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Decentralized H_∞ control of large scale web transport systems

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Abstract - Industrial controllers for web transport systems typically use decentralized PID controllers but these techniques do not achieve good decoupling between tension and speed. Multivariable centralized controllers have been shown to provide better results but are not suitable for large scale systems due to their high orders. This paper presents a multivariable decentralized control strategy applied to a large scale winding system. It is based on overlapping decomposition method applied to the specific case of web transport systems. Simulation results using two degrees of freedom H_∞ controllers show that disturbance rejection and decoupling between web tension and web velocity are significantly improved. *Copyright © 2002 IFAC*

Keywords : Decentralized control, two degrees of freedom H_∞ control, overlapping subsystems, web transport systems

I. INTRODUCTION

The main goal in industrial web transport applications is to increase the web transport velocity as much as possible while controlling the tension of the web. The main concern is the strong coupling between velocity and tension of the web. There exist many sources of velocity disturbances like roller non-circularity, web sliding. Due to the coupling introduced by the elastic web, these disturbances are transmitted to the web tension and may result in a web break or fold. This type of system has recently received a strong interest from the control community. Several studies focus on web handling control (Reid, *et al*., 1993; Wolfermann, 1995; Angermann, *et al*., 2000), using PID, fuzzy or neural approaches. Multivariable control strategies have recently been proposed for industrial metal transport systems in (Geddes, *et al*., 1998; Grimble, *et al.*, 1999) and for flexible webs in (Koç, *et al*., 2000a, 2000c, 2002; Knittel, *et al*., 2001). Some recent studies focus also on robustness analysis, considering uncertainties on radii, inertia and elastic modulus (see Koç, 2000b; Laroche, *et al*., 2001).

Industrial winding systems are strongly coupled systems, generally of large scale. They are a good application for the recent improvements in decentralized control theories (Stankovic, *et al*., 2000). Moreover, traditional control strategies based on PID do not achieve good decoupling (especially for flexible webs). Furthermore, multivariable centralized controllers, recently proposed for this application (Koç, *et al*., 2000c, 2002), are only powerful for reduced size systems, up to three motors. Therefore, decentralized overlapping multivariable control, allowing to highly decrease coupling for large scale systems, is a innovative strategy for winding systems. It is based on multivariable control strategies without the inconvenient of having too many inputs and outputs for one controller.

In this paper we consider a nine motors system. The model presented in section II has been validated on a three motors bench (see figure 1) in our laboratory. The decomposition method, whose principle is given in (Stankovic, *et al*., 2000), is presented in section III and adapted to our special case in section IV. For each subsystem, the controller is designed with a one degree of freedom (1DOF) H_∞ method and a two degrees of freedom (2DOF) H_∞ method. Simulation results are shown in section V. They show the advantage of the overlapping method.

II. PLANT MODEL

The model is briefly presented. More details can be found in recent publications (Koç, 2000b; Koç, *et al*., 2002).

Web modeling is based on physical laws (Brandenburg, 1973; Koç, *et al*., 2002) : Hooke's law allows for web elasticity; Coulomb's law explains contact between web and roll, including friction; mass conservation law allows for coupling between web speed and web tension; the second fundamental relation of dynamics explains variations of rotating roll speed. With adequate hypothesis (Koç, *et al*., 2002), dynamical equations can be found for the tension of each part of the web and for the speed of each roll (equal of web velocity by non sliding hypothesis). For instance, equations giving derivatives of T_1 and V_3 (see figure 2) are as following:

$$L_1 \frac{dT_1}{dt} = V_2\left(ES + T_2\right) - V_1 \frac{\left(ES + T_2\right)^2}{ES + T_1},$$
$$\frac{d}{dt}\left(J_3 \frac{V_3}{R_3}\right) = K_3 U_3 + R_3\left(T_2 - T_3\right) - f_3\left(\frac{V_3}{R_3}\right), \tag{1}$$

where E is the elastic modulus of the web; S is its section; L_1 is the length between the two first rolls; R_3 is the radius of the third roll; J_3 is its inertia; K_3 is the torque per tension ratio of the motor and f_3 is the friction function depending on rotating speed.
The non-linear model is made from the equations above; the first equation being used for each roll; the second one being used for each part of the web between two rolls. The order of the resulting system is then $2n_r$-1 where n_r is the number of rolls.

