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QUALITATIVE ANALYSIS OF COMPETITIVE-COOPERATIVE CELLULAR NEURAL NETWORKS WITH DELAY [1]

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Abstract: This paper studies general Delayed Cellular Neural Networks (DCNNs) with competitive-cooperative configurations. It is demonstrated how such a configuration may be exploited to give a detailed characterization of the fixed point dynamics in DCNNs. Specifically, we show that by dividing the connection weights into inhibitory and excitatory type, it is always possible to embed a competitive-cooperative DCNN into an augmented cooperative delay system, and thus allows for the use of the powerful monotone dynamical system theory. In this way, we derive several simple sufficient conditions on guaranteed trapping regions and guaranteed componentwise (exponential) convergence of DCNNs. The results relate specific decay rate and trajectory bounds to system parameters and are therefore of practical significance in designing a DCNN with desired performance. *Copyright © 2002 IFAC*

Keywords: Cellular neural networks, time delay, competition and cooperation, trapping region, convergence, stability, monotone dynamical system.

1. INTRODUCTION

Since their first introduction by Chua and Yang (1988), cellular neural networks (CNNs) have been successfully applied in signal processing, especially in static image treatment. These applications rely essentially on the stable and convergent dynamics of a CNN. To process moving images, Roska and Chua (1992) introduced delayed cellular neural networks (DCNNs) that take into account time delays in the signal transmitted among the cells. However, time delay increases the dimensionality, and hence complexity, of the system. Qualitative analysis of DCNNs is much more difficult than that of standard CNNs and has therefore attracted considerable interests in recent years (e.g., Roska *et al,* 1992; Civalleri *et al,* 1993; Roska *et al,* 1993; Gill, 1994; Cao, 1999; Arik and Tavsanoglu, 2000; Cao, 2000; Takahashi, 2000; Chu *et al,* 2001 b and the references therein). Most of the studies were based on Lyapunov like arguments. That is, convergence in a DCNN has been characterized by the monotonic decreasing of an auxiliary functional on trajectories of the system. This approach is conceptually simple and intuitive, but the performance of a DCNN in the form of, e.g., the rate of convergence from an initial condition to the final state is normally difficult to assess. In a practical design of a network system, the convergence rate is a critical performance that should be taken into account (Michel *et al,* 1989). So far, only a few results have been established on exponential convergence of DCNNs (Cao, 1999; Cao, 2000; Chu *et al,* 2001 b).

In this paper, we study DCNNs from a new perspective. Namely, we view a DCNN as a general competitive-cooperative system, and propose a decomposition method that exploits the competitive-cooperative connectivity of the network. By competitive connection it is meant the way in which a cell's firing inhibits the firing of other cells. Conversely, cooperative connection means the way in which a cell's firing excites the firing of others. In a DCNN, the output of a cell is typically characterized by a sigmoid function (i.e., a continuous, bounded, and non-decreasing function). The competitive-cooperative connection pattern can thus be recognized by the sign of the weights. Positive weights are due to excitatory coupling, negative weights are due to in-

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hibitory coupling, while a zero weight indicates no interaction at all. It will be demonstrated in the sequel that by dividing the weights into positive and negative parts, one can eventually embed a competitive-cooperative DCNN into an augmented cooperative system. Such a system has a significant order-preserving or monotone property that is useful in the analysis of the original DCNN. Similar ideas have also been employed in Chu and Wang (1998) and Chu (2001) to investigate qualitative properties of delayed Hopfield networks and discrete-time neural networks. The present work further extends the studies to DCNNs. In particular, we will establish some detailed characterization on trajectory behavior of DCNNs in terms of trapping regions and componentwise (exponential) convergence. The results enable one to design a DCNN with a desired decay rate and trajectory bounds and are thus of practical interest.

