \frac{|z+p|^2}{|z-p|^2}\cos^2\phi} \tag{4}$$

where $\phi = \cos^{-1}|y_z^H y_p|$ is the angle between the RHP-zero and RHP-pole. We see that if the RHP-pole and RHP-zero are located close in the complex plane ($z-p$ is small), and they are located in the same direction (i.e., $\cos^2\phi$ is not close to 0, i.e. ϕ is not close to $\pm 90^o$), then the peaks c on S and T will be large, and stabilization is in practice impossible.

5. LIMITATIONS AT THE INPUTS: SOME PRACTICAL EXAMPLES

In the rest of the paper we turn to some more important and less well known results that do not depend on the presence of RHP-zeros, but before considering these let us consider some practical examples in order to develop some insight.

We found above that if the plant is unstable, then for the stabilized system, the closed-loop transfer function KS from outputs (e.g. r) to inputs (u) will have a RHP-zero located at the original RHP-pole p. This limits the achievable input performance. It also provides a method for detecting if the underlying plant G is unstable.

We consider three practical examples:

(1) Consider riding a bicycle, which is obviously an open-loop unstable plant. Here we use the tilt of our body (u) to stabilize the plant and, for example, keep the bicycle in a certain angle (y) relative to its vertical position. (By "tilt" we here mean that our body is not in a vertical position). If we, for example, want the bicycle to lean more over (increase the angle y), then we must first tilt (lean over) our body in that direction to start the movement (see Figure 1b), but we must eventually move it back to counteract the resulting "off-balance" of the bike. Thus, there will be an inverse response in the tilt of our body (u).

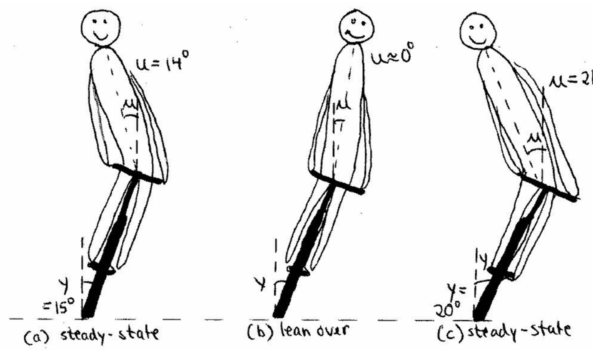

Fig. 1. Inverse response for bicycle caused by underlying instability

(2) Consider the control of an exothermic chemical reactor at an unstable operating point. To stabilize the reactor, we may, for example, use cooling water flow (u) to control reactor temperature (y). There will then be a RHP-zero in the transfer function KS from reactor temperature setpoint (r) to cooling water flow (u). For example, if we want to decrease reactor temperature, then we must initially increase reactor cooling. However, eventually the reactor will approach its new steady-state where cooling actually is decreased. The physical reason is that the lower temperature implies that less products are formed and less heat is generated by the reaction. Thus, there will be an inverse response in the cooling water flow (u).

(3) Consider the control of a distillation column in an unstable operating point (Jacobsen and Skogestad, 1994). In this case reflux flow (u) may be used to control column temperature (y). If we want to decrease the column temperature, then we must initially decrease the colder reflux flow. However, eventually the column will approach its new steady-state where reflux is increased (Jacobsen and Skogestad, 1994). Thus, there will be an inverse response in the reflux flow (u).

6. STABILIZATION AND INPUT USAGE

The fact that there is RHP-zero in KS, or more precisely in S, may be used to derive a lower bound on the norm of KS.

Theorem 1. (Havre and Skogestad, 1997)(Havre and Skogestad, 2001) Consider a plant G with a single unstable (RHP) pole p. The minimum achievable $\mathcal{H}_\infty$-norm of the closed-loop transfer function KS from plant outputs to plant inputs, $u = -KS(G_d d + n)$, is then

$$\min_{K(s)} \|KS(s)\|_\infty =$$

$$\|u_p^H (G_{so}(p))^{-1}\|_2 = \|(G_{si}(p))^{-1} y_p\|_2 \quad (5)$$

Here $S(s) = (I + GK(s))^{-1}$, and G_{so} and G_{si} are the stable versions of G with the RHP-poles mirrored across the imaginary axis and factorized at the output and input, respectively (see Havre and Skogestad (2001) for details), and $\|\cdot\|_2$ denotes the usual Eucledian vector norm. For a plant with multiple RHP-poles we have inequality instead of equality, but the bound the holds for any RHP-pole p.

At least for a SISO plant, a very similar result holds also for the $\mathcal{H}_2$-norm, see Theorem 2 below. If there are saturation limits on the inputs, and the required inputs exceed these, then stabilization is most likely not possible.

