15th Triennial World Congress, Barcelona, Spain

RECONFIGURABLE ROBUST FAULT-TOLERANT CONTROL AND STATE ESTIMATION

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Abstract: In this paper reconfigurable robust Linear Quadratic Regulator (LQR) and Kalman filter (KF) are developed for discrete-time systems subjected to faults, explicitly taking into account uncertainty both in the model of the system and in the estimated faults from the fault detection and diagnosis (FDD) part. For each separate actuator (sensor) with which the system is robustly stabilizable (detectable), a robust LQR gain (robust KF gain) is designed. After each occurrence of faults, a reconfiguration of the LQR (KF) is performed by an appropriate mixing of the pre-designed gains, resulting in the optimal robust LQR (KF) for the current faulty system. The approach is computationally attractive and can handle sensor, actuator and component faults. The approach is tested on an industrial actuator benchmark model.

Keywords: fault-tolerant systems, fault tolerance, robust control, LQR control, Kalman filters

1. INTRODUCTION

The problem of designing fault-tolerant control systems is attracting more and more attention due to the increasing demands imposed on the complex modern control systems, which must have increased survivability and availability One way of achieving this is by making sure that sensor, actuator and component faults are swiftly detected and isolated, and the controller is reconfigured aiming to prevent overall system failures that could lead to huge economical and even human losses (Rauch, 1993).

The problem of controller reconfiguration (CR) has been addressed in the literature in different forms. For an overview the reader is referred to (Astrom *et al.*, n.d.; Patton, 1997). Most of the existing techniques can be classified in the following categories: multiple model control (Griffin and Meybeck, 1997), the pseudo-inverse method (Gao and Antsaklis, 1991; Noura *et al.*, 2000), adaptive control (Bodson and Groszkiewicz, 1997), predictive control (Huzmezan and Maciejowski, 1999), eigenstructure assignment (Konstantopoulos and Antsaklis, 1996). Most of the approaches, however, possess the disadvantage that they require precise information from the FDD scheme. To attack this problem, the robust approach to CT can be utilized (Chen *et al.*, 1999), but then only a certain class of the possible system faults can be addressed. In this paper a new approach is presented to the design of CR schemes that can explicitly take into account uncertainty both in the system representation and in the information provided by FDD scheme. It is applicable to a large class of system faults, namely (partial and total) additive and multiplicative sensor and actuator faults (both total and partial), as well as faults resulting in constant offsets in the state and in the output equations of the state-space model that represents the system. Component faults can be taken care of by designing the closed-loop system robust with respect to them.

In this paper the system is assumed to be uncertain. The uncertainty is allowed to enter the system in a quite general (possibly non-linear) way, and is only restricted to be such that the state-space matrices do not become unbounded. The presented approach is based on an off-line design of a robust LQR gain (robust KF gain) for each separate actuator (sensor) with which the system for which it remains robustly stabilizable (detectable). These gains are then stored into the memory of the CR devise for later reconfiguration of the controller. After the isolation of any combination of faults, the optimal robust LQR and KF gains for the faulty system are swiftly computed by using the pre-designed gains.

The paper is organized as follows. In the next Section the modeling of sensor and actuator faults used throughout the paper is presented, and the representation of uncertainty in the fault signals is discussed. In Section 4 the problem formulation is given, and the reconfiguration strategy is next presented in Section 5. The performance of the proposed approach is illustrated via a case study with an industrial actuator benchmark model. The paper is concluded in Section 6.

2. SENSOR AND ACTUATOR FAULT MODELING

In this Section we present the models of faults considered in the paper.

Actuator Faults Representation: Consider the nominal (fault-free) system

$$\left\{ \begin{array}{rcl} x_{k+1} &=& Ax_k + Bu_k \\ y_k &=& Cx_k, \end{array} \right. \tag{1}$$

with $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, and $y \in \mathbb{R}^p$. Next, consider actuator faults represented by an abrupt change in the control action from u_k to

$$v_k = u_k + \sigma^a(\bar{u} - u_k) \tag{2}$$

where $\bar{u}$ is a vector that cannot be manipulated, and $\sigma^a \triangleq \text{diag}\{[\,\sigma_1^a,\, \sigma_2^a,\, \ldots,\, \sigma_m^a\,]\}$, $\sigma_i^a \in \mathbb{R}$, such that $\sigma_i^a = 1$ represents a total fault of i-th actuator, and $\sigma_i^a = 0$ implies that the i-th actuator operates normally. Then, the faulty system is described as

$$\begin{array}{rcl} x_{k+1}^f &=& Ax_k^f + B(I - \sigma^a)u_k + B\sigma^a\bar{u} \\ y_k &=& Cx_k^f, \end{array} \tag{3}$$