It should be noted that applications of monotone dynamical system theory to qualitative study of pure competitive or pure cooperative neural networks has already been addressed by several authors (e.g., Lemmon and Vijaya Kumar, 1989; Hirsch, 1989; van den Driessche and Zou, 1998 and the references therein). Moreover, in Hirsch (1989) and Chua and Wu (1992), a special type of competitive-cooperative networks satisfying "sign-symmetry" condition were converted into cooperative systems of the same size. For recent works on embedding a general competitive-cooperative neural network systems into a larger cooperative system we also refer to van den Driessche *et al* (2001). Besides, in Chu and Huang (1998), a more general mixed monotone decomposition method was proposed for qualitative analysis of dynamical systems whose dynamics involves both growth and decay effects. The present work may also be related to Chu and Huang (1998).

2. THE DCNN MODEL AND SPECIFICATION

The DCNNs considered in this paper are modelled by the following nonlinear functional differential equation

$$\dot{x}(t) = -Ax(t) + Bs[x(t)] + Cs[x(t-\tau)] + c, \quad (1)$$

where $x(t) \in \mathbf{R}^n$ is the cell's state vector at time t, and $\tau > 0$ is the time delay in the networks; $A = \text{diag}[a_1, ..., a_n]$ with every $a_i > 0$ is the relax matrix, $B = [b_{ij}]$ and $C = [c_{ij}]$ are the $n \times n$ connection matrices associated with delay-free and delayed feedbacks, and the entries of B, C may be positive or negative according to excitation or inhibition nature of the interconnections; the last term c is the constant external input vector to the networks; $s(x) = [s_1(x_1), \ldots, s_n(x_n)]^T$ is a vector-valued output function and it is typically assumed in the theory of CNNs that $s_i(r) = 0.5[|r+1| + |r-1|]$, however, we allow here s_i to be more general sigmoid functions specified merely by the following monotone property and the slope condition:

$$0 \leq \frac{s_i(r_1) - s_i(r_2)}{r_1 - r_2} \leq k_i \quad (2)$$

for $r_1 \neq r_2 \in \mathbf{R}^n$, where $k_i > 0$, $i = 1, ..., n$. This will make our discussion insensitive to the model details. Clearly, the often used sigmoid functions such as $\tanh x_i$ and $0.5[|x_i+1| - |x_i-1|]$ have such a property.

The solution of Eq. (1) depends upon the specification of an initial condition $x(\theta) = \varphi(\theta)$, $\theta \in [-\tau, 0]$. It is usually assumed that the given n vector function φ is continuous, though it need only be measurable for Eq. (1) to be well defined.

The objective of this work is to study in some details the qualitative behavior of the fixed point dynamics of DCNN (1). By definition, for a given constant input vector c, a fixed point or an equilibrium of system (1) is a point $x_e \in \mathbf{R}^n$ having the property that

$$-Ax_e + (B+C)s(x_e) + c = 0. \quad (3)$$

Since $s(x)$ is bounded and continuous, it follows readily from Brouwer's fixed point theorem that there is at least one solution x_e to the above equation for every constant vector c. Further, if x_e is globally asymptotically stable, then it is the unique equilibrium attracting all other trajectories. In this case, to each given input vector c the network associates a unique equilibrium to which it converges irrespectively of the initial conditions. This establishes a one-to-one correspondence between the input space and the steady-state space, which is a desirable property in applications of DCNNs to problems such as signal processing and input patterns classification. One result to be given later provides a global exponential convergence criterion for system (1).

For convenience of discussion, we make the change of variable $z = x - x_e$ and transform Eq. (1) to

$$\dot{z}(t) = -Az(t) + Bf[z(t)] + Cf[z(t-\tau)], \quad (4)$$

where $f(z) = s(z + x_e) - s(x_e)$. It is clear that f belongs to the following sector nonlinear function class $\mathcal{F}$ defined by

(i) $f_i(0) = 0$, and

(ii) $0 \leq \frac{f_i(r_1) - f_i(r_2)}{r_1 - r_2} \leq k_i$, for all i.

In this way, the equilibrium x_e of Eq. (1) is translated into the origin, and thus $z = 0$ is a fixed point of Eq. (4). Henceforth, we will proceed our discussion on Eq. (4).

To characterize the dynamical behavior of system (4), we consider two bounded, continuous, and differentiable functions $\xi(t), \varsigma(t) : [-\tau, +\infty) \to \mathbf{R}^n$ with $\xi(t) > 0$, $\varsigma(t) > 0$, and define the time-variant set

$$\Omega_{\xi,\varsigma}(t) = \{z \in \mathbf{R}^n : -\xi(t) \leq z \leq \varsigma(t)\}, \quad (5)$$

where and throughout inequalities between vectors are in componentwise sense.

