15th Triennial World Congress, Barcelona, Spain

# FDI IN NONLINEAR STOCHASTIC SYSTEMS USING ADAPTIVE MONTE CARLO FILTERS AND LIKELIHOOD RATIO APPROACH

**Ping Li** *,[1] **Visakan Kadirkamanathan** *

* *Department of Automatic Control and Systems Engineering, The University of Sheffield, Sheffield, UK*

Abstract: In this paper, a new method for solving the fault detection and isolation (FDI) problem in general nonlinear stochastic systems is proposed. The proposed method is based on adaptive Monte Carlo filter and likelihood ratio approach. The simulation results on a highly nonlinear system are provided which demonstrate the effectiveness of the proposed method.

Keywords: Fault detection and isolation, Nonlinear systems, Stochastic systems, Monte Carlo filter, Likelihood ratio.

## 1. INTRODUCTION

The increasing complexity and growing demands for reliability of modern control system have stimulated the development of different fault detection and isolation (FDI) approaches, as can be seen from the survey papers (see e.g. Isermann 1984, Basseville 1988, Frank 1990, Patton and Chen 1996). In model-based approaches, the FDI is based on available input-output measurements and a mathematical model of the system to be monitored. One of the main difficulties in FDI of dynamic systems is due to the presence of unknown and unmeasured variables, typically state variables $\mathbf{x}$. Two approaches are commonly used to deal with them: *estimation* and *elimination*. The estimation of $\mathbf{x}$ is usually performed with observers for deterministic systems, or filters for stochastic systems, which lead to observer-based and innovation-based FDI approaches respectively. The elimination of $\mathbf{x}$ directly explores the analytical redundancy embodied in the mathematic model. For the linear system, this leads to the well-known parity space-based FDI approach.

For nonlinear deterministic systems, the nonlinear observer-based approaches have been reviewed in (Garcia and Frank, 1997). However, in comparison with linear systems, the literature addressing model-based FDI for nonlinear stochastic systems is not extensive, the main reason being that the estimation of the state vector of a nonlinear stochastic system is not easy. The model-based FDI for nonlinear stochastic systems is known as a difficult problem and very few results are available.

Recently, the *Monte Carlo filter*, a simulation-based method for nonlinear non-Gaussian state estimation, has attracted much attention (see e.g. Gordon *et al*, 1993, Bolviken *et al*, 2001, Doucet *et al*, 2001). This interest stems from the great advantage of the Monte Carlo filter being able to handle any functional nonlinearity and system or measurement noise of any probability distribution. Our early work (see Kadirkamanathan *et al*, 2000), represents the first attempt to introduce Monte Carlo filter into the field of FDI. More recently, the Monte Carlo filtering based multiple-model and likelihood ratio methods have been developed in (Li and Kadirkamanathan, 2001a,b) for solving the FDI problem for general nonlinear non-Gaussian systems with known sets of possible faults.

[1] Supported by the fundings from UK CVCP ORS award and the Department of Automatic Control and Systems Engineering, University of Sheffield.

In this paper, we generalize further by modelling faults as changes in the parameters, and the adaptive Monte Carlo filters are integrated with the likelihood ratio test for a new approach to FDI in general nonlinear stochastic systems. The paper is organized as follows. In Section 2, the FDI problem is formulated followed by a description of the Monte Carlo filter and adaptive Monte Carlo filter in Section 3. Then, the adaptive Monte Carlo filtering technique is combined with the likelihood ratio (LR) test and a new approach to FDI is developed in Section 4. Experimental results from simulations are provided in Section 5 with conclusions in Section 6.

## 2. PROBLEM STATEMENT

In this paper, the dynamics of the system considered is assumed to be governed by the following discrete time nonlinear state space model:

$$\mathbf{x}_k = \mathbf{f}(\mathbf{x}_{k-1}, \boldsymbol{\theta}, \mathbf{w}_{k-1}) \tag{1}$$

$$\mathbf{y}_k = \mathbf{h}(\mathbf{x}_k, \boldsymbol{\theta}, \mathbf{v}_k) \tag{2}$$

where, $\mathbf{f}(\cdot)$ and $\mathbf{h}(\cdot)$ are the vector-valued nonlinear functions parameterized by vector $\boldsymbol{\theta} = [\theta_1, \theta_2, \cdots, \theta_M]^T$; $\mathbf{x}$ is the state vector, $\mathbf{y}$ is the measurement vector, $\mathbf{w}$ and $\mathbf{v}$ are white noise with known probability density functions (pdfs) which are not necessarily Gaussian.