In addition, we consider also a different class of faults, that act as an additional constant offset to the state equation, independent on the control action u_k and the state x_k^f. We model them by introducing an additional signal ν_k

$$\begin{array}{rcl} x_{k+1}^f &=& Ax_k^f + B(I - \sigma^a)u_k + B\sigma^a\bar{u} + F_\nu\nu_k \\ y_k &=& Cx_k^f, \end{array} \tag{4}$$

Note that actuator faults are represented in the same way even when the (nominal) system (1) is uncertain. In this case we just need to replace the matrices A, B, and C by their uncertain counterparts $A(\Delta)$, $B(\Delta)$, $C(\Delta)$, and $F_v(\Delta)$, $\Delta \in \boldsymbol{\Delta}$.

It is more interesting to consider also the case when the quantities σ^a, $\bar{u}$, and v_k are *uncertain*, which is practically the case when they are provided by an FDD scheme. Suppose, without loss of generosity, that a multiplicative uncertainty representation is used to model these uncertain quantities:

$$\begin{array}{l} (I - \sigma^a) = (I - \hat{\sigma}^a)(I + \Delta_\sigma) \\ \bar{u} = (I + \Delta_u)\hat{u} \\ v_k = (I + \Delta_v)\hat{u}_k \end{array}$$

and that the FDD scheme provides us at each sample with the means $(I - \hat{\sigma}^a)$, $\hat{u}$, and $\hat{v}_k$. Then the state-equation of the faulty system (4) becomes

$$\begin{array}{l} x_{k+1}^f = A(\Delta)x_k^f + B(\Delta)(I + \Delta_\sigma)(I - \hat{\sigma}^a)u_k + \\ B(\Delta)(I - (I - \hat{\sigma}^a)(I + \Delta_\sigma))(I + \Delta_u)\hat{u} + \\ F_v(\Delta)(I + \Delta_v)\hat{v}_k. \end{array} \tag{5}$$

This can be rewritten in the form

$$\begin{array}{l} x_{k+1}^f = A(\Delta)x_k^f + B_f(\Delta, \Delta_\sigma)(I - \hat{\sigma}^a)u_k + \\ b(\Delta, \Delta_\sigma, \Delta_u, \Delta_v, \hat{\sigma}^a, \hat{u}, \hat{v}_k). \end{array} \tag{6}$$

Thus, in what follows, it is assumed that the additional uncertainties Δ_σ, Δ_u, and Δ_v are included into Δ, and we will not discuss them any more. It is important to note here that $B_f(\Delta, \Delta_\sigma)$ is independent on the means $\hat{\sigma}^a$, $\hat{u}$, and $\hat{v}_k$, and thus remains the same after fault occurrences.

Sensor Faults Representation: Again, we consider the nominal, fault-free, system (1). Sensor faults, occurring in the system (1) represent incorrect reading from the sensors, i.e. the real output of the system y_k is different from what is measured. Let the measurement after sensor faults be denoted as y_k^f ($y_k^f \neq y_k$). In this paper sensor faults are modeled as $y_k^f = y_k + \sigma^s(\bar{y} - y_k)$, where

$$\begin{array}{l} \sigma^s \triangleq \text{diag}\{[\,\sigma_1^s,\, \sigma_2^s,\, \ldots,\, \sigma_p^s\,]\},\ \sigma_i^s \in \mathbb{R}, \\ s.t. \left\{ \begin{array}{l} \sigma_i^s = 1 \rightarrow \text{total fault of } i\text{-th sensor} \\ \sigma_i^s = 0 \rightarrow i\text{-th sensor operates normally} \end{array} \right. \end{array}$$

Then the faulty system with sensor faults is described as

$$\textbf{faulty meas.:}\ y_k^f = (I_p - \sigma^s)Cx_k + \bar{c}, \tag{7}$$

with $\bar{c} = \sigma^s\bar{y}$ acting as a constant offset. Uncertainties in σ^s and $\bar{c}$ can be taken care of in the same way as in the case of actuator faults, described above.