Definition 1. The set $\Omega_{\xi,\varsigma}(t)$ is said to be a *guaranteed trapping region* for system (4) provided that for every $f \in \mathcal{F}$, and for any $t_0 \geq 0$, the solution of Eq. (4) satisfies $z(t) \in \Omega_{\xi,\varsigma}(t)$ for $t \geq t_0$ whenever $z(t_0+\theta) \in \Omega_{\xi,\varsigma}(t_0+\theta)$ for $\theta \in [-\tau, 0]$.

In addition, if we further impose certain contractivity on the set $\Omega_{\xi,\varsigma}(t)$ by letting

$$\lim_{t\to\infty}\xi(t) = 0 = \lim_{t\to\infty}\varsigma(t), \qquad (6)$$

then we can define a special type of convergence properties for DCNNs.

Definition 2. System (4) is called *guaranteed componentwise convergent with respect to* $\Omega_{\xi,\varsigma}(t)$ [$\Omega_{\xi,\varsigma}(t)$-GCC] if $\Omega_{\xi,\varsigma}(t)$ is a guaranteed trapping region and is contractive by conditions (6). Particularly, if

$$\xi(t) = \alpha e^{-\sigma t}, \quad \varsigma(t) = \beta e^{-\sigma t} \qquad (7)$$

for some scalar $\sigma > 0$ and two constant vectors $\alpha, \beta \in \mathbf{R}^n$ with $\alpha, \beta > 0$, then the system is called *guaranteed componentwise exponentially convergent* (GCEC).

These concepts enable us to characterize the dynamical behavior of the DCNNs in a more detailed manner. Moreover, the above properties are insensitive to the details of the output functions (i.e., valid for the whole class $\mathcal{F}$). This feature of robustness against change in the model details is of basic physical significance.

In the sequel, by $\Omega_{\xi,\varsigma}(t)$-GCC and GCEC we will also mean the guaranteed componentwise convergence w.r.t. $\Omega_{\xi,\varsigma}(t)$ and guaranteed componentwise exponential convergence of system (4). This would not raise any confusion according to the context.

3. A COMPETITION-COOPERATION DECOMPOSITION METHOD

In order to examine the above specified properties, we apply the ideas in Chu and Wang (1998) and Chu (2001) to system (4) and develop in this section a decomposition approach for the DCNNs. This approach takes advantage of the competitive-cooperative connectivity inherent in a neural network in a natural way, without requiring any additional assumption (e.g., "sign-symmetry") on the connection weights.

Following Chu and Wang (1998) and Chu (2001), we split the connection matrices B, C into two parts:

$$B = B^+ - B^-, \quad C = C^+ - C^-,$$

where $b_{ij}^+ = \max\{b_{ij}, 0\}$ correspond to the excitatory weights and $b_{ij}^- = \max\{-b_{ij}, 0\}$ the inhibitory weights, c_{ij}^+ and c_{ij}^- are similarly defined. Then system (4) can be rewritten as

$$\begin{aligned}\dot{z}(t) &= -Az(t) + (B^+ - B^-)f[z(t)] \\ &\quad + (C^+ - C^-)f[z(t-\tau)]. \end{aligned} \qquad (8)$$

We refer to it as a decomposition of competitive-cooperative connectivity of network (4). This division of the interaction into cooperative/competitive parts is a reasonable requirement on any model that purports to have some relation to biological systems. It also leads to an intuitive description of the temporal evolution of a neural network. According to Eq. (8) the transient states of a network evolve under the actions of cooperation and competition effects, and at a steady state or an equilibrium, say z_e, these two opposite actions achieve a balance:

$$(B^+ + C^+)f(z_e) = Az_e + (B^- + C^-)f(z_e).$$

Moreover, at a stable steady-state such a balance will persist in the presence of certain extent disturbances in the network, whereas for an unstable one the balance might be disrupted by an even very small disturbance. Hence, the former corresponding to a stable balance between cooperation and competition actions in a neural network and the latter to an unstable balance. This suggests that one might expect to ascertain stability of a neural network (4) by examining the temporal evolution of the deviation from the competition-cooperation balance of the network. We will pursue this suggestion elsewhere.