Throughout this paper, the faults are modelled as changes in the parameter vector $\boldsymbol{\theta}$. It is also assumed that the nominal parameter value $\boldsymbol{\theta}^0$ characterizing the fault-free behavior of the system is known. With the system model (1), (2) and following above description, the FDI problem can be stated as follows:

*Problem 1.* (Problem of FD). : Fault detection is to decide between two hypotheses:

$$\mathbf{H}_0 : \boldsymbol{\theta} \in \boldsymbol{\Theta}^0 \quad \text{and} \quad \mathbf{H}_1 : \boldsymbol{\theta} \notin \boldsymbol{\Theta}^0$$

where: $\boldsymbol{\Theta}^0 = \{\boldsymbol{\theta} | \parallel \boldsymbol{\theta} - \boldsymbol{\theta}^0 \parallel < \epsilon\}$

*Problem 2.* (Problem of FI). : Fault isolation is to determine which component or subset of the parameter vector $\boldsymbol{\theta}$ has changed.

## 3. MONTE CARLO FILTER AND ADAPTIVE MONTE CARLO FILTER

The Bayesian solution to the dynamic state estimation problem involves the construction of the pdf of the current state $\mathbf{x}_k$, given the measurements up to time $k$. If $\mathcal{Z}_k$ is denoted to be the set of measurements up to time $k$, i.e. $\mathcal{Z}_k = \{\mathbf{y}_1, \mathbf{y}_2, , \cdots, \mathbf{y}_k\}$, then the Bayesian solution would be to calculate the pdf $p(\mathbf{x}_k|\mathcal{Z}_k)$. This pdf will encapsulate all the information about the state $\mathbf{x}_k$ which is contained in the measurements $\mathcal{Z}_k$ and the prior pdf of $\mathbf{x}_0$.

The key to calculating the conditional pdf $p(\mathbf{x}_k|\mathcal{Z}_k)$ is Bayes law, the recursive formulas for the estimation of the pdf $p(\mathbf{x}_k|\mathcal{Z}_k)$ are formed by the following two steps:

- *prediction:* assuming knowledge of the posterior pdf for the state at time $k-1$: $p(\mathbf{x}_{k-1}|\mathcal{Z}_{k-1})$, the one-step ahead predictive pdf at time $k-1$, $p(\mathbf{x}_k|\mathcal{Z}_{k-1})$ can be obtained by:

$$p(\mathbf{x}_k|\mathcal{Z}_{k-1}) = \int p(\mathbf{x}_k|\mathbf{x}_{k-1}) \, p(\mathbf{x}_{k-1}|\mathcal{Z}_{k-1}) d\mathbf{x}_{k-1} \tag{3}$$

  where $p(\mathbf{x}_k|\mathbf{x}_{k-1})$ is defined by the system model (1) and the known statistics of $\mathbf{w}_{k-1}$.
- *filtering:* based on predictive pdf $p(\mathbf{x}_k|\mathcal{Z}_{k-1})$, the posterior pdf at time $k$ given measurement $y_k$, $p(\mathbf{x}_k|\mathcal{Z}_k)$ can be computed *via* Bayes rule:

$$p(\mathbf{x}_k|\mathcal{Z}_k) = \frac{p(\mathbf{y}_k|\mathbf{x}_k)p(\mathbf{x}_k|\mathcal{Z}_{k-1})}{\int p(\mathbf{y}_k|\mathbf{x}_k)p(\mathbf{x}_k|\mathcal{Z}_{k-1})d\mathbf{x}_k} \tag{4}$$

  where the conditional pdf $p(\mathbf{y}_k|\mathbf{x}_k)$ is defined by the measurement model (2) and the known statistics of $\mathbf{v}_k$.

