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NONLINEAR OBSERVERS FOR STATE AND PARAMETER ESTIMATION IN BIOCHEMICAL PROCESSES

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Abstract: A systematic approach for the on-line estimation of the non measured component concentrations and the reaction rates inside chemical and biochemical reactors is presented. Two appealing features of the presented approach are worth to be mentioned. Firstly, the estimators of the reaction rates are easy to implement and in particular to calibrate. Secondly, the estimation of these as well as that of the non measured component concentrations does not necessitate any change of coordinates and it fully takes advantage of the process balance model. Simulation results related to a biotechnological process are given in order to illustrate the performances of the proposed estimators.

Keywords: nonlinear observer, modelling, estimation, chemical reactor, bioprocess

1. INTRODUCTION

The lack of cheap and reliable instrumentation for the on-line measurement of the relevant variables in many processes de£nitely constitutes a serious obstacle for the development of the latter. One way to overcome this problem is to use "software sensors". Over the last two decades, there has been a growing and widespread development of software sensors for the estimation of component concentrations and reaction rates inside chemical reactors (see e.g. (Bonvin *et al.*, 1989), (Schuler and Schmidt, 1992), (Farza *et al.*, 1999)) and inside biochemical reactors (see e.g. (Shimizu *et al.*, 1989), (Wang and Stephanopoulos, 1984), (Bastin and Dochain, 1990), (Farza *et al.*, 1998)). The estimation of the component concentrations and in particular that of the reaction rates is interesting since these rates are very complex functions of the operating conditions and the state of the process.

Many works in the development of software sensors for chemical and biochemical reactors are based on an Extended Kalman Filter (EKF) approach which generally leads to complex non-linear algorithms. Moreover, it is well known that the EKF may give biased or even diverging estimates if it is not well initialized. Another approach for developing software sensors for bioreactors is that proposed by Bastin and Dochain (Bastin and Dochain, 1990) which consists of estimat-

ing the component concentrations using a so-called asymptotic observer and the reaction rates using an observer-based estimator. Many improvements of the observer-based estimator have been proposed later in order to facilitate the choice of the tuning parameters ((Oliveira *et al.*, 1996), (Perrier *et al.*, 2000)).

In (Farza *et al.*, 1998), the authors proposed a simple nonlinear observer for the on-line estimation of the reaction rates. In these works, the authors assumed that all the states (component concentrations) were measured. However, only a part of these measurements are used to update the reaction rates whereas the remaining measurements have been treated as known signals and were simply injected into the process model. Moreover, no algorithm was given to estimate the non-measured component concentrations.

In the present paper, we £rstly propose nonlinear observers for the estimation of the reaction rates, in chemical and biochemical reactors, from the measurement of some component concentrations (plus the temperature if energy balance is considered). Then, we show that the provided estimates can be used in order to design asymptotic observers for the estimation of the non measured component concentrations.

An outline of this paper is as follows: in the next section, we propose an observer synthesis for a class of nonlinear systems including biochemical process models. In section 3, we show that the proposed observer can be used for the estimation of the reaction rates in biochemical reactors. We also show that the so-provided reaction rate estimates can be used for the on-line estimation of the non-measured component concentrations. Section 4 is devoted to simulation where the performances of the proposed estimator are demonstrated.

2. OBSERVER SYNTHESIS FOR A CLASS OF NONLINEAR SYSTEMS

We consider multi-output nonlinear systems which can be described as follows:

$$\begin{cases} \dot{x} = F(s,x)x + G(u,s,x) + \varepsilon(t) \\ y = Cx \end{cases} \quad (1)$$

where

$$x = \begin{pmatrix} x^{(1)} \\ x^{(2)} \\ \vdots \\ x^{(q)} \end{pmatrix} \in \mathbb{R}^n \text{ with } x^{(k)} \in \mathbb{R}^{n_k},\ k = 1,\dots,q$$

and $p = n_1 \geq n_2 \geq \ldots \geq n_q$, $\sum_{k=1}^{q} n_k = n$; the input $u \in U$ is a compact subset of $\mathbb{R}^m$, the output $y \in \mathbb{R}^p$, $s(t)$ is a known signal. The functions G and F assume triangular forms, i.e.