Fig. 1. Experimental setup

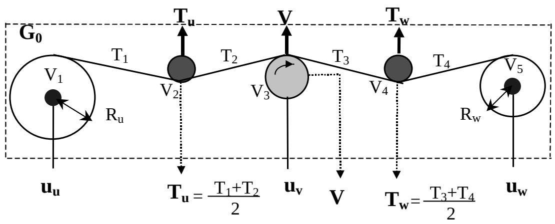

Fig. 2. Scheme of the three motors plant

From the physical system, a linear model is obtained around an operating point:

$$E(t)\,\frac{dX}{dt} = A(t)X + B(t)U \qquad Y = C\,X \tag{2}$$

where state matrices E, A and B are parameter dependant. State vectors and state matrices for a three motors plant are shown in Appendix 1.
This model has been already validated on a three motors system (see Koç, 2000b, Koç, *et al.*, 2002).
A nine motors system is build from the physical model for simulation purpose.

III. DECENTRALIZED OVERLAPPING CONTROL STRATEGY

Overlapping in decentralized control gives extra degrees of freedom that improves performances compared to disjointed decomposition (Ikeda, *et al.*, 1986).

3.1 Expansion

Firstly, the initial system model is decomposed with an appropriate input, output and state *expansion* that respects the *inclusion principle* (Ikeda, *et al.*, 1986; Stankovic, *et al.*, 2000) : the system with coupling is *expanded* into a new space, called *expanded* space, where the subsystems are disjointed.
For instance, let us consider a linear system S of the state model given in (3).

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \\ \frac{dx_3}{dt} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A33 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} B_{11} & 0 & 0 \\ 0 & B_{22} & 0 \\ 0 & 0 & B_{33} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$
$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} C_{11} & 0 & 0 \\ 0 & C_{22} & 0 \\ 0 & 0 & C_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{3}$$

Considering the overlapped subsystems *S1* and *S2* indicated by dashed lines in the model above, system S is expanded into *Se* (4). System *Se* has to respect the *inclusion principle* and the *restriction principle* as mentioned in (Ikeda, *et al.*, 1986) :

$$\left\{\begin{aligned} \begin{bmatrix} \frac{dx_{1e}}{dt} \\ \frac{dx_{2e}}{dt} \end{bmatrix} &= \begin{bmatrix} A_{11} & A_{12} & 0 & A_{13} \\ A_{21} & A_{22} & 0 & A_{23} \\ A_{21} & 0 & A_{22} & A_{23} \\ A_{31} & 0 & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} x_{1e} \\ x_{2e} \end{bmatrix} + \begin{bmatrix} B_{11} & 0 & 0 & 0 \\ 0 & B_{22} & 0 & 0 \\ 0 & 0 & B_{22} & 0 \\ 0 & 0 & 0 & B_{33} \end{bmatrix} \begin{bmatrix} u_{1e} \\ u_{2e} \end{bmatrix} \\ \begin{bmatrix} y_{1e} \\ y_{2e} \end{bmatrix} &= \begin{bmatrix} C_{11} & 0 & 0 & 0 \\ 0 & C_{22} & 0 & 0 \\ 0 & 0 & C_{22} & 0 \\ 0 & 0 & 0 & C_{33} \end{bmatrix} \begin{bmatrix} x_{1e} \\ x_{2e} \end{bmatrix} \end{aligned}\right. \tag{4}$$

with : $x_{1e} = [x_1^T\ x_2^T]^T$, $x_{2e} = [x_2^T\ x_3^T]^T$, $u_{1e} = [u_1^T\ u_2^T]^T$ $u_{2e} = [u_2^T\ u_3^T]^T$, $y_{1e} = [y_1^T\ y_2^T]^T$, $y_{2e} = [y_2^T\ y_3^T]^T$