The above Bayes recursive estimation (3) and (4) can only be analytically solved for a small class of problems. The most important examples of such a class of problems are those with linear system and measurement equations, and Gaussian additive noise, in which the pdf can be summarized by means and covariances. In such cases, the Kalman filter is used to propagate and update the means and covariances of the pdf. For general nonlinear, non-Gaussian systems described by (1) and (2), there is no simple way to proceed.

*Monte Carlo filters* (also known as *particle filters*) were proposed as a new way of representing and recursively generating an approximation to the conditional pdf $p(\mathbf{x}_k|\mathcal{Z}_k)$ (see e.g. Gordon *et al*, 1993). The key idea is to represent the required pdf by a swarm of points called "particles", rather than by a function over the state space. For example, the predictive pdf $p(\mathbf{x}_k|\mathcal{Z}_{k-1})$ is expressed or approximated by a set of $N$ particles $\{\mathbf{x}_{k|k-1}(i) : i = 1, \cdots, N\}$, and $p(\mathbf{x}_k|\mathcal{Z}_k)$ is approximated by a set of $N$ particles $\{\mathbf{x}_k(i) : i = 1, \cdots, N\}$. These partilces can be considered as the realizations or random samples from the required pdfs and, as the number of particles increases, they effectively provide a good approximation to the required pdf.

It can be shown that these particles can be obtained recursively by the following filtering algorithm (see e.g. Gordon *et al*, 1993):

**Initialization:** The random samples (particles) $\{\mathbf{x}_0(i) : i = 1, 2, \cdots, N\}$ are drawn from the pdf $p(\mathbf{x}_0)$.
**Repeat the following steps for each time step $k$ ($k = 1, 2, \cdots$)**

(1) Draw $N$ samples $\{\mathbf{w}_{k-1}(i) : i = 1, 2, \cdots, N\}$ from the pdf of system noise $\mathbf{w}_{k-1}$.
(2) Generate $N$ samples (particles) $\{\mathbf{x}_{k|k-1}(i) : i = 1, 2, \cdots, N\}$, which approximate the predictive distribution $p(\mathbf{x}_k|\mathcal{Z}_{k-1})$, *via* state equation (1):

$$\mathbf{x}_{k|k-1}(i) = \mathbf{f}(\mathbf{x}_{k-1}(i), \boldsymbol{\theta}, \mathbf{w}_{k-1}(i)) \quad (5)$$

(3) On receipt of measurement $\mathbf{y}_k$, compute the importance weights associated with each predictive samples or particles by:

$$\begin{aligned} &\tilde{\alpha}_k(i) = p(\mathbf{y}_k|\mathbf{x}_{k|k-1}(i)) \\ &\alpha_k(i) = \frac{\tilde{\alpha}_k(i)}{\sum_{j=1}^{N} \tilde{\alpha}_k(j)} \\ &(i = 1, 2, \cdots, N) \end{aligned}$$

(4) Obtain $N$ particles $\{\mathbf{x}_k(i) : i = 1, 2, \cdots, N\}$, which approximate the filter distribution $p(\mathbf{x}_k|\mathcal{Z}_k)$ by the resampling of $\{\mathbf{x}_{k|k-1}(i) : i = 1, 2, \cdots, N\}$ with sampling probabilities satisfying:

$$\Pr\{\mathbf{x}_k(i) = \mathbf{x}_{k|k-1}(j)\} = \alpha_k(j)$$

The resampling mentioned above is carried out by following algorithm:

- Generate a uniform distributed random variable $u_i \in [0, 1]$ for $i = 1, 2, \cdots, N$.
- For $i = 1, 2, \cdots, N$, set $\mathbf{x}_k(i) = \mathbf{x}_{k|k-1}(j)$ if $\sum_{l=0}^{j-1} \alpha_k(l) < u_i \leq \sum_{l=0}^{j} \alpha_k(l)$ with $\alpha_k(0) = 0$.

