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RESEARCH ON PREDICTING CHAOTIC SYSTEMS BASED ON FUZZY NEURAL NETWORK

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Abstract: A new fuzzy neural network, with optimized structure, is presented in the paper, as well as an effective algorithm to train the fuzzy neural network. It is demonstrated that the new fuzzy neural network has a higher training speed and is more accurate in predicting chaotic systems than the general neural network by simulating experiments both on prediction of the one dimensional Logistic map standard chaotic system and on dealing with the chaotic time series corrupted by noise signal. *Copyright © 2002 IFAC*

Keywords: Fuzzy neural networks; Chaotic systems; Lyapunov exponent; Correlation dimension

1. INTRODUCTION

Fuzzy neural networks have spurned much interest among many researchers. By now, there have been many applications for fuzzy neural networks to be used. In 1989, Casdagli studied the prediction of chaotic time series by use of a general fuzzy neural network(1989,Casdagli). For example, Wolf(1985) and Jones(1990) discussed the performance of function approximation and dynamic invariants such as Lyapunov exponent, correlation dimension and so on in chaos prediction(Wolf,1985). In 1992, A.M.Albano studied the capability of general fuzzy neural networks in predicting chaotic data corrupted by noise. However, since the chaotic systems have complicated geometric structure and high sensitivity to initial conditions, the result of forecasting by general fuzzy neural network is not good enough.

Although some progress has been made in chaos prediction using general fuzzy neural networks, which can make short-term prediction at some degree, or that such dynamic invariants as Lyapunov exponent and correlation dimension can reach some accuracy level, the prediction accuracy and the training speed of adaptive linear systems and general neural networks are not high. Consider that the fuzzy logic inference rules can be reduced to be simple, our goal in this paper is to use the new fuzzy neural network that has less net nodes as the prediction model to improve the training speed and prediction accuracy. As a special neural network, the new fuzzy neural network needs less time to learn. Therefore, good results will be achieved if the new fuzzy neural network can be used in predicting chaotic systems. Based on the consideration above, a new fuzzy neural network model with an effective algorithm is presented in this paper. The simulating experiments on one dimensional Logistic map standard chaotic system are made, then the result obtained is compared with that of the general neural network. Finally, its capability of dealing with chaotic data corrupted by noise is studied. It is shown that the proposed fuzzy neural network is more accurate than the general neural network in chaos prediction according to the simulating results.

2 MODEL OF THE NEW FUZZY NEURAL NETWORK

To demonstrate the algorithm of the new fuzzy neural network, its structure is introduced firstly.

2.1 Architecture of the new fuzzy neural network

As shown in Figure 1, it is a four-layer network. Compared with the general fuzzy neural network (Stick, J. , 1993), it has additional parameters in membership functions generating layer to improve the convergence speed by modifying the parameters of membership functions on line.

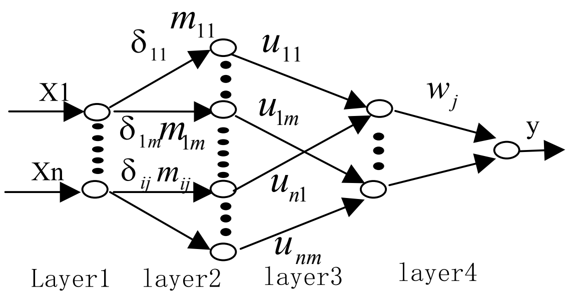

Fig. 1. Architecture of the fuzzy neural network (layer1: input layer, layer2: membership functions

generating layer, layer3: inferring layer, layer4: defuzzy output layer)

The fuzzy neural network has four layers, including input layer, membership functions generating layer, inferring layer and defuzzy output layer, where：

Input layer：n nodes correspond to n input variables；When predicting logistic map chaotic system, number n equals 1 and the input is the time series.

Membership functions generating layer： It has m*n nodes, where m denotes the partition number of input variable x_i (i=1…n)(when predicting logistic map chaotic system，the partition number of each input variable will be 3) and each connection have two adaptable parameters m_{ij} and δ_{ij} (where $1 \le i \le n, 1 \le j \le m$), which are the additional parameters of membership functions compared to the general fuzzy neural network in the reference(Stick,1993). The membership functions are selected as follows：$u_{ij} = \exp[-(x_i - m_{ij})^2 / \delta_{ij}^2]$，where u_{ij} is respectively the output of nodes in the membership functions generating layer.

Inferring Layer： It has m nodes，the output Γ_i is of the following form：$\Gamma_i = u_{1i} \cdot u_{2i} \cdots u_{ni} = \prod_{j=1}^{n} u_{ji}$

Output Layer： The number of the nodes represents the number of the output. Each weight w_j （j=1…m）represents the connection between the output layer and the inference layer. The output is computed by: $y = w_1\Gamma_1 + w_2\Gamma_2 + \cdots + w_m\Gamma_m$；

In view of output of the model，it is simpler than that of the general neural network.