Now take the symmetric transformation $y = -z$. From Eq. (8), it follows that

$$\begin{aligned}\dot{y}(t) &= -Ay(t) + B^+g[y(t)] + B^-f[z(t)] \\ &\quad + C^+g[y(t-\tau)] + C^-f[z(t-\tau)] \quad (9) \\ \dot{z}(t) &= -Az(t) + B^-g[y(t)] + B^+f[z(t)] \\ &\quad + C^-g[y(t-\tau)] + C^+f[z(t-\tau)] \quad (10)\end{aligned}$$

where $g(u) = -f(-u) \in \mathcal{F}$. Accordingly, we introduce the following augmented system:

$$\dot{d}(t) = -\Lambda d(t) + \Pi h[d(t)] + \Xi h[d(t-\tau)], \qquad (11)$$

where

$$d(t) = \begin{bmatrix} p(t) \\ q(t) \end{bmatrix}, \quad h[d(t)] = \begin{bmatrix} g[p(t)] \\ f[q(t)] \end{bmatrix},$$

$$\Lambda = \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix}, \ \Pi = \begin{bmatrix} B^+ & B^- \\ B^- & B^+ \end{bmatrix},$$

$$\Xi = \begin{bmatrix} C^+ & C^- \\ C^- & C^+ \end{bmatrix}.$$

Noticing the (elementwise) non-negativity of Π and Ξ, system (11) itself is cooperative and hence possesses the following important *order-preserving property*.

Lemma 1. Let $u(t)$ and $v(t)$ be solutions of Eq. (11). Then $u(t_0+\theta) \leq v(t_0+\theta)$ for $\theta \in [-\tau, 0]$ implies $u(t) \leq v(t)$ for $t \geq t_0 \geq 0$. Moreover, if $w(t)$ satisfies

$$\dot{w}(t) \geq -\Lambda w(t) + \Pi h[w(t)] + \Xi h[w(t-\tau)] \qquad (12)$$

for $t \geq t_0$, then $u(t_0+\theta) \leq w(t_0+\theta)$ for $\theta \in [-\tau, 0]$ implies $u(t) \leq w(t)$ for $t \geq t_0 \geq 0$.

This is a specialization of general results (e.g., Ohta, 1981; Chu *et al*, 2001b) on monotone dynamics of cooperative delay differential systems to Eq. (11). It indicates that the states of a cooperative system will retain for all time their initial relationship, a partial ordering induced by the subset of non-negative state vectors of the state space. In the literature, such results are also referred to as comparison principle for delay systems.

Remark 1. Let $z(t)$ be a solution of Eq. (4), it is clear that $p(t) = -z(t)$, $q(t) = z(t)$ constitute a solution to Eq. (11). That is, the evolution of system (4) is restricted to the manifold $p = -q$ in the state space of system (11). In this way system (4) is embedded into an augmented cooperative system (11). On the other hand, it should be noted that for a solution $[p(t)^T \; q(t)^T]^T$ of system (11), neither $p(t)$ nor $q(t)$ should satisfy Eq. (4) unless $-p(t) = q(t)$ [in this case $z(t) = -p(t) = q(t)$ is a solution of system (4)]. In general, the two systems may be related by the following *two-sided comparison principle.*

Lemma 2. Assume for Eqs. (4) and (11) that the initial condition $-p(t_0+\theta) \leq z(t_0+\theta) \leq q(t_0+\theta)$ holds for $\theta \in [-\tau, 0]$. Then $-p(t) \leq z(t) \leq q(t)$ for $t \geq t_0 \geq 0$.

This follows readily from Eqs. (9), (10), and Lemma 1. Hence, a solution of Eq. (11) may provide a two-sided constraint on that of Eq. (4). This enables one to deduce qualitative properties of system (4) from a related cooperative system (11). In the sequel, we will apply this idea to study the dynamical behavior of system (4).

4. TRAPPING REGIONS

We present here sufficient conditions for trapping regions of system (4).