The use of adaptive Monte Carlo filter for simultaneously estimating the states and the parameters in general nonlinear non-Gaussian systems has been proposed in (Kitagawa, 1998). The method is also termed as the "self-organizing state space model" since the method automatically yields the distribution of the parameters to be estimated. The idea is to view the parameters as additional states, or more precisely, to augment the state vector $\mathbf{x}$ with the parameter vector $\boldsymbol{\theta}$ as $\mathbf{z}_k = [\mathbf{x}_k^T, \boldsymbol{\theta}^T]^T$ and rewrite the state space model in terms of $\mathbf{z}_k$:

$$\begin{bmatrix} \mathbf{x}_k \\ \boldsymbol{\theta}_k \end{bmatrix} = \begin{bmatrix} \mathbf{f}(\mathbf{x}_{k-1}, \boldsymbol{\theta}_{k-1}, \mathbf{w}_{k-1}) \\ \boldsymbol{\theta}_{k-1} + \mathbf{w}'_{k-1} \end{bmatrix} \quad (6)$$

$$\mathbf{y}_k = \mathbf{h}(\mathbf{x}_k, \boldsymbol{\theta}_k, \mathbf{v}_k) \quad (7)$$

where, a random walk model $\boldsymbol{\theta}_k = \boldsymbol{\theta}_{k-1} + \mathbf{w}'_{k-1}$, with $\mathbf{w}'_{k-1}$ a zero mean Gaussian white noise, is introduced for parameter evolution to allow the exploration of the parameter space.

Given the above state space representation (6) and (7), the Monte Carlo filter outlined in this section can be used to obtain the sample-based joint pdf of the state $\mathbf{x}$ and parameter vector $\boldsymbol{\theta}$. In the sequel, the aforementioned Monte Carlo filtering technique is integrated with the LR test and a new FDI method for general nonlinear stochastic systems is developed.

## 4. FDI VIA ADAPTIVE MONTE CARLO FILTERS AND LR APPROACH

### 4.1 *Design of adaptive Monte Carlo filters*

To simplify presentation, we first restrict the faults to those leading to changes in a single component of $\boldsymbol{\theta}$. Extensions of the general case can be readily made. Let $\boldsymbol{\theta}^0 \in \Re^M$ be the known nominal parameter vector. A Monte Carlo filter is implemented based on the following nominal system model:

$$\begin{aligned} &\text{Nominal M-C filter } \mathbf{M}_0 : \\ &\mathbf{x}_k^{(0)} = \mathbf{f}(\mathbf{x}_{k-1}^{(0)}, \boldsymbol{\theta}^0, \mathbf{w}_{k-1}) \\ &\mathbf{y}_k = \mathbf{h}(\mathbf{x}_k^{(0)}, \boldsymbol{\theta}^0, \mathbf{v}_k) \end{aligned}$$

We then further implement a set of $M$ adaptive Monte Carlo filters with each monitoring only one parameter (*i.e.* one component of $\boldsymbol{\theta}$). These adaptive Monte Carlo filters are based on following augmented state space models:

$$\text{Adaptive M-C filter } \mathbf{M}_1 :$$
$$\begin{bmatrix} \mathbf{x}_k^{(1)} \\ \theta_{1,k} \end{bmatrix} = \begin{bmatrix} \mathbf{f}(\mathbf{x}_{k-1}^{(1)}, \boldsymbol{\theta}_1^0, \theta_{1,k-1}, \mathbf{w}_{k-1}) \\ \theta_{1,k-1} + w'_1 \end{bmatrix}$$
$$\mathbf{y}_k = \mathbf{h}(\mathbf{x}_k^{(1)}, \boldsymbol{\theta}_1^0, \theta_{1,k}, \mathbf{v}_k)$$
$$\bullet$$
$$\bullet$$
$$\bullet$$
$$\text{Adaptive M-C filter } \mathbf{M}_M :$$
$$\begin{bmatrix} \mathbf{x}_k^{(M)} \\ \theta_{M,k} \end{bmatrix} = \begin{bmatrix} \mathbf{f}(\mathbf{x}_{k-1}^{(M)}, \boldsymbol{\theta}_M^0, \theta_{M,k-1}, \mathbf{w}_{k-1}) \\ \theta_{l,k-1} + w'_l \end{bmatrix}$$
$$\mathbf{y}_k = \mathbf{h}(\mathbf{x}_k^{(M)}, \boldsymbol{\theta}_M^0, \theta_{M,k}, \mathbf{v}_k)$$

where, for $j = 1, \cdots, M$, $\boldsymbol{\theta}_j^0$ is the part of $\boldsymbol{\theta}^0$ complementary to its $j$th component, $\theta_{j,k}$ is the value of the $j$th component of $\boldsymbol{\theta}$ at time instant $k$. It can be seen that the state space models for each adaptive Monte Carlo filter are essentially the same as that for nominal Monte Carlo filter, but the state vector is augmented by a different component of $\boldsymbol{\theta}$.