Example *Consider a SISO plant* $G(s) = \frac{1}{s-10}$ *with a single unstable pole* $p = 10$. *We obtain* $G_s(s) = \frac{1}{s+10}$. *For any linear feedback controller* K, *we then have that the lower bound*

$$\|KS\|_\infty \geq |G_s^{-1}(p)| = 2p = 20$$

must be satisfied. Thus, if we require that the plant inputs are bounded with $\|u\|_\infty \leq 1$, *then we cannot allow the magnitude of measurement noise to exceed* $\|n\|_\infty = 1/20 = 0.05$.

7. POLE VECTORS AND STABILIZATION

We usually start the controller design by designing a (lower-layer) controller to stabilize the plant. The issue is then: Which outputs (measurements) and inputs (manipulatons) should be used for stabilization? We should clearly avoid saturation of the inputs, because this makes the system effective open-loop and stabilization is impossible. A reasonable objective is therefore to minimize the required input usage of the stabilizing control system. It turns out that this is achieved, for a single unstable mode, by selecting the output (measurement) and input (manipulation) corresponding to the largest elements in the output and input pole vectors (y_p and u_p), respectively.[2] More precisely, this choice minimizes the lower bound on both the $\mathcal{H}_2$ and $\mathcal{H}_\infty$-norms of the transfer function KS from measurement (output) noise to input:

Theorem 2. **(Stabilizing SISO Control with minimum $\mathcal{H}_2$ and $\mathcal{H}_\infty$input usage).** (Havre, 1998)(Havre and Skogestad, 1998*b*) (Havre and Skogestad, 2002) Consider a plant G with a single unstable pole p. The minimum achievable $\mathcal{H}_2$- and $\mathcal{H}_\infty$-norm of the closed-loop transfer function $K_{jk}S_{kk}$ from output y_k to the input u_j is then

$$\min_{K_{jk}(s)} \|K_{jk}S_{kk}(s)\|_\infty = \frac{1}{\sqrt{|2p|}} \min_{K_{jk}(s)} \|K_{jk}S_{kk}(s)\|_2$$

$$= |(G_{kj})_s^{-1}(p)| = \frac{|2p| \cdot |q^H t|}{|u_{p,j}| \cdot |y_{p,k}|} \quad (6)$$

where $u_{p,j}$ is the j'th element in the input pole vector, $y_{p,k}$ is the k'th element in the output pole vector, v and t are the normalized left and right eigenvectors of A corresponding to the pole p, $S_{kk}(s) = (1 + G_{kj}K_{jk}(s))^{-1}$, and the notation $(G_{kj})_s^{-1}(p)$ means: Find the stable version of G_{kj} with the RHP-poles mirrored across the imagninary axis, i.e., $(G_{kj}(s))_s = \frac{s-p}{s+p} G_{kj}(s)$, take its inverse, i.e. $(G_{kj}(s))_s^{-1} = ((G_{kj}(s))_s)^{-1}$, and evaluate $(G_{kj(s)})_s^{-1}$ at $s = p$.

Remarks.

(1) When minimizing the input usage, both in terms of the $\mathcal{H}_2$-norm and the $\mathcal{H}_\infty$-norm, the unstable open-loop pole p is mirrored into the left half plane.
(2) In general, the values of the $\mathcal{H}_2$- and $\mathcal{H}_\infty$-norms of KS for a given system (with a given controller) may be arbitrary far apart. It is then somewhat surprising that the minimum of $\mathcal{H}_2$-norm and $\mathcal{H}_\infty$-norms differ by a constant factor of $\sqrt{2p}$ (although the two controllers achieving these two minimum values are of course different).
(3) The $\mathcal{H}_\infty$-controller that achieves the bound in (6) is in general improper.

In the following example we design for a simple SISO plant $\mathcal{H}_2$- and $\mathcal{H}_\infty$-optimal controllers that achieve the lower bounds on the input usage.

Example. *Consider the SISO plant*

$$G(s) \overset{s}{=} \left[\begin{array}{cc|c} -10 & 0 & \sqrt{120/11} \\ 0 & 1 & \sqrt{10/11} \\ \hline \sqrt{120/11} & -\sqrt{10/11} & 0 \end{array}\right]$$

$$= \frac{s-2}{(0.1s+1)(s-1)}$$

with an unstable (RHP) pole at $p = 1$ *and a RHP-zero at* $z = 2$. *With the above realization, the eigenvectors and pole "vectors" corresponding to the unstable pole are*

$$t = q = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad u_p = 0.9535 \quad \textit{and} \quad y_p = -0.9535$$

The $\mathcal{H}_2$-norm of KS is minimized with the following LQG controller:

$$K_{\text{LQG}}(s) = -44 \frac{0.1s + 1}{s^2 + 13s + 78}$$

[2] Notation: In the rest of the paper we consider a single unstable pole $p = p_i$, so to simplify we will drop the subscript i.