2.2 Training Algorithm

The aim of prediction is to minimize the difference between the expected value and the actual output. The difference can be measured by mean square error. The proposed algorithm can be summarized as following:

According to the BP algorithm of neural networks, the objective function of the network can be defined by $E_p = (y - Y)^2 / 2$, where Y is the expected value and y is actual output of the network. During the process of learning, parameters m_{ij}, δ_{ij} and w_j are needed to be modified. Given the learning rate η >0, the parameters can be adjusted by :

$$m_{ij}(n+1) - m_{ij}(n) = -\eta(\partial E_p / \partial m_{ij}) \quad (1)$$

$$\delta_{ij}(n+1) - \delta_{ij}(n) = -\eta(\partial E_p / \partial \delta_{ij}) \quad (2)$$

$$w_j(n+1) - w_j(n) = -\eta(\partial E_p / \partial w_j) \quad (3)$$

where：for equation (1)：

$$m_{ij}(n+1) - m_{ij}(n) = -\eta(\partial Ep / \partial m_{ij})$$

$$= -\eta(\partial E_p / \partial y)\partial y / \partial m_{ij} = -\eta(x - Y)\partial y / \partial m_{ij}$$

$$= -\eta(y - Y)w_j \prod_{L=1, L\ne i}^{n} u_{Li}$$

$$\cdot 2\exp[-(x_i - m_{ji})^2 / \delta_{ij}^2](x_i - m_{ij}) / \delta_{ij}^2$$

$$= -2\eta(y - Y)w_j \prod_{k=1}^{n} \mu_{ki} \cdot (x_i - m_{ij}) / \delta_{ij}^2$$

for equation（2）：

$$m_{ij}(n+1) - m_{ij}(n) = -\eta(\partial Ep / \partial m_{ij})$$

$$= -\eta(\partial E_p / \partial y)\partial y / \partial m_{ij} = -\eta(x - Y)\partial y / \partial m_{ij}$$

$$= -\eta(y - Y)w_j \prod_{L=1, L\ne i}^{n} u_{Li}$$

$$\cdot 2\exp[-(x_i - m_{ji})^2 / \delta_{ij}^2](x_i - m_{ij}) / \delta_{ij}^2$$

$$= -2\eta(y - Y)w_j \prod_{k=1}^{n} \mu_{ki} \cdot (x_i - m_{ij}) / \delta_{ij}^2$$

for equation (3)：

$$w_j(n+1) - w_j(n) = -\eta(y - Y)\Gamma_j$$

3 SIMULATION RESULTS

In the simulation, we select a one dimensional standard chaotic system: Logistic map $x_{n+1} = \lambda x_n(1 - x_n)$, where λ =4 means the standard chaotic behavior. The fuzzy neural network and a general network are trained with the training data. When two networks are trained well, they are used to forecast the one dimensional standard chaotic system. Then its statistic characters : correlation dimension and Lyapunov exponent, are analyzed.

3.1 Comparison with the general neural network

In the simulation，the fuzzy neural network has an input layer and an output layer, each of which contains one node(the input node denotes time series n and the output node denotes the predicted output of the chaotic system), a membership functions generating layer and a inferring layer, each of which contains three nodes(the number of partition equals 3); The general neural network has three layers, one node in the input layer and the output layer and four nodes in the hidden layer.

Because the Back-Propagation training algorithm is often used to train the general neural network and good results are often gained, it is also utilized to train the general neural network in the paper.

With the same training samples(4000 training samples), the training speed can be indicated by the number of iteration or times of adjusting the adaptable parameters. Therefore, the training speeds of the fuzzy neural network and the general neural network are compared in Figure 2 and Figure 3 by the number of iteration:

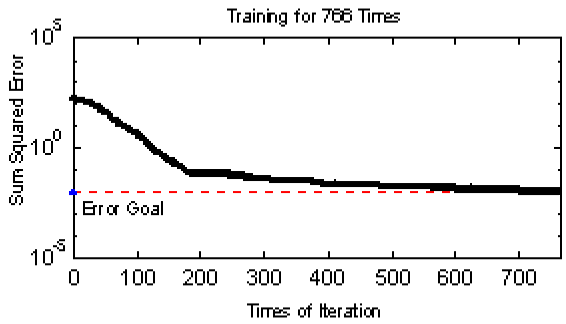

Fig.2. The training of the fuzzy neural network

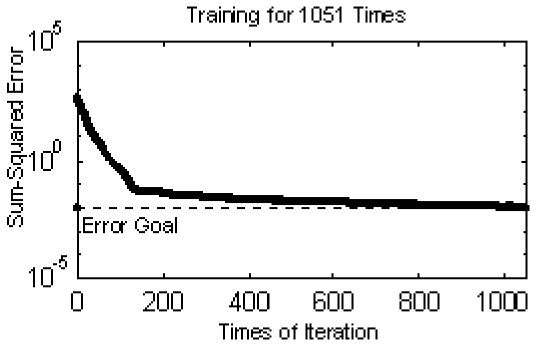

Fig.3. The training of the general neural network

In Figure 2 and Figure 3, the new fuzzy neural network needs less computation to converge than the general neural network in the training.