As we have $M$ adaptive Monte Carlo filters running in parallel and each estimates one parameter as described in section 3, it seems that the

FDI could be achieved using these $M$ parameter estimates. Unfortunately, because each adaptive Monte Carlo filter estimates only one of the parameters and assumes that the other parameters are known and constant, a change in a single parameter affects all the models and hence the parameter estimates. It is thus not straightforward to determine which parameter has really changed. This motivates the development of the following FDI method.

### 4.2 *FDI based on adaptive Monte Carlo filters and likelihood ratio test*

The starting point for the LR approach is the logarithm of the likelihood ratio (LLR), which is a function of a random variable $y$, defined by:

$$s(y) = \ln \frac{p_{\theta_1}(y)}{p_{\theta_0}(y)} \tag{8}$$

where $p_{\theta_i}(y)(i = 0, 1)$ is a pdf parameterized by $\theta_i$. The key statistical property of this ratio is as follows (see e.g. Basseville and Nikiforov, 1993): Let $\mathbf{E}_{\theta_0}$ and $\mathbf{E}_{\theta_1}$ denote the expectations of the random variables with distributions $p_{\theta_0}$ and $p_{\theta_1}$ respectively, then:

$$\mathbf{E}_{\theta_0}(s) < 0 \quad \text{and} \quad \mathbf{E}_{\theta_1}(s) > 0$$

In other words, any change in parameter $\theta$ is reflected as a change in the sign of the mean value of the log-likelihood ratio (LLR). If the observations $y_k(k = 1, 2, \cdots)$ with a pdf $p_\theta(y)$ are independent of each other, the joint LLR for the observations from $y_j$ to $y_k$ can be expressed as:

$$S_j^k = \sum_{i=j}^{k} s_i \quad \text{and} \quad s_i = \ln \frac{p_{\theta_1}(y_i)}{p_{\theta_0}(y_i)} \tag{9}$$

Suppose $\theta = \theta_0$ before change, and $\theta = \theta_1 \neq \theta_0$ after change, then the typical behavior of this joint or cumulative LLR $S_1^k$ shows, on average, a negative drift before change, and a positive drift after change. This behavior can be used for detecting any change between two known pdf $p_{\theta_0}$ and $p_{\theta_1}$, and several detection algorithms based on the LLR test have been developed (*e.g.* SPRT, CUSUM, GLR etc.) (see e.g. Willsky and Jones, 1976, Basseville and Nikiforov, 1993). In the rest of this section, a new method for FDI in general nonlinear stochastic systems is proposed by combining the adaptive Monte Carlo filters described previously with the LLR test.

The key idea of the new method is to compute the joint likelihood of the measurements based on each (adaptive) Monte Carlo filters *via* Monte Carlo integration which uses the complete sample-based pdf information provided by (adaptive) Monte Carlo filters, and then activating in parallel $M$ LLR tests for $\mathbf{M}_m(m = 1, 2, \cdots, M)$ versus $\mathbf{M}_0$. More specifically, the joint LLR to be computed in the present case is actually as follows:

$$S_j^k(m) = \sum_{r=j}^{k} \ln \frac{p(\mathbf{y}_r|\mathbf{M}_m, \mathcal{Z}_{r-1})}{p(\mathbf{y}_r|\mathbf{M}_0, \mathcal{Z}_{r-1})} \tag{10}$$

where the likelihood $p(\mathbf{y}_r|\mathbf{M}_m, \mathcal{Z}_{r-1})$, is precisely the one step ahead output prediction density based on the $m$th Monte Carlo filter $\mathbf{M}_m$ which can be expressed as:

$$\begin{aligned} l_r^{(m)} &= p(\mathbf{y}_r|\mathbf{M}_m, \mathcal{Z}_{r-1}) \\ &= \int p(\mathbf{y}_r|\mathbf{x}_r^{(m)}, \theta_{m,r}) p(\mathbf{x}_r^{(m)}, \theta_{m,r}|\mathcal{Z}_{r-1}) d\mathbf{x}_r \\ &\quad (m = 0, 1, \cdots, l) \end{aligned} \tag{11}$$