Theorem 1. The set $\Omega_{\xi,\varsigma}(t)$ is a guaranteed trapping region for system (4) if

$$\dot{\gamma}(t) \geq (\Pi\Sigma - \Lambda)\gamma(t) + \Xi\Sigma\gamma(t-\tau), \; t \geq 0, \quad (13)$$

where $\gamma(t) = [\xi(t)^T \; \varsigma(t)^T]^T$, $\Sigma = \mathrm{diag}[k_1, ..., k_n, k_1, ..., k_n]$.

Proof. From the definition of class $\mathcal{F}$, it is easy to see that $h[\gamma(t)] \leq \Sigma\gamma(t)$. Then by noticing the non-negativity of the entries of matrices Π, Ξ, it follows that $\Pi h[\gamma(t)] \leq \Pi\Sigma\gamma(t)$ and $\Xi h[\gamma(t-\tau)] \leq \Xi\Sigma\gamma(t-\tau)$. Hence, if condition (13) holds, then we have, for $t \geq 0$,

$$\dot{\gamma}(t) \geq -\Lambda\gamma(t) + \Pi h[\gamma(t)] + \Xi h[\gamma(t-\tau)]. \quad (14)$$

Now consider an arbitrary $f \in \mathcal{F}$ and let $z(t)$ be the solution of the corresponding Eq. (4) with the initial value satisfying $-\xi(\theta) \leq z(\theta) \leq \varsigma(\theta)$ for $\theta \in [-\tau, 0]$. Take in Eq. (11) $p(\theta) = \xi(\theta)$, $q(\theta) = \varsigma(\theta)$, and without loss of generality, let the initial time $t_0 = 0$, then by condition (14) and Lemma 2,

$$-p(t) \leq z(t) \leq q(t), \;\; t \geq 0. \quad (15)$$

Meanwhile, let $u(\theta) = [p(\theta)^T \; q(\theta)^T]^T = \gamma(\theta)$. From condition (14) and Lemma 1,

$$u(t) \leq \gamma(t), \;\; t \geq 0.$$

This and condition (15) yield

$$-\xi(t) \leq z(t) \leq \varsigma(t), \;\; t \geq 0.$$

The theorem thus follows. □

Remark 2. Assume that every $b_{ii} \geq 0$, then similar to the proof of Theorem 1 in Chu *et al* (2001a), one can show that condition (13) is also necessary.

By taking $\xi(t)$ and $\varsigma(t)$ to be two constant vectors, we obtain a special guaranteed trapping region.

Corollary 1. For two constant vectors $\alpha, \beta \in \mathbf{R}^n$ with $\alpha > 0$, $\beta > 0$. The set $\Omega_{\alpha,\beta} = \{x \in \mathbf{R}^n : -\alpha \leq x \leq \beta\}$ is a guaranteed trapping region for system (4) if

$$[\Lambda - (\Pi + \Xi)\Sigma]\eta \geq 0, \quad (16)$$

where $\eta = (\alpha^T \; \beta^T)^T$.

Remark 3. The above results depend only on the slopes k_i $(i = 1, ..., n)$ of the output function s and thus are applicable to the whole set of $\mathcal{F}$. For a network (4) with a given sigmoid s, one can similarly conclude that the set $\Omega_{\xi,\varsigma}(t)$ is a trapping region if

$$\dot{\gamma}(t) \geq -\Lambda\gamma(t) + \Pi h[\gamma(t)] + \Xi h[\gamma(t-\tau)] \quad (17)$$

for $t \geq 0$, where $h[\gamma(t)]$ and $\gamma(t)$ are specified as in Eqs. (11) and (13). Also, for the set $\Omega_{\alpha,\beta}$ in Corollary 1, condition (17) now reads

$$\Lambda\eta - (\Pi + \Xi)h(\eta) \geq 0.$$

Due to the boundedness of $h(\cdot)$, one can pick a constant vector $\eta > 0$ to fulfill the condition. This indicates that a DCNN always has a trapping region.

5. COMPONENTWISE CONVERGENCE

By further assuming contractivity of the set $\Omega_{\xi,\varsigma}(t)$ in Theorem 1, we obtain the following componentwise convergence result.