$$G(u,s,x) = \begin{pmatrix} G^{(1)}(u,s,x^{(1)}) \\ G^{(2)}(u,s,x^{(1)},x^{(2)}) \\ \vdots \\ G^{(q-1)}(u,s,x^{(1)},\dots,x^{(q-1)}) \\ G^{(q)}(u,s,x) \end{pmatrix}$$

with $G^{(k)}(u,s,x) \in \mathbb{R}^{n_k}$, $k = 1,\dots,q$;

$$F(s,x) = \begin{bmatrix} 0 & F_1(s,x^{(1)}) & 0 & \dots & 0 \\ 0 & 0 & F_2(s,x^{(1)},x^{(2)}) & \ddots & \vdots \\ \vdots & & & \ddots & 0 \\ 0 & & \ddots & 0 & F_{q-1}(s,x) \\ 0 & \dots & & \dots & 0 \end{bmatrix}$$

is block diagonal and each F_k, $k = 1,\dots,q-1$, denotes a $n_k \times n_{k+1}$ rectangular matrix;

$$\varepsilon(t) = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ \varepsilon^{(q)} \end{pmatrix} \text{ with } \varepsilon^{(q)} = \begin{pmatrix} \varepsilon_1^{(q)} \\ \varepsilon_2^{(q)} \\ \vdots \\ \varepsilon_{n_q}^{(q)} \end{pmatrix} \in \mathbb{R}^{n_q}$$

and each $\varepsilon_i^{(q)}$, $i = 1,\dots,n_q$ is an unknown bounded real-valued function which may depend on x, s, u, uncertain parameters, etc.; $C = [I_p, 0, \dots, 0]$ where I_p is the $p \times p$ identity matrix.

Note that the class of systems (1) is more general than those we considered in previous works ((Farza *et al.*, 1998; Busawon *et al.*, 1998)) since the matrices F_i are rectangular and are not assumed to be square.
The observation problem we are concerned with is well posed if the following set of assumptions holds:
(A1) There exist a set of controls $\mathcal{U} \in \mathcal{C}^\infty(U, \mathbb{R}^m)$ and two compact sets $K_1 \subset K_2$ such that every trajectory $x(t)$ associated to any $u \in \mathcal{U}$ and issued from K_1 lies in K_2. This means, in particular, that we only deal with bounded inputs and bounded trajectories.
(A2) There exist two positive constants α, β such that for every $k \in 1,\dots,q-1$, $\forall u \in \mathbb{R}^m$, $\forall x \in \mathbb{R}^n$, $\forall t \geq 0$,

$$0 < \alpha^2 I_{n_k+1} \leq F_k(s,x)^T F_k(s,x) \leq \beta^2 I_{n_k+1}$$

where I_{n_k+1} is the $(n_k+1)\times(n_k+1)$ identity matrix.
(A3) The function ε is bounded.
(A4) The signal $s(t)$ and its time derivative $\dot{s}(t)$ are bounded.
(A5) The functions $G^{(i)}(u,s,x)$, $i = 1,\dots,q$ are global Lipschitz with respect to x uniformly in u and s.
(A6) The functions $F_i(s,x)$,$i = 1,\dots,q-1$ are global Lipschitz with respect to x uniformly in u.

Before giving our main theorem, we shall make some remarks and introduce notations used hereafter.
(1) For every $\xi \in \mathbb{R}^n$, $t \geq 0$, let $\Lambda(s(t),\xi)$ be the block diagonal matrix de£ned by

$$\Lambda(s(t),\xi) = diag\,[I_{n_1}, F_1(s(t),\xi), F_1(s(t),\xi)F_2(s(t),\xi),\ldots,\prod_{i=1}^{q-1}F_i(s(t),\xi)\Big] \quad (2)$$

where I_{n_1} denotes the $n_1 \times n_1$ identity matrix.
By Assumption $(A2)$, $\Lambda(s(t),\xi)$ is left invertible. Indeed, its left-inverse shall be denoted by $\Lambda^{+}(s(t),\xi)$ in the sequel.
(2) Let S be the unique solution of the algebraic Lyapunov equation :