3.2 Contraction

The next step consists in designing controllers for each disjointed subsystem, leading to a decentralized controller for the *expanded* model. Let's assume the controller has the following form:

C1: $$\dot{z}_1 = F_1\, z_1 + \begin{bmatrix} G_1^{\,1} & G_2^{\,1} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$
$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} H_1^{\,1} \\ H_2^{\,1} \end{bmatrix} z_1 + \begin{bmatrix} K_{11}^{\,1} & K_{12}^{\,1} \\ K_{21}^{\,1} & K_{22}^{\,1} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \tag{5}$$

C2: $$\dot{z}_2 = F_2\, z_2 + \begin{bmatrix} G_1^{\,2} & G_2^{\,2} \end{bmatrix} \begin{bmatrix} y_2 \\ y_3 \end{bmatrix}$$
$$\begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} H_1^{\,2} \\ H_2^{\,2} \end{bmatrix} z_2 + \begin{bmatrix} K_{11}^{\,2} & K_{12}^{\,2} \\ K_{21}^{\,2} & K_{22}^{\,2} \end{bmatrix} \begin{bmatrix} y_2 \\ y_3 \end{bmatrix} \tag{6}$$

The simple controller composed diagonally of controllers *C1* and *C2* do not have the *contractibility* property. State model is then rearranged in order to be *contractible* (7). This model includes blocks called low coupling.

$$\begin{bmatrix} \dot{Z}_{1e} \\ \dot{Z}_{2e} \end{bmatrix} = \begin{bmatrix} F_1 & 0 \\ 0 & F_2 \end{bmatrix} \begin{bmatrix} Z_{1e} \\ Z_{2e} \end{bmatrix} + \begin{bmatrix} G_1^{\,1} & G_2^{\,1} & 0 & 0 \\ 0 & 0 & G_1^{\,2} & G_2^{\,2} \end{bmatrix} \begin{bmatrix} Y_{1e} \\ Y_{2e} \end{bmatrix}$$

$$
\begin{bmatrix} U_{1e} \\ U_{2e} \end{bmatrix} = \left[\begin{array}{c:c} H_1^{\,1} & 0 \\ H_2^{\,1} & H_1^{\,2} \\ \hdashline H_2^{\,1} & H_1^{\,2} \\ 0 & H_2^{\,2} \end{array}\right] \begin{bmatrix} Z_{1e} \\ Z_{2e} \end{bmatrix} + \qquad (7)
$$

$$
\left[\begin{array}{cc:cc} K_{11}^{\,1} & K_{11}^{\,1} & 0 & 0 \\ K_{21}^{\,1} & (K_{22}^{\,1}+K_{11}^{\,2})/2 & 0 & K_{12}^{\,2} \\ \hdashline K_{21}^{\,1} & 0 & (K_{11}^{\,2}+K_{22}^{\,1})/2 & K_{12}^{\,2} \\ 0 & 0 & K_{21}^{\,2} & K_{22}^{\,2} \end{array}\right] \begin{bmatrix} Y_{1e} \\ Y_{2e} \end{bmatrix}
$$

low coupling

Under the *inclusion* condition, this controller is *contracted* into the initial space, leading to the implementation controller (8). This controller includes entries called high coupling (coefficient 1/2 in (8))

$$
\begin{bmatrix} \dot{Z}_1 \\ \dot{Z}_2 \end{bmatrix} = \left[\begin{array}{c:c} F_1 & 0 \\ \hdashline 0 & F_2 \end{array}\right] \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} + \left[\begin{array}{ccc} G_1^1 & G_2^1 & 0 \\ \hdashline 0 & G_1^2 & G_2^2 \end{array}\right] \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}
$$

$$
\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \left[\begin{array}{c:c} H_1^1 & 0 \\ H_2^1 & H_1^2 \\ \hdashline 0 & H_2^2 \end{array}\right] \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} K_{11}^1 & K_{12}^1 & 0 \\ K_{21}^1 & \dfrac{(K_{22}^1 + K_{11}^2)}{2} & K_{12}^2 \\ 0 & K_{21}^2 & K_{22}^2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \qquad (8)
$$

3.3 Stability proofs

Under the *inclusion* condition of the controller, stability of the closed loop in the *extended* space leads stability in the initial state. As stability in the *expanded* state is easier to determine, this theorem is generally used and a standard Lyapunov vector function method can be used, as in (Ikeda, *et al.*, 1986).