3. PROBLEM FORMULATION

Consider the fault-free discrete-time system

$$\begin{array}{rcl} x_{k+1} &=& A(\Delta)x_k + B(\Delta)u_k + T\xi_k \\ y_k &=& C(\Delta)x_k + \eta_k \end{array} \tag{8}$$

where $\xi_k \in \mathbb{R}^{n_\xi}$ and $\eta_k \in \mathbb{R}^p$ are zero-mean white process and measurement noises with covariances $E\{T\xi\xi^T T^T\} = Q_{KF} = Q_{KF}^T > 0$ and $E\{\eta\eta^T\} = R_{KF} = R_{KF}^T > 0$, respectively, and $\Delta \in \mathbf{\Delta}$ is uncertainty that represents plant-model mismatch, possible component faults that could unexpectedly enter the system during its operation, as well as imprecise information from the FDD part (see previous Section).

Next, consider the performance index

$$J_k(A, B, Q, R) \triangleq \sum_{i=1}^{\infty} \|x_{k+i}\|_Q^2 + \|u_{k+i}\|_R^2, \quad (9)$$

with $Q = Q^T > 0$ and $R = R^T > 0$, and let the model describing both sensor and actuator faults be

$$\begin{aligned} x_{k+1}^f &= A(\Delta)x_k + B(\Delta)(I - \sigma^a)u_k + b + \xi_k \\ y_k^f &= (I - \sigma^s)C(\Delta)x_k^f + c + \eta_k, \end{aligned} \quad (10)$$

for some diagonal matrices σ^s, σ^a, and constant, but uncertain, vectors b and c. Below, two separate problems are considered, namely the LQR and the KF reconfiguration problems. For the state-feedback controller design (i.e. the LQR problem) we consider only the state equation of the model (10), while for the state-estimator design (the KF problem) we consider $u = 0$ and $b = 0$ (to make sure that the state estimates are unbiased). Clearly then, sensor faults affect the KF, but not the LQR, while actuator faults affect only the LQR, and not the KF.

Consider the following reconfigurable LQR and KF:

Structure of the Reconfigurable KF

$$\begin{aligned} &\textbf{state: } \hat{x}_{k+1}^f = A_0\hat{x}_k^f + L_R e_k^f, \\ &\textbf{innovation: } e_k^f = y_k^f - (I - \sigma^s)C_0\hat{x}_k^f - c_0, \end{aligned} \quad (11)$$

Structure of the Reconfigurable LQR

$$\textbf{controller: } u_k = F_R x_k^f + \tilde{u}_k + r_k. \quad (12)$$

where r_k is a reference signal, and where A_0, B_0, C_0, b_0, and c_0 are the mean values of the uncertain quantities A, B, C, b, and c. The additional signal $\tilde{u}_k$ is introduced so that later on we can take care of the offset b. The problem is then defined as follows.

Problem Formulation: Reconfigure (redesign) the LQR in cases of actuator faults (i.e. design F_R and $\tilde{u}_k$), and the KF in cases of sensor faults (i.e. design the gain L_R) in a computationally effective way, such that

$$F_R = \arg\min_{F_R} \max_{\Delta \in \mathbf{\Delta}} J_k(A(\Delta), B(\Delta)(I - \sigma^a), Q, R),$$

$$\tilde{u}_k = \arg\min_{\tilde{u}_k} \max_{\Delta \in \mathbf{\Delta}} \|B(\Delta)(I - \sigma^a)\tilde{u}_k - b\|_2$$

$$L_R = \arg\min_{L_R} \max_{\Delta \in \mathbf{\Delta}} J_k(A(\Delta), C^T(\Delta)(I - \sigma^s), Q_{KF}, R_{KF}).$$

It can be shown that the last optimization problem leads to an unbiased state estimator. However, minimum variance estimation error is only achieved in the case when no uncertainty is present in the system.