The controller is strictly proper with LHP-poles at $-6.5 \pm 5.98j$ and a LHP-zero at -10 which cancels the open-loop stable pole at -10 in the plant. With this controller the closed-loop poles of the minimal realization are located at $\{-1, -1\}$, and we achieve:

$$\|K_{\rm LQG}S_{\rm LQG}(s)\|_2 = \frac{\sqrt{8p}\cdot|q^H t|}{|u_p|\cdot|y_p|}$$

$$= \frac{\sqrt{8\cdot 1}\cdot 1}{0.9535\cdot 0.9535} = 3.11$$

The $\mathcal{H}_\infty$-norm of KS is minimized with the following controller

$$K_\infty(s) = -2.2\,\frac{0.1s+1}{0.1s+3.4}$$

The controller is semi-proper, with a LHP-pole at -34 and a LHP-zero at -10 which cancels the corresponding stable pole in G. With this controller the closed-loop pole of the minimal realization of KS is located at -1, and we achieve:

$$\|K_\infty S_\infty(s)\|_\infty = \frac{3.11}{\sqrt{2p}} = 2.2$$

which as expected is equal to

$$|G_s^{-1}(p)| = \left|\frac{(0.1s+1)(s+1)}{s-2}\right|_{s=1} = \left|\frac{1.1\cdot 2}{-1}\right| = 2.2$$

Note that $K_\infty S_\infty(s) = -2.2\frac{s-1}{s+1}$ is semi-proper (it remains flat at magnitude 2.2 at all frequencies) so its $\mathcal{H}_2$-norm is infinite.

Note that the scalar $|2p|\cdot|q^H t|$ in (6) is independent of j and k. We then have the following important result:

> The input usage required for stabilization, both in terms of the $\mathcal{H}_2$- and $\mathcal{H}_\infty$-norms, is minimized by selecting the input (actuator) u_j corresponding to the largest element in the input pole vector u_p, and the output (measurement) y_k corresponding to the largest element in the output pole vector.

The pole vectors thus provide a very simple tool for selecting inputs and outputs for stabilizing control, and requires only a single calculation. Also note that the input (actuator) and output (measurement) are selected independently. The correct use of the pole vectors assumes that the plant has been scaled such that output noise is similar in all outputs, and such that a given input variation means the same for all inputs.

The main limitations in the use of the pole vectors for stabilizing control is that only allows for the consideration of one unstable pole at a time. The implications for stabilizing complex poles is therefore not clear, although the pole vectors remain a useful tool (e.g. see the application to the Tennessee-Eastman process in (Havre and Skogestad, 1998*b*)).

8. PERFORMANCE LIMITATIONS FOR A STABILIZED PLANT

Above we discussed the possible limitations in stabilizing an unstable plant. Assume now that we have managed to design a feedback controller that stabilizes the original plant (G). That is, the "new" plant (P) with these feedback loops closed is stable. We want to answer the following question: Does the presence of the original instability in the orginal plant limit the control performance of the "new" partially controlled plant?

First note that the there are some unavoidable limitations on complementary sensitivity T, for example, the $\mathcal{H}_\infty$-norm of T must exceed 1 if we want integral action and high-frequency roll-off, and there are also time domain limitations as discussed by Seron *et al.* (1997). However, this does not imply a fundamental limitation in terms of performance, since we may prefilter the reference change, such that the resulting transfer function $T_r = TF$ from references r to outputs y has any desired shape.

Again, we find that fundamental limitations only appear when we consider the plant inputs. To derive this, consider a case where the primary objective is to use the plant inputs u to control the primary outputs y_1, and we have available the secondary measurements y_2 for stabilizing control. The transfer function of the original plant is then

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} G_1 \\ G_2 \end{pmatrix} u \tag{7}$$

The plant $\begin{pmatrix} G_1 \\ G_2 \end{pmatrix}$ is unstable and we design a stabilizing control system $u = K_2(r_2 - y_2)$ involving the inputs u and outputs y_2. The resulting closed-loop system (the "new" plant) is then

$$y_1 = \underbrace{G_1K_2(I+G_2K_2)^{-1}}_{P}\, r_2 \tag{8}$$

Comment: The number of control degrees of freedom for the "new" plant P is the same as for the original plant G, since the reference values (setpoints) r_2 for the stabilized outputs replace the original plant inputs u as degrees of freedom.