The predicted correlation dimension of original data, the fuzzy neural network (4000 training samples) and the general neural network (4000 training samples) are listed in Figure 4 and Figure 5.

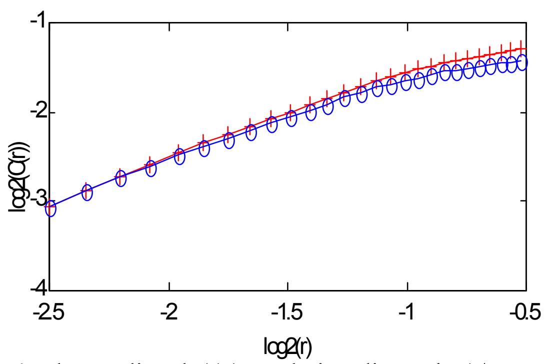

Fig.4. The predictedc(r)(correlation dimension)/r (radium) log figure with the fuzzy neural network.

" + " denotes original data correlation dimension/radium log figure of logistic map standard chaotic system.

"o" denotes the predicted correlation dimension/radium log figure of the fuzzy neural network.

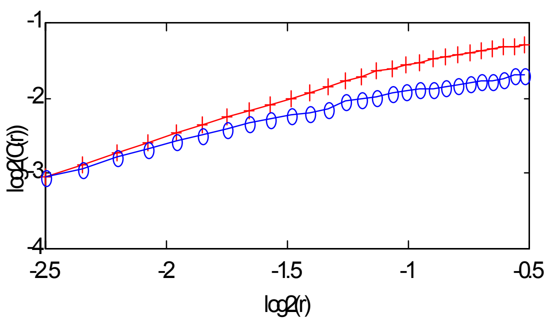

Fig.5. The predicted c(r)/r log figure with the general neural network.

" + " denotes original data correlation dimension/radium log of logistic map chaotic system.

"o" denotes the predicted correlation dimension/radium log figure with the general neural network.

In Figures 4 and 5, it is shown that the dynamic invariant as correlation dimension predicted with the fuzzy neural network given in the paper is more accurate than that predicted with the general neural network.

Now for different training points, the Lyapunov exponents of original data/the fuzzy neural network/the general neural network are listed as in Table 1 and Figure 6:

Table1. Lyapunov exponent of different points

Point N	Original Lyapunov Exponent λ	Lyapunov Exponent of FNN λ_1	Lyapunov Exponent of NN λ_2
1000	0.4704	0.4648	0.4514
2000	0.4653	0.4562	0.4821
3000	0.4608	0.4644	0.4688
4000	0.4510	0.4546	0.4386
5000	0.4512	0.4599	0.4659
6000	0.4462	0.4439	0.4429
7000	0.4461	0.4465	0.4354
8000	0.4430	0.4496	0.4292
9000	0.4443	0.4350	0.4538
10000	0.4453	0.4364	0.4280

In Table 1: max N=10000 and embedding dimension m=1, $\overline{\lambda}_i$ is defined as Lyapunov exponent ($\overline{\lambda}$: original data, $\overline{\lambda_1}$: the fuzzy neural network, $\overline{\lambda_2}$: the neural network) and $\delta_i^{\ 2}$ is defined as the standard square error of Lyapunov exponent ($\delta_1^{\ 2}$: the fuzzy neural network, $\delta_2^{\ 2}$: the neural network), then:

$\overline{\lambda}$ =0.45236, $\overline{\lambda_1}$ =0.45213, $\overline{\lambda_2}$ =0.44902 and $\delta_1^{\ 2}$=0.010702, $\delta_2^{\ 2}$ =0.018140

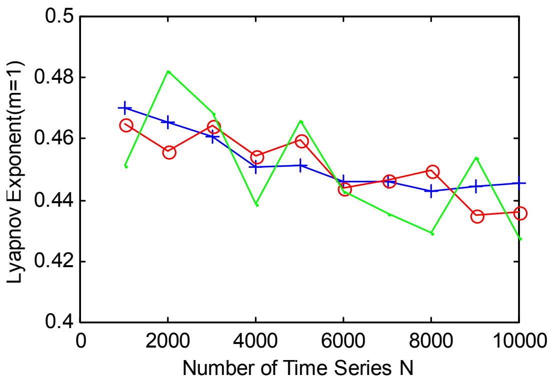

Fig. 6. Lyapunov exponent of different points

"+"denotes the original data figure of logistic map standard chaotic system.