$$S + A^T S + SA - C^T C = 0 \quad (3)$$

$$\text{where } A = \begin{bmatrix} 0 & I_{n_1} & 0 & \ldots & 0 \\ \vdots & \ddots & I_{n_1} & \ddots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ \vdots & & & \ddots & I_{n_1} \\ 0 & \ldots & & \ldots & 0 \end{bmatrix}$$

is a $n_1 q \times n_1 q$ square matrix and $C = [I_{n_1}, 0 \ldots, 0]$ is $n_1 \times n_1 q$. It can be shown that the explicit solution of (3) is given by

$$S(i,j) = (-1)^{i+j} C_{i+j-2}^{j-1} I_{n_1}, \quad for\ 1 \le i,j \le q$$

where $C_j^i = \dfrac{j!}{i!(j-i)!}$ and that S is symmetric positive de£nite. Moreover, Cholesky's decomposition of S is given by $S = U^T U$ where the upper triangular matrix U is given by:

$$U(i,j) = (-1)^{i+j} C_{j-1}^{i-1} I_{n_1},\ for\ 1 \le i \le j \le q$$
$$\text{and } U^{-1}(i,j) = C_{j-1}^{i-1} I_{n_1}, \qquad for\ 1 \le i \le j \le q$$

3) Set $\bar{S}(s(t),\xi) = \Lambda^T(s(t),\xi) S \Lambda(s(t),\xi)$ and $\bar{C} = C\Lambda(s(t),\xi)$. On one hand, it is easy to see that $\bar{S}$ is an invertible square matrix and one can show that: $\bar{S}^{-1} = \Lambda^{+} U^{-1} \Lambda \Lambda^{+} (U^{-1})^T (\Lambda^{+})^T$
On the other hand, we note that $\bar{C}$ is constant:

$$\bar{C} = [I_{n_1}, 0_{n_1 \times n_2}, 0_{n1 \times n_3}, \ldots, 0_{n_1 \times n_{q-1}}] \quad (4)$$

where $0_{n_1 \times n_k}$ denotes the $n_1 \times n_k$ null matrix, $k = 2, \ldots, q-1$.
Our candidate observer for system (1) under assumptions (A1) to (A6) is given by :

$$\begin{aligned} \dot{\hat{x}} &= F(s(t),\hat{x})\hat{x} + G(u,s(t),\hat{x}) \\ &\quad - \theta \Delta_\theta \bar{S}(s(t),\hat{x})^{-1} \bar{C}^T (\bar{C}\hat{x} - y) \end{aligned} \quad (5)$$

where $\bar{S}$ and $\bar{C}$ are given above and
$\Delta_\theta = diag\left[I_{n_1}, \theta I_{n_2}, \theta^2 I_{n_3}, \ldots, \theta^{q-1} I_{n_q}\right]$ for some $\theta > 0$ and I_{n_k} denotes the $n_k \times n_k$ identity matrix.

We now state the following :
Theorem. *Consider system (1) under assumptions (A1) to (A6) and system (5). Then, one has:*

$$\exists \theta_0 > 0;\ \forall \theta > \theta_0;\ \exists \lambda_\theta > 0;\ \exists \mu_\theta > 0;\ \exists M_\theta > 0;$$
$$\forall u \in U;\ \forall \hat{x}(0) \in R^n;\ \textit{one has}\ :$$
$$\|\hat{x}(t) - x(t)\| \le \lambda_\theta e^{-\mu_\theta t} \|\hat{x}(0) - x(0)\| + M_\theta \delta$$

where δ is the upper bound of $\|\varepsilon\|$. Moreover, one has $\lim_{\theta \to \infty} \mu_\theta = +\infty$ *and* $\lim_{\theta \to \infty} M_\theta = 0$.

Remark: Note, that for $\varepsilon(t) = 0$, the convergence of the estimation error is exponential. In the case where $\|\varepsilon(t)\| \neq 0$ but bounded by δ, the asymptotic estimation error is bounded and the corresponding upper bound is as small as δ. Moreover, this bound can be made as small as desired by choosing values of θ high enough. It is worth noticing that the scalar θ has to be speci£ed bearing in mind the unavoidable compromise between fast convergence and noise sensitivity.