The (rephrased) question is then: *Does the presence of an unstable pole p in the original plant G_2 limit the achievable control performance of the "new" (stable) plant P from r_2 to y_1?*

The answer is quite simple, and depends on whether G_1 contains the unstable pole p or not (Larsson, 2000):

- *If the instability is detectable in the primary outputs y_1 (i.e. if G_1 contains the instability),* *then the answer is* no*: there is no performance limitations.*

- *If the instability is not detectable in the primary outputs y_1 (i.e. if G_1 is stable), then the answer is* yes*: The "new" plant P has an unstable zero located at p which imposes a performance limitation on how well y_1 can be controlled.*

In summary, a performance limitation is introduced if we want to use r_2 to control variables not containing the instability. The reason for the performance limitation is that the transfer function $K_2S_2 = K_2(I + G_2K_2)^{-1}$ from r_2 to u must contain a RHP-zero at the location of the RHP-pole p, and if G_1 does not contain this RHP-pole, then the transfer function $P = G_1K_2S_2$ from r_2 to y_1 must contain a RHP-zero at p which invariably limits the achievable performance.

Special case: Control objective at the input (i.e., $y_1 = u$ and $G_1 = I$). This is a fairly common situation. For example, recall the bicycle example, and assume that the main objective is that the tilt u of the upper body should be in a given position ($y_1 = u$ and $y_2 = y$ in Figure 1). If we want to make a setpoint change in the tilt ($r = u$), then there will be an unavoidable inverse response with a time constant given by the location of the (original) unstable pole. This will set a definite limit on how well you can perform in a possible World Championship in bicycle tilting.

9. CONCLUSION

Instability (RHP-poles) requires feedback control with active use of the plant inputs. In the paper we have reviewed some of the main limitation imposed by the presence of a RHP-pole. The most important is probably that there is a minimum bound on the transfer function KS from plant outputs (noise and disturbances) to plant inputs, and this may result in instability if there are constraints on the allowed inputs.

10. REFERENCES

Åström, K.J. (1997). Limitations on control system performance. *Proc. European Control Conference, ECC 97, July 1-4, 1997, Brussels, Belgium* pp. TU–E E4. See C97-3, V1.

Boyd, S. and C. A. Desoer (1985). Subharmonic functions and performance bounds in linear time-invariant feedback systems. *IMA J. Math. Contr. and Info.* **2**, 153–170.

Doyle, J. C. (1986). Redondo Beach lecture notes. Internal Report, Caltech, Pasadena.

Doyle, J. C., B. Francis and A. Tannenbaum (1992). *Feedback Control Theory.* Macmillan Publishing Company.

Havre, K. (1998). Studies on Controllability analysis and control structure design. PhD thesis. Norwegian University of Science and Technology, Trondheim.

Havre, K. and S. Skogestad (1997). Limitations imposed by RHP zeros/poles in multivariable systems. In: *Proc. European Control Conference, Brussels.* number Tu-A H1.

Havre, K. and S. Skogestad (1998*a*). Effect of RHP zeros and poles on the sensitivity functions in multivariable systems. *J. of Process Control* **8**, 155–164.

Havre, K. and S. Skogestad (1998*b*). Selection of variables for regulatory control using pole vectors. In: *Proc. IFAC symposium DYCOPS-5, Corfu, Greece.* pp. 614–619.

Havre, K. and S. Skogestad (2001). Achievable performance of multivariable systems with unstable zeros and poles. *Int. J. Control* **74**, 1131–1139.

Jacobsen, E.W. and S. Skogestad (1994). Instability of distillation columns. *AIChE Journal* **40**(9), 1466–1478.

Kwakernaak, H. (1995). Symmetries in control system design. *Trends in Control - A European Perspective, Alberto Isidori (ed.)* pp. 17–51. See B95-1.

Middleton, R.H. (1991). Trade-offs in linear control system design. *Automatica* **27**(2), 281–292.

Seron, M.M., J.H. Braslavsky and G.C. Goodwin (1997). *Fundamental limitations in filtering and control.* Springer.

Skogestad, S. and I. Postlethwaite (1996). *Multivariable Feedback Control.* John Wiley & Sons.

Zames, G. (1981). Feedback and optimal sensitivity: model reference transformations, multiplicative seminorms, and approximate inverse. *IEEE Transactions on Automatic Control* **AC-26**, 301–320.

Zhou, K., J. C. Doyle and K. Glover (1996). *Robust and Optimal Control.* Prentice-Hall.