4. RECONFIGURATION STRATEGIES

Actuator Faults: In this section we will present a possible reconfiguration strategy, that achieves a optimal closed-loop performance after the occurrence of any combination of actuator faults for which the system remains stabilizable. Denote

$$B(\Delta) = \left[\, b_1(\Delta),\ b_2(\Delta),\ \ldots,\ b_m(\Delta) \,\right],$$

with $b_i(\Delta) \in \mathbb{R}^{n\times 1}$. Initially, it is assumed that the pairs $(A(\Delta), b_i(\Delta))$, $i = 1, \ldots, m$, are controllable for all $\Delta \in \mathbf{\Delta}$, which will later on be released.

In (Kanev and Verhaegen, 2001) it was shown that defining

$$U(A(\Delta), B(\Delta), Q, R, X, Y, \gamma) = \begin{bmatrix} X & (A(\Delta)X + BY)^T & XQ^{1/2} & Y^T R^{1/2} \\ A(\Delta)X + BY & X & 0 & 0 \\ Q^{1/2}X & 0 & \gamma I & 0 \\ R^{1/2}Y & 0 & 0 & \gamma I \end{bmatrix} \quad (13)$$

then given the state-space model (A, B, C, D), the control action $u_k = YX^{-1}x^k$, where $X = X^T > 0$ and Y are such that $U(A(\Delta), B(\Delta), Q, R, X, Y, \gamma) \geq 0$ for all $\Delta \in \mathbf{\Delta}$, minimizes the cost function (9), achieving $\max_{\Delta\in\mathbf{\Delta}} J_k(A, B, Q, R) = \gamma x_0^T X^{-1} x_0$.

Consider the matrix representing total faults in all actuators but the i-th:

$$B_i(\Delta) = \left[\, 0,\ \ldots,\ 0,\ b_i(\Delta),\ 0,\ \ldots,\ 0 \,\right].$$

Noting that $U(A, B, Q, R, X, Y, \gamma) \geq 0$ is equivalent to $U(A, B, Q, R, X/\gamma, Y/\gamma, 1) \geq 0$, we assume that for each B_i, $i = 1, \ldots, m$, we have found matrices $X_i = X_i^T > 0$ and Y_i, such that for all $\Delta \in \mathbf{\Delta}$ it holds that

$$U(A(\Delta), B_i(\Delta), Q, R_i, X_i, Y_i, 1) \geq 0, \quad (14)$$

where $R_i = \mathbf{diag}\{0, \ldots, r_i, \ldots, 0\}$, $r_i > 0$. This then implies that given the faulty model

$$x_{k+1}^f = Ax_k^f + B_i u_k \quad (15)$$

the state feedback $u_k = Y_i X_i^{-1} x_k^f$ quadratically stabilizes the system and, moreover, optimizes the cost function $J_k(A, B_i, Q, R_i)$, defined in (9), so that

$$J_k(A, B_i, Q, R_i) = (x_k^f)^T X_i^{-1} x_k^f. \quad (16)$$

We will first show how one can combine the locally optimal state-feedback gains $F_i = Y_i X_i^{-1}$ to form the global optimal state-feedback F. Later on we will use this to present the reconfiguration rule that achieves an optimal state-feedback gain in cases of actuator faults.

Theorem 4.1. (RLQR design). Consider the faulty systems (15) for $i = 1, \ldots, m$, each of them coupled with a state-feedback gain matrix $F_i = Y_i X_i^{-1}$, such that (14) holds for all $\Delta \in \mathbf{\Delta}$, guaranteeing bound on the cost functions as in (16). Then the state-feedback control

$$u_k = \Big(\sum_{i=1}^{m} E_i Y_i\Big)\Big(\sum_{i=1}^{m} X_i\Big)^{-1} x_k \quad (17)$$

where $E_i \triangleq B_i^+ B_i$ is a matrix that has zeros everywhere except in entry (i,i) where it has a one, quadratically stabilizes the nominal system (8) and optimizes the performance index achieving

$$\max_{\Delta\in\boldsymbol{\Delta}} \min J_k(A,B,Q,R) = m(x_k^f)^T(\sum_{i=1}^m X_i)^{-1}x_k^f, \quad (18)$$

for $R = \sum_{i=1}^m R_i = \text{diag}\{r_1,\ldots,r_m\}$.