Theorem 2. Suppose condition (6) is satisfied. Then system (4) is $\Omega_{\xi,\varsigma}(t)$-GCC if condition (13) holds.

Particularly, inserting special $\xi(t)$ and $\varsigma(t)$ specified by Eqs. (7) into condition (13) leads immediately to the following.

Theorem 3. System (4) is GCEC if there are two constant vectors $\alpha, \beta \in \mathbf{R}^n$ with $\alpha, \beta > 0$, and a scalar $\sigma > 0$ such that

$$[\sigma I - \Lambda + (\Pi + e^{\sigma\tau}\Xi)\Sigma]\eta \leq 0, \quad (18)$$

where $\eta = (\alpha^T \ \beta^T)^T$ and I is an identity matrix with appropriate dimensions. It is easily verifiable that this condition can be rewritten as

$$\mu_\infty\{\Gamma^{-1}[\Lambda + (\Pi + e^{\sigma\tau}\Xi)\Sigma]\Gamma\} \geq \sigma > 0,$$

where $\Gamma = \text{diag}[\alpha_1, ..., \alpha_n, \beta_1, ..., \beta_n]$ and $\mu_\infty(\cdot)$ is the infinity matrix measure defined by $\mu_\infty(M) = \max_{1\leq i\leq \ell}\{m_{ii} + \sum_{j\neq i}|m_{ij}|\}$ for a matrix $M = [m_{ij}]_{\ell\times\ell}$.

This result relates the specific exponential decay rate and trajectory bound to system parameters and thereby provides a way to design a DCNN with desired converging performance. These parameters only involve the connection weights and the cell gains, regardless of the exact features of the cells. This merit facilitates application of the criterion in a wide range.

Remark 4. Observe that condition (18) remains valid with $\kappa\alpha, \kappa\beta$ in place of α, β for any constant $\kappa > 0$. Meanwhile, given an arbitrary initial condition $\varphi(\theta)$ of system (4), one can always pick a $\kappa > 0$ such that $-\kappa\alpha \leq \varphi(\theta) \leq \kappa\beta$ for $\theta \in [-\tau, 0]$. Therefore by Theorem 3,

$$-\kappa\alpha e^{-\sigma t} \leq z(t) \leq \kappa\beta e^{-\sigma t}, \quad t \geq 0. \tag{19}$$

This shows that condition (18) actually gives a *global exponential convergence* criterion for network (4), and Eq. (19) provides a trajectory estimate.

Next, we make some comments on the symmetrical case $\alpha = \beta$. In this case, condition (18) is reduced to

$$[\sigma I - A + (|B| + e^{\sigma\tau}|C|)K]\alpha \leq 0, \tag{20}$$

where $|B| = [|b_{ij}|] = B^+ + B^-, |C| = [|c_{ij}|] = C^+ + C^-$. Clearly, it is equivalent to the existence of a constant vector $\alpha > 0$ such that

$$[A - (|B| + |C|)K]\alpha > 0. \tag{21}$$

Noticing the non-positivity of every off-diagonal entries of matrix $A - (|B| + |C|)K$, condition (21) is in turn tantamount to the matrix $A - (|B| + |C|)K$ being an M-matrix (Berman and Plemmons, 1979). Further, by the properties of M-matrices, it is also equivalent to

$$\begin{vmatrix} h_{11} & \cdots & h_{1i} \\ \vdots & \cdots & \vdots \\ h_{i1} & \cdots & h_{ii} \end{vmatrix} > 0, \quad i = 1, ..., n, \tag{22}$$

where

$$h_{ij} = \begin{cases} a_i - k_i(|b_{ii}| + |c_{ii}|), & i = j; \\ -k_j(|b_{ij}| + |c_{ij}|), & i \neq j. \end{cases}$$

Remark 5. Condition (20) has been obtained in Cao (1999) using Lyapunov method. Also, letting each $\alpha_i = 1$ in (21) yields a result similar to the "dominant template" condition presented in Roska *et al* (1993) by means of Lyapunov-Razumikhin method. Nevertheless, the present results provide, in addition to global exponential convergence, more subtle estimate on trajectory bounds and decay rates for DCNNs.