The calculation of the quantity defined by (11) can not be performed analytically for general nonlinear non-Gaussian model. However, with the adaptive Monte Carlo filter described above, this quantity can be estimated by utilizing the complete pdf information of the predictive augmented state $[\mathbf{x}_{r|r-1}^{(m)}, \theta_{m,r|r-1}]^T$ represented by a swarm of particles, here $[\mathbf{x}_{r|r-1}^{(m)},\ \theta_{m,r|r-1}]^T$ denotes the one step ahead prediction of the augmented state given $\mathcal{Z}_{r-1}$ and based on $m$th (adaptive) Monte Carlo filter. This is achieved by reusing the likelihood of each predictive augmented state particle computed during Monte Carlo filtering. More specifically, since $\{[\mathbf{x}_{r|r-1}^{(m)}(i),\ \theta_{m,r|r-1}(i)]^T : i = 1, \cdots, N\}$ can be considered as the samples from $p(\mathbf{x}_r^{(m)}, \theta_{m,r}|\mathcal{Z}_{r-1})$, the required quantity defined by (11) can be estimated *via* Monte Carlo integration as follows:

$$l_r^{(m)} \approx \frac{1}{N} \sum_{i=1}^{N} p(\mathbf{y}_r|\mathbf{x}_{r|r-1}^{(m)}(i), \theta_{m,r|r-1}(i)) \tag{12}$$

where the likelihood of each predictive augmented state sample $[\mathbf{x}_{r|r-1}^{(m)}(i), \theta_{m,r|r-1}(i)]^T (i = 1, \cdots, N)$ from (adaptive) Monte Carlo filter is defined by the measurement equation and the known statistics of the measurement noise $\mathbf{v}_r$.

Prior to the occurrence of any fault, the output predictions given by the filters described above are all filtered versions of the actual measurement $\mathbf{y}$, therefore, after initial transition, they are identical up to filtering errors resulting only from the intrinsic uncertainty in the system model. Consequently, the likelihood of the one step ahead output prediction based on each Monte Carlo filter are all close to each other. Thus, the joint LLRs $S_1^k(m)(m = 1, \cdots, M)$ defined by (10) are all close to zero after an initial period of transition.

In the presence of a fault, due to different adaptations of the above Monte Carlo filters, the output predictions behave differently, and the likelihood of the output predictions based on different adaptive Monte Carlo filters and the nominal Monte Carlo filter are no longer close to each other and the joint LLRs $S_1^k(m)(m = 1, \cdots, M)$ defined by (10) will drift away from zero. Intuitively, the output prediction obtained with the adaptive Monte Carlo filter estimating the changed parameter should be closer to the measured output $\mathbf{y}$ than the output prediction of any other adaptive Monte Carlo filters and the nominal Monte Carlo filter, or more specifically, if $\theta_f(1 \leq f \leq M)$ is the changed parameter, then the likelihood $p(\mathbf{y}_k|\mathbf{M}_f, \mathcal{Z}_{k-1})$ based on the $f$th adaptive Monte Carlo filter will be greater than the likelihood based on any other adaptive Monte Carlo filters and the nominal Monte Carlo filter. Thus $S_1^k(f)$ will positively drift away from zero as $k$ increases and take the greatest value among the $M$ LLRs defined by (10).

Based on the above argument, define the decision function for fault detection as:

$$d_k = \max_{1 \leq m \leq M} \max_{1 \leq j \leq k} S_j^k(m) \tag{13}$$

then $d_k$ is close to zero in the fault-free condition and increases positively after the occurrence of a fault. Fault detection could thus be achieved by thresholding $d_k$ and a fault alarm is set at the time $t_a$ determined by:

$$t_a = \min\{k : d_k > \lambda\} \tag{14}$$

where $\lambda > 0$ is a threshold which depends on the noise level in the monitored system and can be determined by simulation. The fault isolation is achieved, after fault detection, by finding out the index $f$ of the faulty parameter which is given by:

$$f = \arg \max_{1 \leq m \leq M} S_{t_a}^k(m) \tag{15}$$

In the implementation of the proposed method, the maximization of the joint LLR over the time index $j$ is constrained in a sliding window with width $W$ to avoid the linearly growing computation. In practice, $W$ can be chosen sufficiently large to insure detection and isolation of all important faults subject to the computation limit, the decision function is then given by:

$$d_k = \max_{1 \leq m \leq M} \max_{k-W+1 \leq j \leq k} S_j^k(m) \tag{16}$$

In the above discussion, the single fault condition (single parameter change) is assumed, the extension to the general case (i.e. the faults leading to simultaneous changes in several parameters) is straightforward. In such a case, a similar method would require adaptive Monte Carlo filter estimating multiple parameters, i.e. the state vectors of the adaptive Monte Carlo filters described in §4.1 are augmented by the different subsets of $\boldsymbol{\theta}$ corresponding to the different faults, instead of a different component of $\boldsymbol{\theta}$.

## 5. NUMERICAL EXAMPLE

An example is presented in this section to illustrate the operation of the new FDI method proposed in this paper.

The considered system is described by following state space model:

$$\begin{aligned} x_k &= 0.5x_{k-1} + a\frac{x_{k-1}}{(1+x_{k-1}^2)} \\ &\quad + b\cos(1.2(k-1)) + w_{k-1} \\ y_k &= cx_k^2 + v_k \end{aligned}$$

where $w_k$ and $v_k$ are uncorrelated zero mean Gaussian white noise with variance $Q_w = 1$ and $Q_v = 10$ respectively. The parameters to be monitored are collected in the vector $\boldsymbol{\theta} = [a, b, c]^T$. The nominal value of $\boldsymbol{\theta}$ is $\boldsymbol{\theta}^0 = [a_0, b_0, c_0]^T = [25, 8, 0.05]^T$ which are taken from (Gordon *et al*, 1993).

Three kinds of fault are considered in simulations, the component fault is modelled by a change in the parameter $a$ of system state equation, the actuator fault is modelled by a change in parameter $b$ and the sensor fault is modelled as a change in the parameter $c$ of the measurement equation.

Two Monte Carlo simulation experiments have been carried out. In the first experiment, the component fault is simulated to occur at time $k = 201$ at which the parameter $a$ jumps from the nominal value $a_0$ to $0.5a_0$. In the second experiment, the sensor fault is simulated to occur at time $k = 201$ at which the parameter $c$ is shifted from $c_0$ to $0.5c_0$. The new method for FDI proposed in this paper is used to detect and isolate these faults.

In the two experiments, the sample sizes for nominal Monte Carlo filter and adaptive Monte Carlo filters are chosen as $N = 1000$, the width of sliding window for maximizing the joint LLRs defined by (10) is chosen as $W = 50$, the threshold in (14) is chosen as $\lambda = 2.5$. The decision function $d_k$ defined by (16) and the cumulative LLRs $S_{t_a}^k(m)$ from these two experiments are shown in Fig.1 and Fig.2 respectively. The component fault is detected at time $t_a = 223$ and the sensor fault is detected at time $t_a = 218$. We can see, from these figures, that $d_k$ remains steady around zero before the fault takes place, and drifts positively away from zero after the fault occurs and the cumula-

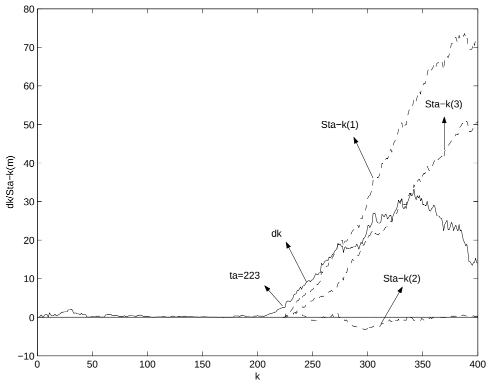


Fig. 1. $d_k$ and $S_{t_a}^k(m)(m = 1, 2, 3)$ from the first experiment (parameter $a$ changes).