IV. APPLICATION TO WEB WINDING SYSTEMS

4.1 Decomposition

The decomposition step is of high importance. Indeed, the final result, i.e. the properties of the system with the final controller, highly depends on the decomposition. One simple rule may be that the decomposition should respect as much as possible the couplings: subsystems should have low coupling with each others. Of course, the decomposition must make sense. In our case, several trials have been done (respecting the *expansion* and *restriction* principles); only the best solutions are presented.

The number of subsystems comes from a trade-off on their order. In our case, subsystems including 3 motors appeared to be a good trade-off.

4.2 1DOF H_∞ controller design method

The H_∞ method is now used to control complex systems. This approach has specifically shown good results in designing multivariable controllers for two and three motors winding systems (Koç, *et al.*, 2000a, 2002; Koç, 2000b). We then naturally use it for local controller design. The mixed sensitivity method is shortly presented in the Appendix 2; more information can be found in (Zhou, *et al.*, 1995).
The obtained controller (using this method) for a 3 motors system or subsystem is of order 15.

4.3 2DOF H_∞ controller design method

In the mixed-sensitivity methods, disturbance rejections and tracking properties are interdependent. To consider these two issues separately, we have chosen a 2DOF H_∞ control strategy. Typically the two parts of such controller $K = [K_f \quad K_b]^T$ (figure 3) are designed in two steps: disturbances rejection is optimized with K_b and tracking specifications are improved with K_f . However, as we use 2DOF H_∞ method, these two parts of the controller are computed in one step.
In our application, a 2DOF H_∞ controller is designed with output weighting and model matching (figure 3). Model M_o is the desired transfer function T_{yr}. Compared to 1DOF strategy, the order of the 2DOF controller is only increased by the order of M_o. In our case, M_o is of order 2.

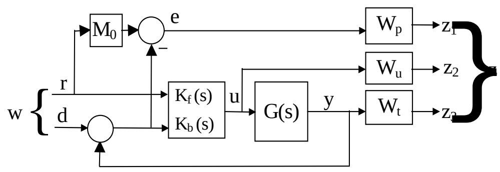

Fig. 3. Scheme of weighted model matching for 2DOF H_∞ controller design

The weighting functions W_p, W_u, and W_t appear in the closed loop transfer matrix:

$$
T_{zw} := \begin{bmatrix} W_p(M_0 - T_{yr}) & W_p S_{enb} \\ W_u S_u K_f & W_u K_b S_{enb} \\ W_t T_{yr} & W_t T_{enb} \end{bmatrix}
$$

where $S_{enb} = (I + GK_b)^{-1}$ is the sensitivity function: and $T_{enb} = I - S_{enb}$ is the complementary sensitivity function:
To compare 1DOF and 2DOF strategies, we use the same weighting functions.
The order of the resulting controller is 17 (15 for the 1DOF controller).

V. SIMULATION RESULTS

Figure 4 shows the tension signal at the middle of the web in the case of two control strategies: the first one does not include any overlapping whereas the second one, includes overlapping. All the local controllers are 1DOF H_∞ controllers. Whereas the tension reference remains constant equal to 1.5 kg, speed reference changes by steps every 10 seconds, leading to tension perturbations. In the case of overlapping control, the perturbations are largely attenuated.

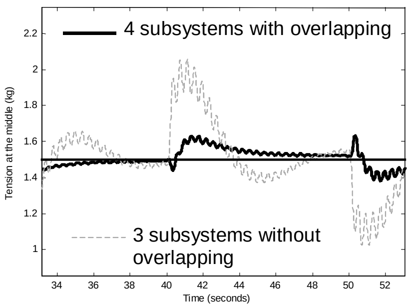

Fig. 4. Comparison between responses with and without overlapping

Simulation results of the decentralized overlapping control are shown on figures 5 to 7 during a velocity decrease in the cases of 2DOF H_∞ controllers and standard H_∞ approach.