"o" denotes the figure of logistic map standard chaotic system with fuzzy neural network.

"." denotes the prediction error of the logistic map standard chaotic system with the general neural network.

From Table 1 and Figure 6, it is shown that the dynamic invariant as Lyapunov exponent predicted with the fuzzy neural network is more accurate than that predicted with the general neural network.

The prediction errors are compared as in Figure 7 and Figure 8 more clearly(10000 training samples in each network):

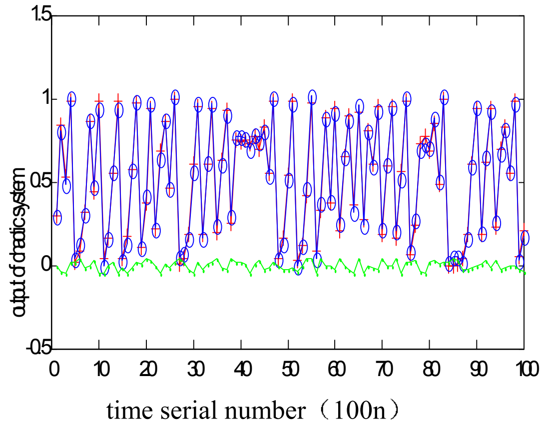

Fig. 7. Prediction error with fuzzy neural network

"+" denotes the original data figure of logistic map standard chaotic system.

"o" denotes the figure of logistic map standard chaotic system with fuzzy neural network.

"." denotes the prediction error of the chaotic system with the general neural network.

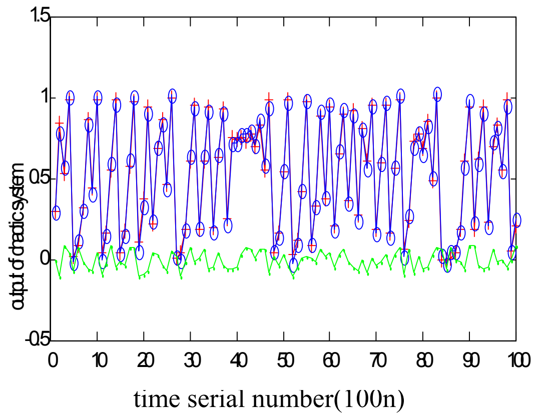

Fig. 8. Prediction error with general neural network

"+" denotes the original data of the logistic map standard chaotic system.

"o" denotes the output of the logistic map standard chaotic system with fuzzy neural network.

"." denotes the prediction error of the logistic map standard chaotic system with the general neural network.

From Figure 7 and Figure 8, it is shown that the prediction error of the fuzzy neural network is smaller than that of the general neural network.

3.2 Prediction for chaotic systems with noise

Now we further study its capability of dealing with chaotic data corrupted by noise. The noise is very similar with chaotic data，therefore, it is very difficult to predict chaotic systems with noise. It is not only because the innate characters between them are very similar, but also because noise brings great difficulty to calculate statistic constants such as correlation dimension，in figure of which noise will make "scope of dimension" obscure. So over-fitting often appears in constructing model for prediction. If the filtering technology is involved, then it will throw serious influence on chaotic data, but the fuzzy neural network has strong capability of generalizing and tolerating. In the simulation, a random variable as noise is added into chaotic data, and then the prediction is made as in table 2:

Table 2. Respective squared errors with noises at different level

Noise level	sample Number N_l	$\delta_1^{\ 2}$ of FNN	$\delta_2^{\ 2}$ of NN
10%	1000	0.012	0.0215
10%	2000	0.013	0.0274
25%	1000	0.043	0.0763
25%	2000	0.047	0.0801
50%	1000	0.092	0.1631
50%	2000	0.098	0.1832

In Table 2, noise level is defined as the ratio of standard square error of noise to that of chaotic data, where the embedding dimension equals one； N_l is the number of learning samples； $\delta_1^{\ 2}$ and $\delta_2^{\ 2}$ are respectively the standard square errors of the fuzzy neural network and the general neural network.From the Table 2, we can see that the prediction error of chaotic time series corrupted by noise at different level with the fuzzy neural network is smaller than that with the general neural network. Moreover, the prediction error also reflects the noise level approximately. So it is confirmed that the fuzzy neural network can predict the chaotic time series corrupted by noise to some extent.

4. CONCLUSION

In the simulation experiment on one dimensional Logistic map chaotic system，the correlation dimension and Lyapunov exponent obtained with the new fuzzy neural network are all more accurate than that with the general neural network. It is also shown that the fuzzy neural network can deal with the chaotic data corrupted by noise. Therefore, the modified fuzzy neural network has better performance than the general neural network in prediction of chaotic systems.

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