3. APPLICATION TO BIOREACTORS AND CHEMICAL REACTORS

We will illustrate throughout this section the use of the above observers for the on-line estimation of the reaction rates and component concentrations in chemical and biochemical reactors. Indeed, the mass and energy balance model of reaction systems, including chemical and biochemical reactors, can be written as follows (Farza *et al.*, 1999):

$$\dot{\xi} = KH\alpha - D\xi + W \quad (6)$$

where $\xi = (\xi_1, \ldots, \xi_N)^T$ is the vector of process component concentrations (g/l) (plus the temperature (K) if the energy balance is considered); K is the $N \times M$ stoichiometric (or yield coef£cient) matrix; H is a $M \times M$ diagonal matrix and each of its diagonal terms corresponds to a subset (possibly all) of the measured component concentrations; the vector $\alpha = (\alpha_1, \ldots, \alpha_M)^T$ is the vector of (speci£c) reaction rates (g/l/h), D is the $N \times N$ dilution rate matrix (h^{-1}) and £nally W is the in¤ow/out¤ow rate vector (g/l/h).

As stated before, the modelling of the (speci£c) reaction rates α_j's is a dif£cult and hazardous task. Thus, they will be considered as completely unknown time-varying parameters which have to be estimated. However, as no balances are available for these kinetics, we will assume that their second time derivatives are described by unknown and bounded functions. At a £rst glance, such a stance of modelling might be surprising since most of works related to the estimation of the reaction rates did not take into account the second time derivatives. Indeed, the most adopted hypothesis consists in assuming the boundedness of the reaction rate dynamics (£rst derivatives). Nevertheless, the second derivatives have been considered in some estimation algorithms (Wang and Stephanopoulos, 1984), (Takiguchi *et al.*, 1997). In our work, the motivation behind the consideration of the second derivatives lies in the fact that the upper bounds of these derivatives are generally much smaller than those of the £rst ones.

By considering the second time derivatives, we obtained a model where the uncertain term correspond to these functions and not to the £rst time derivatives. This allows to obtain more accurate estimates since the estimation error related to the observer we propose is bounded by the upper bound of the unknown term, as stated in the theorem.

In order to estimate all reaction rates and all non measured component concentrations, the following hypothesis are required:

(C1) K, D, W are known.

(C2) Suppose that ξ is partitioned into the set of measured variables $\xi^{(1)}$ and into the set of non measured variables $\xi^{(2)}$ as follows: $\xi = \begin{pmatrix} \xi^{(1)} \\ \xi^{(2)} \end{pmatrix}$ and accordingly, $K = \begin{pmatrix} K^{(1)} \\ K^{(2)} \end{pmatrix}$, then we have $rank(K^{(1)}) = rank(K) = M$.

As stated before, as no balances are available for the reactions rates, the time derivatives of their dynamics will be supposed bounded. Thus, system (6) can be decomposed into two subsystems which respectively account for the measured and non measured component concentrations :

$$\begin{cases} \dot{\xi}^{(1)} = K^{(1)} H(\xi^{(1)})\alpha(t) - D^{(1)}\xi^{(1)} + W^{(1)} \\ \dot{\alpha} = \eta(t) \\ \dot{\eta} = \varepsilon(t) \\ y = \xi^{(1)} \quad \text{and} \end{cases} \tag{7}$$

$$\dot{\xi}^{(2)} = -D^{(2)}\xi^{(2)} + K^{(2)} H(\xi^{(1)})\alpha(t) + W^{(2)} \tag{8}$$

where $\begin{pmatrix} D^{(1)} \\ D^{(2)} \end{pmatrix}$ and $\begin{pmatrix} W^{(1)} \\ W^{(2)} \end{pmatrix}$ respectively denote the partitions of D and W induced by the partition of K under the form $K = \begin{pmatrix} K^{(1)} \\ K^{(2)} \end{pmatrix}$; η is α's time derivative and ε is an unknown bounded function which may depend on the concentrations, the temperature, the inputs, the parameter uncertainties, etc.