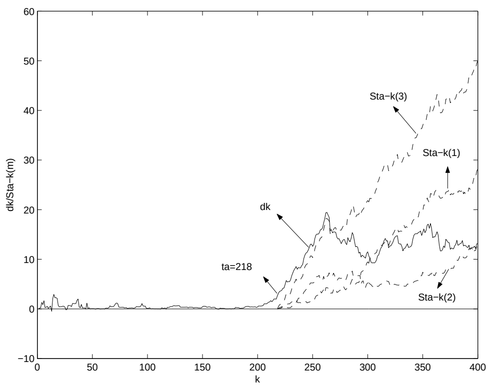


Fig. 2. $d_k$ and $S_{t_a}^k(m)(m = 1, 2, 3)$ from the second experiment (parameter $c$ changes).

tive LLR $S_{t_a}^k(m)$ corresponding to the changed parameter (e.g. $S_{t_a}^k(1)$ in Fig.1 and $S_{t_a}^k(3)$ in Fig.2) takes the greatest value. These results show that the proposed FDI method is able to detect the faults in time and to isolate the faults correctly.

## 6. CONCLUSION

A new method for FDI in general nonlinear stochastic dynamic systems has been proposed by combining adaptive Monte Carlo filters with the likelihood ratio approach, the FDI performance of the new method is illustrated with a highly nonlinear stochastic system. The results from the simulations show that the proposed method is capable of detecting the fault in time and isolating faults correctly. The proposed method provides an uniform framework for model-based FDI in general nonlinear systems with non-Gaussian system disturbance and measurement noise.

## 7. REFERENCES


Basseville, M. (1988). Detecting changes in signals and systems – A survey. *Automatica*, **24**(3): 309-326.

Basseville, M and Nikiforov, I. (1993). *Detection of abrupt changes — Theory and Application.* Prentice-Hall, Inc.

Bolviken, E., Acklam, P. J., Christophersen, N. and Stordal, J-M. (2001). Monte Carlo filters for non-linear state estimation, *Automatica*, **37**(2): pp177-183.

Doucet, A. and Freitas, N.de and Gordon, N. (2001) *Sequential Monte Carlo Methods in Practice* Springer-Verlag, New York.

Frank, P. M. (1990) Fault diagnosis in dynamic systems using analytical knowledge based redundancy – A survey and new results. *Automatica*, **26**: 459-474.

Garcia, E. A and Frank, P. M. (1997) Deterministic nonlinear observer-based approaches to fault diagnosis: A survey. *IFAC J. of Control Eng. Practice*, **5**(5): pp663-670.

Gordon, N. J., Salmond, D. J. and Smith, A. F. M. (1993). Novel approach to nonlinear/non-Gaussian Bayesian state estimation. *IEE Proceedings-F*, **140**(2): 107-113.

Isermann, R. (1984) Process fault detection based on modeling and estimation methods – A survey. *Automatica*, **20**: 387-404.

Kadirkamanathan, V., Li, P., Jaward, M. H and Fabri, S. G. (2000) A sequential Monte Carlo filtering approach to fault detection and isolation in nonlinear systems. In *Proc. IEEE conference on Decision and Control'2000*, pp.4341-4346, Sydney, Australia.

Kitagawa, G. (1998) A self-organizing state-space model. *Journal of the American Statistical Association*, **93**(443): 1203-1215.

Li, P. and Kadirkamanathan, V. (2001a) Particle filtering based likelihood ratio approach to fault diagnosis in nonlinear stochastic systems. *IEEE Trans. on Systems, Man and Cybernetics—Part C Applications and Reviews*, **31**(3), pp337-343.

Li, P. and Kadirkamanathan, V. (2001b) Particle filtering based multiple model approach to fault diagnosis in nonlinear stochastic systems. in *Proceedings of European Control Conference ECC2001*, pp. 1378-1383. Porto, Portugal.

Patton, R. J. and Chen, J. (1996) Robust fault detection and isolation (FDI) systems. *Control and Dynamic Systems*, **74**: pp171-224. Academic Press. Inc.

Willsky, A. S. and Jones, H. L. A generalized likelihood ratio approach to the detection and estimation of jumps in linear systems, *IEEE Trans. on Automatic Control*, **AC-21**, pp108-122.