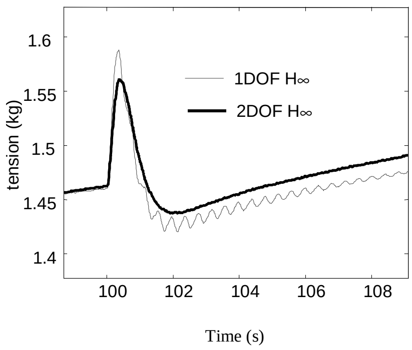

Fig. 5. Unwinding tension of the overlapping system

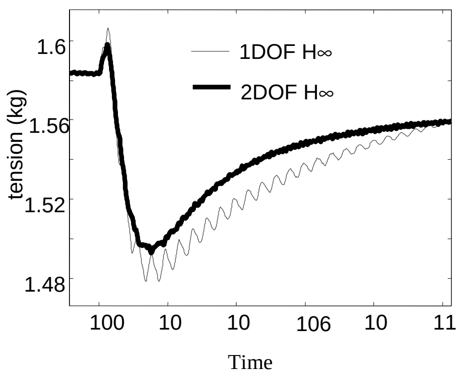

Fig. 6. Winding tension of the overlapping system

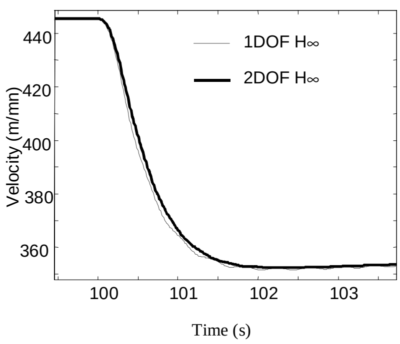

Fig. 7 . Web velocity of the overlapping system

Performances are slightly improved using a 2DOF H_∞ design, but not as much as the effect of the overlapping control. Indeed, overlapping already improves decoupling The overlapping control strategy has already taken into account the coupling between two successive subsystems. Furthermore, one can notice that tension signal is substantially smoothed, which means that the stress on the system is reduced. Moreover, robustness to variation of elasticity modulus has been tested in simulation. It appears that the control strategy with overlapping has significantly improved robustness with respect to variations of the Young modulus of the web (Gigan, 2001).

VI. CONCLUSIONS

Decentralized control with different overlapping strategies has been applied to a nine motors web handling system, in which the estimated model is of order 33. Each subsystem includes 3 motors; and has its own local controller of order 17.

Decentralized control strategies without overlapping are unable to achieve proper decoupling between tension and velocity. When introducing overlapping, it becomes possible to obtain a good decoupling whereas tracking is also of high quality.

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APPENDIX 1 – Linear model of the 3 motors plant

$$E(t)\,\frac{dX}{dt} = A(t)X + B(t)U \qquad Y = C\,X \qquad Y^T = (T_u\ V_3\ T_w) \qquad X^T = (V_1\ T_1\ V_2\ T_2\ V_3\ T_3\ V_4\ T_4\ V_5)$$

$$U^T = (u_u\ u_v\ u_w)$$

$$A(t) = \begin{bmatrix} -f_1(t) & R_u^2(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -E_0 & -V_0 & E_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -R_2^2 & -f_2 & R_2^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & V_0 & -E_0 & -V_0 & E_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -R_3^2 & -f_3 & R_3^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & V_0 & -E_0 & -V_0 & E_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -R_4^2 & -f_4 & R_4^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & V_0 & -E_0 & -V_0 & E_0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -R_w^2(t) & -f_5(t) \end{bmatrix} \qquad C = \begin{bmatrix} 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \end{bmatrix}$$

$$E(t)=\begin{bmatrix} J_1(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & J_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & L_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & J_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & L_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & J_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & L_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & J_5(t) \end{bmatrix} \qquad B(t)=\begin{bmatrix} -K_u R_u(t) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & K_t R_3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & K_w R_w(t) \end{bmatrix}$$

V_i, R_i, J_i and f_i are the linear velocity, the radius, the inertia and the viscous friction coefficient of the roll i respectively. T_i and L_i are the web tension and the web length between the roll i and the roll $i+1$. K_u, K_t, K_w are the torque constants of each motor. V_0 is the nominal linear web velocity. E_0 is a parameter depending on elasticity modulus E, on web section S and on nominal tension T_0 : $E_0 = ES + T_0$. All parameters varying during the winding process are expressed as functions of time.