The proof to Theorem 4.1 can be found in (Kanev and Verhaegen, 2001) and is not listed here due to space limitations.

This theorem is very useful for the purposes of controller reconfiguration in cases of actuator faults. Indeed, let us consider the faulty system (10) (note that the constant offset term b does not affect the solution), and thus we consider initially $\bar{u} = 0$), and let Ω be the set of the indexes of all actuators that are not completely lost, i.e.

$$\Omega \triangleq \{i:\ i\in\{1,2,\ldots,m\},\ \sigma^a(i,i)\neq 1\}. \quad (19)$$

Then we have the result.

Theorem 4.2. (RLQR Controller reconfiguration). Consider the faulty systems (15) coupled with LQR regulators with gains $F_i = Y_i X_i^{-1}$. The control action

$$u_k \triangleq (I-\sigma^a)^\dagger\Big(\sum_{i\in\Omega} E_i Y_i\Big)\Big(\sum_{i\in\Omega} X_i\Big)^{-1} x_k^f \quad (20)$$

where $E_i = B_i^\dagger B_i$, applied to the faulty system (10) with $b=0$ quadratically stabilizes the system for any $\Delta\in\boldsymbol{\Delta}$ and optimizes the performance index, achieving

$$\min\max_{\Delta\in\boldsymbol{\Delta}} J_k = \mathbf{vol}(\Omega)x_k^T\Big(\sum_{i\in\Omega} X_i\Big)^{-1} x_k, \quad (21)$$

where $R_f = (\sum_{i\in\Omega} R_i)(I-\sigma^a)^2$, and $\mathbf{vol}(\Omega)$ represents the number of elements in the set (19).

The proof can be found in (Kanev and Verhaegen, 2001).

Note that non-zero offset term in Equation (10) could be viewed as just changing the initial conditions, which does not affect the the solution to the optimal control problem (20). Though it cannot destabilize the system, it can lead to an unwanted steady-state tracking error in cases of reference trajectory tracking problems. Its effect could be reduced by using the additional signal $\tilde{u}_k$ in Equation (12). For this purpose one must solve the robust least-squares problem

$$\tilde{u}_k = \arg\min_{\tilde{u}_k}\max_{\Delta\in\boldsymbol{\Delta}} \|B(\Delta)(I-\sigma^a)\tilde{u}_k - b\|_2. \quad (22)$$

For solving this problem one could, for example, make use of the approach propose in (**?**). It should be noted, that a simple analytical solution exists to the optimization problem (22) whenever $v_k = 0$ and there is no uncertainty in the estimation of σ^a and $\bar{u}$ (consult equation (4)). In this case the offset term reduces to $b = B(\Delta)\sigma^a\bar{u}$, and the optimal $\tilde{u}_k$ can be computed as follows

$$\begin{aligned}\tilde{u}_k &= -\Big[B(\Delta)(I-\sigma^a)\Big]^\dagger B(\Delta)\sigma^a\bar{u}\\ &= -(I-\sigma^a)^\dagger B(\Delta)^\dagger B(\Delta)\sigma^a\bar{u}\end{aligned}$$

or $\tilde{u}_k = -(I-\sigma^a)^\dagger\sigma^a\bar{u}$.

Next we release the strong Assumption introduced in the beginning of this Section and consider the general case when the system is not controllable by each separate actuator. Denote the set $\mathcal{S}_B$ of all faulty B matrices for which the system remains controllable as

$$\mathcal{S}_B \triangleq \{B(I-\sigma^a):\ (A,B(I-\sigma^a)) \text{ is controllable}\},$$

and the index set $\mathcal{I}$ representing all columns of the matrix B for which the system is controllable, i.e. $\mathcal{I}\triangleq\{i:\ (A,B_i) \text{ is controllable}\}$. Further, let $B = B_T + B_P$ with $B_T\in\mathcal{S}_B$ and $B_P\notin\mathcal{S}_B$. Finally, define the set

$$\mathcal{S}_{B_P} = \mathcal{S}_B \cap \{\sum_{i\notin\mathcal{I}} B_i\alpha_i:\ \alpha_i\in\{0,1\}\}.$$