Now, it is easy to see that subsystem (7) is under form (1) with : $x^{(1)} \triangleq \xi^{(1)}$; $x^{(2)} \triangleq \alpha$ and $x^{(3)} \triangleq \eta$. Moreover, one can easily show that assumptions (A1) to (A6) are satis£ed by (7) (see e.g. (Farza *et al.*, 1999)). As a result, a nonlinear observer of the form (5) can be used for the on-line estimation of the reaction rates. This observer takes the following form:

$$\begin{cases} \dot{\hat{\xi}}^{(1)} = K^{(1)} H\left(\hat{\xi}^{(1)}\right)\hat{\alpha} - D^{(1)}\hat{\xi}^{(1)} + W^{(1)} \\ \quad - 3\theta\left(I_M + 2K^{(1)}\left(K^{(1)}\right)^{+}\right)(\hat{\xi}^{(1)} - \xi^{(1)}) \\ \dot{\hat{\alpha}} = \hat{\eta} - 3\theta^2 H^{-1}\left(K^{(1)}\right)^{+}(\hat{\xi}^{(1)} - \xi^{(1)}) \\ \dot{\hat{\eta}} = -\theta^3 H^{-1}\left(K^{(1)}\right)^{+}(\hat{\xi}^{(1)} - \xi^{(1)}) \end{cases} \tag{9}$$

The so-estimated reaction rates can now be used in order to obtain on-line estimates, $\hat{\xi}^{(2)}$, of the non measured component concentrations, $\xi^{(2)}$. Indeed, it suf£ces to replace in equation (8) the vector of unknown reactions rates α by its estimate, $\hat{\alpha}$, provided by observer (9). The resulting system is:

$$\dot{\hat{\xi}}^{(2)} = -D^{(2)}(t)\hat{\xi}^{(2)} + K^{(2)} H(\xi^{(1)})\hat{\alpha} + W^{(2)} \tag{10}$$

Now, it is easy to see that the convergence of (10) is guaranteed as soon as the (time-varying) matrix $(-D^{(2)})$ is stable.

4. EXAMPLE

In this section, the performances of the proposed observer are illustrated through a microbial culture which involves a single biomass X growing on a single substrate S and yielding two £nal products P_1 and P_2. The bioprocess is supposed to be continuous with a scalar dilution rate D and an input substrate concentration S_{in}. It should be appreciated that the example was chosen mainly for its simplicity and illustrative properties. Remember that the theory presented in this paper can be applied to the large class of biochemical processes described by the general model (6). The mathematical dynamical model of the process is :

$$\begin{cases} \dot{S} = -k_1 r_1 - k_2 r_2 + D(S_{in} - S) \\ \dot{P}_1 = r_2 - DP_1 \\ \dot{X} = r_1 - DX \\ \dot{P}_2 = k_3 r_2 - DP_2 \end{cases} \tag{11}$$

where r_1 and r_2 respectively denote the growth and the biosynthesis reaction rates; k_1, k_2 and k_3 are yield coef£cients. We suppose that the substrate S and the product P_1 are measured and our objective consists in estimating the reaction rates r_1 and r_2 as well as the concentrations X and P_2. This has been achieved using an observer of the form (9) and (10) which specialized as follows:

$$\begin{cases} \dot{\hat{S}} = -k_1\hat{r}_1 - k_2\hat{r}_2 + D(S_{in} - \hat{S}) - 3\theta(\hat{S} - S) \\ \dot{\hat{P}}_1 = \hat{r}_2 - D\hat{P}_1 - 3\theta(\hat{P} - P) \\ \hat{r}_1 = \hat{\eta}_1 - 3\theta^2(\hat{P} - P) \\ \hat{r}_2 = \hat{\eta}_2 + 3\dfrac{\theta^2}{k_2}\left((\hat{P} - P) + k_1(\hat{S} - S)\right) \\ \dot{\hat{\eta}}_1 = -\theta^3(\hat{P} - P) \\ \dot{\hat{\eta}}_2 = \dfrac{\theta^3}{k_2}\left((\hat{P} - P) + k_1(\hat{S} - S)\right) \\ \dot{\hat{X}} = \hat{r}_1 - D\hat{X} \\ \dot{\hat{P}}_2 = k_3\hat{r}_2 - D\hat{P}_2 \end{cases} \tag{12}$$