Observe that, although (18), (20), (21), and (22) are all sufficient GCEC conditions, the trajectory behavior that they can yield for a DCNN may be quite different. The first two conditions can guarantee a network to be convergent with a prescribed exponential decay rate and trajectory bounds, described by σ and α, β, respectively, while the last two only ensure exponential convergence in a network, saying nothing about decay rate explicitly (condition (21) also provides an estimate of the trajectory bound). On the other hand, it should be noted that conditions (21) and (22) are delay independent. This is of practical significance in the case where time delays exist but their magnitudes could not be evaluated accurately. In addition note that condition (22) involves no free parameters and can be checked directly according to the system under consideration.

Finally, we point out that, from an asymmetric exponential constraint (7), one can always get a symmetric one. Indeed, rewrite condition (18) as

$$\begin{aligned} &[\sigma I - A + (B^+ + e^{\sigma\tau}C^+)K]\alpha \\ &\qquad\qquad + (B^- + e^{\sigma\tau}C^-)K\beta \leq 0, \\ &[\sigma I - A + (B^+ + e^{\sigma\tau}C^+)K]\beta \\ &\qquad\qquad + (B^- + e^{\sigma\tau}C^-)K\alpha \leq 0. \end{aligned}$$

Adding them gives

$$[\sigma I - A + (|B| + e^{\sigma\tau}|C|)K]\rho \leq 0, \tag{23}$$

where $\rho = \alpha + \beta > 0$. By referring to condition (20), this corresponds to a symmetric constraint on the state of system (4). Therefore, the existence of a $2n$-dimensional positive vector η satisfying condition (18) is equivalent to that of an n-dimensional positive vector ρ satisfying condition (23). Hence, as far as for qualitatively ascertaining componentwise exponential convergence of system (4), one may simply check the symmetric case for convenience, without loss of any generality. Of course, it is evident that an asymmetric constraint may give more accurate trajectory behavior than does a symmetric one.

Example. To illustrate the above results, we consider a DCNN (1) with n identical neurons that satisfy condition (2) with the maximum slope $k_i = 1$. Also let $A = I$ (the identity matrix) and B, C are constant matrices to be specified. Then, as stated above, corresponding to a given constant input vector c, the network has at least one equilibrium x_e. Further, by Theorem 3 (for simplicity, here we only consider symmetric case and set each $\alpha_i = 1$), condition

$$\sigma - 1 + \sum_{j=1}^{n}(|b_{ij}| + e^{\sigma\tau}|c_{ij}|) \leq 0, \ i = 1, ..., n,$$

ensures the system to be convergent to x_e in terms of

$$|x_i(t) - x_e| \leq \kappa e^{-\sigma t},\ t \geq 0,$$

whenever $|x_i(\theta)| \leq \kappa$ ($\theta \in [-\tau, 0]$) with $\kappa > 0$ a constant depending on the initial condition $x_i(\theta)$, $i = 1, ..., n$. Moreover, according to Remark 4, the convergence is in fact global and x_e is uniquely determined by the input c. This ensures a consistent response in the network to input signals, and precludes the possibility of sustained oscillation or chaos in the network. If one is merely interested in confirming GCEC or global exponential convergence of the system, it may be convenient to use an even simpler criterion

$$\sum_{j=1}^{n}(|b_{ij}| + |c_{ij}|) < 1,$$

according to condition (21) with each $\alpha_i = 1$.

6. CONCLUSION

We have developed a decomposition method for the study of the fixed point dynamics in competitive-cooperative DCNNs. The method consists in embedding a competitive-cooperative DCNN into an augmented cooperative delay system through dividing the connection weights into inhibitory and excitatory types. This allows for the use of the powerful theory of monotone dynamical systems, and the explicitly division of the connectivity into the two different types offers a higher potential for relating formal neural network models to neurophysiology. Simple conditions on guaranteed trapping regions and guaranteed componentwise (exponential) convergence have been established using the method, which relate the system parameters to desired convergent performance, and are therefore of practical significance in applications. The results show the efficiency of the proposed method. Extension of the method to neural networks with distributed delays is straightforward.

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