Thus, for any $B_F\in\mathcal{S}_{B_P}$, the pair (A,B_F) is controllable, but the indexes of the non-zero columns are not in the set $\mathcal{I}$. Let the number of elements of $\mathcal{S}_{B_P}$ be $\mathbf{vol}(\mathcal{S}_{B_P})$. Then the following state-feedback gains are designed off-line:
(i) For all $i\in\mathcal{I}$ we design gains $F_i = Y_i X_i^{-1}$ for the (controllable) systems (15).
(ii) For all $B_F\in\mathcal{S}_{B_P}$ we design a state-feedback gains $\tilde{F}_i = \tilde{Y}_i\tilde{X}_i^{-1}$, $i=1,\ldots,\mathbf{vol}(\mathcal{S}_{B_P})$.
The number of matrices that need to be stored in general, assuming that $q=\mathbf{vol}(\mathcal{I})$, cannot exceed the number $2(q+2^{m-q})$.

Consider the general fault scenario

$$B(I-\sigma^a) = B_T(I-\sigma^{A,T}) + B_P(I-\sigma^{A,P}),$$

with

$$\begin{aligned}\sigma^{A,T} &\in \{\text{diag}(\sigma_1^{A,T},\ldots,\sigma_m^{A,T}):\ \sigma_i^{A,T} = 1 \text{ for all } i\notin\mathcal{I}\}\\ \sigma^{A,P} &\in \{\text{diag}(\sigma_1^{A,P},\ldots,\sigma_m^{A,P}):\ \sigma_i^{A,P} = 1 \text{ for all } i\in\mathcal{I}\}\end{aligned}$$

Case $\sigma^{A,T}\neq I$ and $B_P(I-\sigma^{A,P})(I-\sigma^{A,P})^\dagger\in\mathcal{S}_{B_P}$
The controller gain should be taken as $F_R = (I-\sigma^{A,P})\tilde{F}_P$, where $\tilde{F}_P$ is the optimal gain designed for the system with $B_F = B_P(I-\sigma^{A,P})(I-\sigma^{A,P})^\dagger\in\mathcal{S}_{B_P}$.
Case $(\sigma^{A,T}\neq I)$ and $B_P(I-\sigma^{A,P})(I-\sigma^{A,P})^\dagger\in\mathcal{S}_{B_P}$ take

$$F_R = (I-\sigma^{A,T})^\dagger\left((I-\sigma^{A,P})\tilde{Y}_P + \sum_{i\in\mathcal{I}} E_i Y_i\right)\left(\tilde{X}_P + \sum_{i\in\mathcal{I}} X_i\right)^{-1}.$$

Case $(\sigma^{A,T}\neq I)$ and $B_P(I-\sigma^{A,P})(I-\sigma^{A,P})^\dagger\notin\mathcal{S}_{B_P}$ then the optimal controller for the system cannot be formed from the off-line designed state-feedback gains. However, in this case a stabilizing controller is

$$F_R = (I - \sigma^{A,T})^{\dagger} \left(\sum_{i \in \mathcal{I}} E_i Y_i \right) \left(\sum_{i \in \mathcal{I}} X_i \right)^{-1},$$

which could be used until the optimal controller is being designed on-line.

Sensor Faults: Next, we will derive similar results to the actuator-faults case, but now in the case of sensor faults. The reconfiguration strategy that will be proposed uses the faulty innovation $e_k^f = y_k^f - C\hat{x}_k^f$ and reconfigures the Kalman filter gain in such a way, that consistent state estimate is obtained. Using the duality between the Kalman Filter and the LQ regulator, the results that follow are equivalent to those for LQR controller reconfiguration and are just briefly discussed.

Consider the system (8) with $u_k = 0$, and denote

$$C(\Delta) = \left[\, c_1^T(\Delta), \, c_2^T(\Delta), \, \ldots, \, c_p^T(\Delta) \, \right]^T,$$

with $c_i \in \mathbb{R}^{1 \times n}$, $i = 1, \ldots, p$. Next, let $C_i(\Delta) = \left[\, 0, \, \ldots, \, 0, \, c_i^T(\Delta), \, 0, \, \ldots, \, 0 \, \right]^T$ represent total faults in all sensors, but the i-th. Consider the set of faulty systems