APPENDIX 2 – Mixed sensitivity method

Refering to figure 8 where $G(s)$ is a LTI model of the plant, the controller $K(s)$ is computed in order to minimizes H_∞ norm of transfer between reference r and fictive output z. By tuning weights $W_e(s)$, $W_u(s)$ and $W_y(s)$, transfer functions $S_{en} = (I + G K)^{-1}$, $K S_{en}$ and $T = I - S_{en}$ are forced to have adequate shapes.

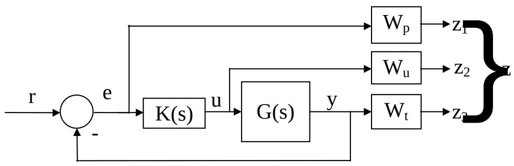

Fig. 8. Weighted model of the system

Weighting function W_p has high gain at low frequency in order to reject low frequency disturbances. The form of W_p is as following [10]: $W_p(s) = \dfrac{\dfrac{s}{M} + \omega_B}{s + \omega_B \varepsilon_0}$ where M is the maximum peak magnitude of S_{en} : $\|S_{en}\|_\infty \le M$,

(11)

and ω_B is the required frequency bandwidth, ε_0 is the steady-state error allowed. W_u is used to avoid the large control signals and W_t tunes the roll-off.

APPENDIX 2 – Overlapping control scheme

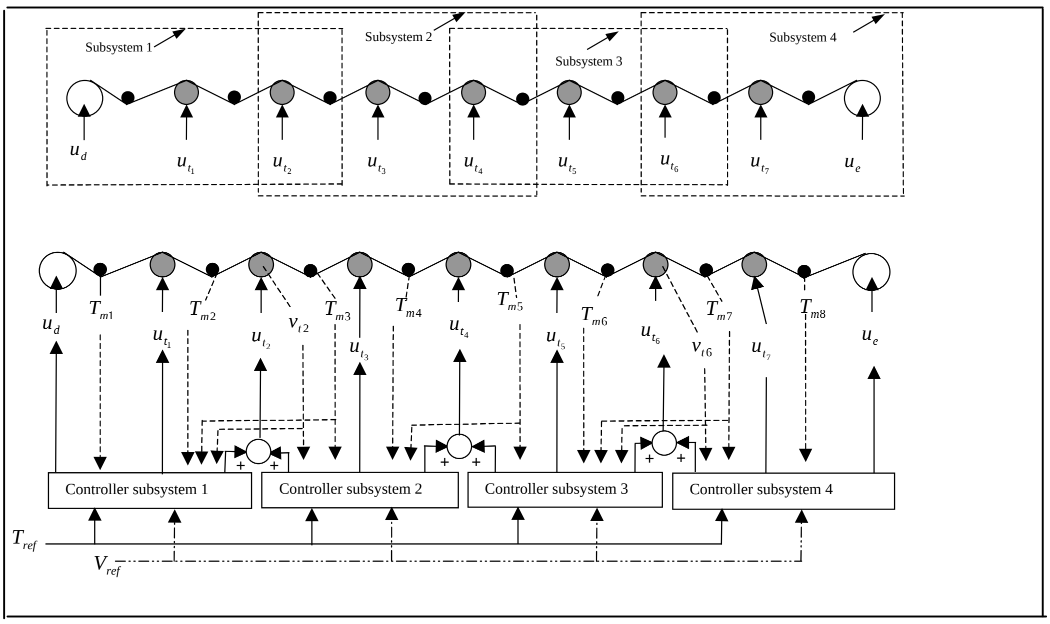

Fig. 9. Partitioning in 4 overlapping subsystems