In order to illustrate the observer performances, we have compared corresponding results with data issued from model simulation. The following kinetic expressions have been used for simulation purposes:

$$\begin{cases} r_1 = \mu_{max} \dfrac{SX}{(K_{S1} + S + \frac{S^2}{K_I})} \\ r_2 = \nu_{max} \dfrac{SX}{(K_{S2} + S)} \dfrac{K_{P_1}}{(K_{P_1} + P_1)} \end{cases} \quad (13)$$

where μ_{max}, ν_{max}, K_{S1} and K_{S2} are constant kinetic parameters. The model's simulation was performed under the following initial conditions:
$X(0) = 0.5g/l$, $S(0) = 15g/l$, $P_1(0) = 0.25g/l$, $P_2(0) = 2g/l$
The dilution rate varies as a trapezoidal signal from 0.1 to $0.2\ l/h$ and the parameter values used in simulation are :
$k_1 = 5.0$, $k_2 = 10$, $k_3 = 4$, $\mu_{max} = 0.25h^{-1}$, $K_{S_1} = 2g/l$, $K_I = 91(g/l)^2$, $\nu_{max} = 0.1h^{-1}$, $K_{S2} = 1.84g/l$, $K_{P_1} = 10g/l$, $S_{in} = 20g/l$
The estimator's simulation was performed under similar operating conditions as the model and the value of θ was equal to 1.5.
Before being used by the estimator, the measurements of S and P_1 have been corrupted by an additive noisy signal as shown on £gures 1 and 2. Estimation results are reported in £gures 3, 4, 5 and 6. We remark that the estimates of r_1 and r_2 as well as those of X and P_2 quickly converge to the (true) simulated time evolutions. These results clearly show that the reaction rates can be accurately estimated by the proposed estimators even when these parameters are subject to abrupt variations. We also note the good agreement between estimates of X and P_2 and their values issued from model simulation. Moreover, these estimates clearly highlight the good behaviour of the proposed approach in dealing with noise rejection.