$$\begin{aligned} x_{k+1} &= A(\Delta)x_k + \xi_k \\ y_i^f(k) &= C_i(\Delta)x_k + \eta_{i,k} \end{aligned}, \text{ for } i = 1, \ldots, p, \quad (23)$$

with their corresponding robust Kalman filters

$$\hat{x}_{k+1}^f = A_0 x_k^f + L_i(y_i^f(k) - C_i^0 \hat{x}_k), \qquad (24)$$

where $L_{KF,i} = X_{KF,i}^{-1} Y_{KF,i}$, and $X_{KF,i}^T = X_{KF,i} > 0$, and $Y_{KF,i}$ are such that for all $\Delta \in \boldsymbol{\Delta}$ the following LMIs are feasible:

$$U(A^T, C_i^T, Q_{KF}, R_{KF,i}, X_{KF,i}, Y_{KF,i}, 1) \geq 0$$

for $R_{KF,i} = \text{diag}\{0, \ldots, 0, r_{KF,i}, 0, \ldots, 0\}, r_{KF,i} > 0$. Now, assume that a combination of sensor faults have occurred, resulting in the faulty system (10). Define

$$\Omega_S \triangleq \{i : \; i \in \{1, 2, \ldots, p\}, \; \sigma^s(i,i) \neq 1\}$$

as the set of the indexes of all sensors that are not completely lost. Then we have the following result.

Theorem 4.3. (Reconfigurable RKF). Consider the faulty systems (23) for $i = 1, \ldots, p$, each of them coupled with a Kalman filter (24). Then the Kalman filter

$$\hat{x}_{k+1}^f = A_0 x_k^f + L_R(y^f(k) - (I - \sigma^s)C_0 \hat{x}_k^f - \bar{c}_0), \qquad (25)$$

with gain

$$L_R = \Big(\sum_{i \in \Omega_S} X_{KF,i} \Big)^{-1} \Big(\sum_{i \in \Omega_S} E_i Y_{KF,i} \Big)^T (I - \sigma^s)^{\dagger} \quad (26)$$

where $E_i = C_i C_i^{\dagger}$, provides an unbiased state estimate for the faulty system (10) with $u_k = 0$, with process noise covariance matrix Q_{KF} and measurement noise covariance matrix

$$R_{KF} = \Big(\sum_{i \in \Omega_S} R_{KF,i} \Big) (I - \sigma^s)^2,$$

and minimizes the variance of the estimation error $e_k^f = x_k^f - \hat{x}_k^f$.

For proof consult (Kanev and Verhaegen, 2001).

Notice, that here detectability of the system with each separate measurement is assumed, which assumption can similarly be released as in the actuator faults case.

5. EXPERIMENTAL PART

In this section we will illustrate the developed approach via a case study with an industrial benchmark example, taken from (Blanke *et al.*, 1995). A linear, continuous-time model of the system can be written in state-space as

$$\begin{aligned} \dot{x}(t) &= A_c x(t) + B_c u(t) + F_{ac} f_a(t) + T_c \xi(t) \\ y(t) &= C_c x(t) + D_c u(t) + F_{sc} f_s(t) + \eta(t), \end{aligned} \qquad (27)$$

where $f_a(t) = [f_a^1, f_a^2]^T(t)$ is the vector of actuator faults, $f_s(t) = [f_s^1, f_s^2]^T(t)$ is the vector of sensor faults, $\xi(t)$ is a (estimated) disturbance signal (regarded as fault), and $\eta^T = [\eta_1, \; \eta_2]$ is a vector of the measurement noises. The matrices in Equation (27) are given by

$$A_c(\theta) = \begin{bmatrix} 0 & -\frac{K_v}{T_v} & 0 \\ \frac{K_q \eta}{I_{tot}} & -\frac{f_{tot} + K_v K_q \eta}{I_{tot}} & 0 \\ 0 & \frac{1}{N} & 0 \end{bmatrix}, \; B_c(\theta) = \begin{bmatrix} \frac{K_v}{T_v} \\ \frac{K_v K_q \eta}{I_{tot}} \\ 0 \end{bmatrix},$$

$$C_c(\theta) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & \alpha_s \end{bmatrix}, \; D_c(\theta) = 0_{2 \times 3}, \; T_c(\theta) = \begin{bmatrix} 0 \\ \frac{1}{N I_{tot}} \\ 0 \end{bmatrix},$$

$$F_{ac}(\theta) = \begin{bmatrix} \frac{K_v}{T_v} & 0 \\ \frac{K_q K_v \eta}{I_{tot}} & \frac{K_q \eta}{I_{tot}} \\ 0 & 0 \end{bmatrix}, \; F_{sc}(\theta) = \begin{bmatrix} 1 & 0 \\ 0 & \alpha_s \end{bmatrix}.$$

The nominal values of the parameters are $f_{tot} = 19,7.10^{-3}$, $I_{tot} = 2,53.10^{-3}$, $K_q = 0,54$, $K_v = 0,9$, $N = 89$, $\alpha_s = 0,987$, $\eta = 0,85$, $T_v = 8,8.10^{-3}$. The parameter f_{tot} can vary from $+200\%$ to -50%, I_{tot} can vary in the interval $\pm 15\%$, K_q can vary $\pm 5\%$, and η can take values from 0.70 to 0.85. Note that the faulty model from Equation (27) can immediately be recast into the form (10) due to the multiplicative nature of the considered faults. The model is discretized with sampling frequency $1/T_s = 100$ $[Hz]$.

The fault scenario used in the simulation is given in Table 1. Two simulations have been performed to demonstrate the capabilities of the reconfigurable LQR and KF. The objective of the first simulation is that the second output tracks a constant reference trajectory $r = 1$, while for the second simulation the goal is that the state estimation error remains "small". The results from

Table 1. Simulated fault scenario.

fault	start/stop time	value
f_s^2	$[0,7\ 0,9]\ [s]$	$x_3(0,7T_s) - x_3(tT_s)$
ξ	$[1,2\ 2,3]\ [s]$	5 [Nm]
f_a^2	$[2,7\ 3,0]\ [s]$	1 [A]
f_a^1	$[3,3\ 3,9]\ [s]$	$-0.5u(tT_s) + u(3,3T_s)$
f_s^1	$[4,2\ 4,7][s]$	$-y_1(tT_s) + y_1(4,2T_s)$

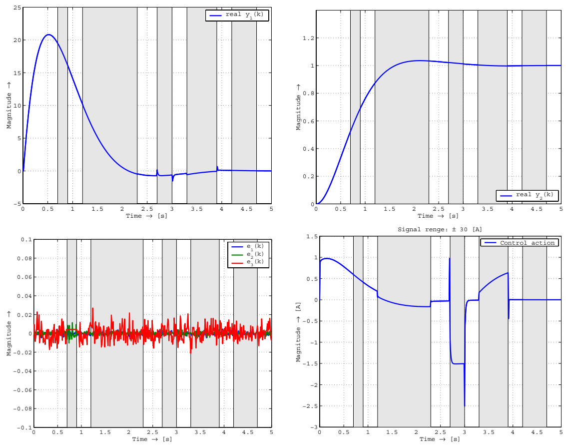

Fig. 1. Simulation results: above: the two outputs $y_1(k)$ and $y_2(k)$, below: the estimation errors and the control action.

the simulation are presented in Figure 1, where control action $u(k)$ and the two system outputs $y_1(k)$ and $y_2(k)$ are reported for the first simulation, and the state estimation errors are depicted for the second simulation. It can clearly be seen from the plots how after each occurrence of the faults f_a and ξ, the control action abruptly changes to compensate for the effect of the fault on the output, and as a result the two outputs remain practically insensitive to the faults. The control action, however, does not change due to occurrences of sensor faults (f_s), in which case the KF is reconfigured.

6. CONCLUSIONS

In this paper we develop reconfiguration strategies for automatic robust LQR/KF redesign in cases of faults in the sensors, actuators, or other system components. For each separate actuator (sensor) with which the system is robustly stabilizable (detectable), a robust LQR controller gain (robust Kalman filter gain) is pre-computed, and then an appropriate mixing of these gains is invoked after fault occurrences, so that optimality is preserved for the faulty system. Component faults can be taken care of by designing the LQR (KF) robust with respect to them. The main advantage of the proposed method is that it can explicitly deal with uncertainty in the information provided by the FDD scheme. The approach is illustrated in a case study with an industrial actuator benchmark model.

7. REFERENCES

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