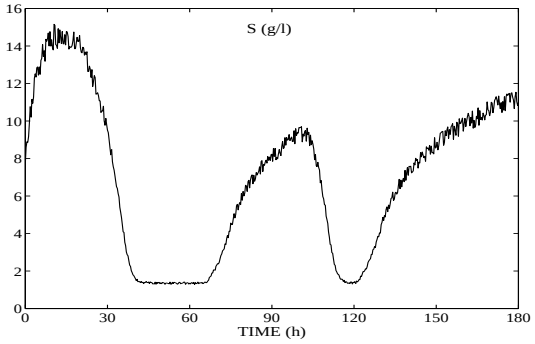

Fig. 1. Noisy measurements of S

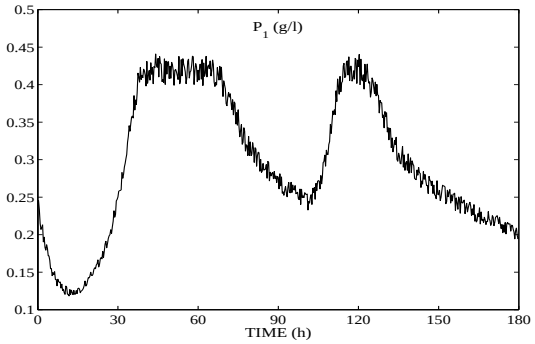

Fig. 2. Noisy measurements of P_1

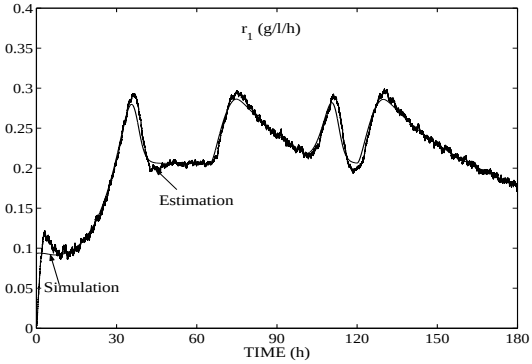

Fig. 3. Estimated and simulated r_1

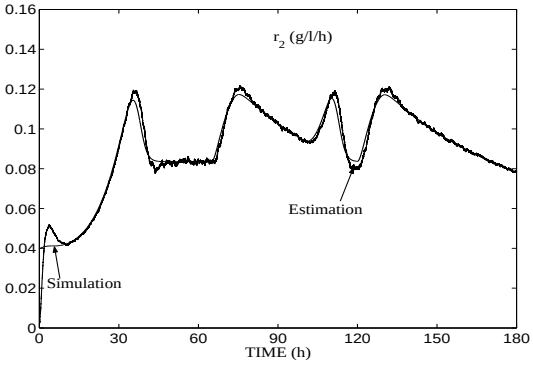

Fig. 4. Estimated and simulated r_2

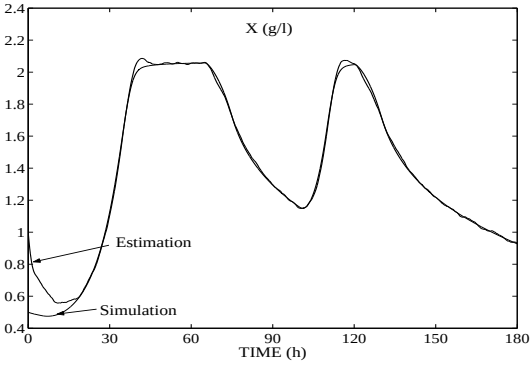

Fig. 5. Estimated and simulated X

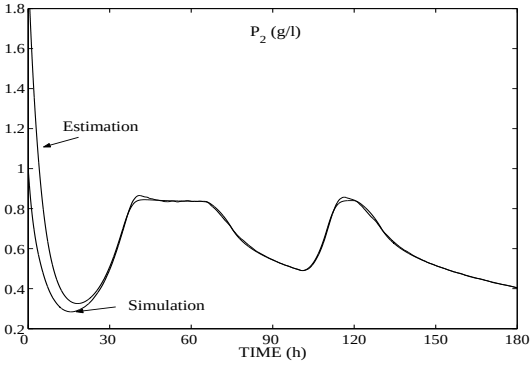

Fig. 6. Estimated and simulated P_2

Conclusion : Simple nonlinear observers for the on-line estimation of the reaction rates and the non measured component concentrations inside biochemical reactors have been presented. The reaction rates are updated through nonlinear observers whose implementation and in particular calibration is simple and easy to carry out. The so-obtained estimates are then exploited to synthesize an asymptotic observer to on-line estimate the non measured component concentrations. Simulation results have been given and they have demonstrated the good performances of the given estimators in coping with nonlinearities and parameter uncertainties in chemical and biochemical systems.

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APPENDIX

The proof is similar to that given in (Busawon *et al.*, 1998). Nevertheless, the submatrices F_i in the matrix F (and Λ) are here assumed to be rectangular and not necessary square as in (Busawon *et al.*, 1998). The main outlines of the proof are given hereafter.
Since $\Lambda(\hat{z},s)F(s,\hat{x}) = A\Lambda(s,\hat{x})$, one can show that:
$\theta\bar{S} + F^T\bar{S} + \bar{S}F - \bar{C}^T\bar{C} = 0.$
where $\bar{S}(\hat{x},s) = \Lambda^T(\hat{x},s)S\Lambda(\hat{x},s)$ and $\bar{C}$ is the constant matrix de£ned by (4).
Set $e(t) = \hat{x}(t) - x(t)$and $\bar{e} = \Delta_\theta^{-1}e$, then :

$$\begin{aligned}\dot{\bar{e}} &= \theta\left(F(s,\hat{x}) - \bar{S}(s,\hat{x})^{-1}\bar{C}^T\bar{C}\right)\bar{e}\\ &+ \Delta_\theta^{-1}\left(F(s,\hat{x})x - F(s,x)x\right)\\ &+ \Delta_\theta^{-1}\left(G(u,s,\hat{x}) - G(u,s,x)\right) - \Delta_\theta^{-1}\varepsilon(t)\end{aligned}$$

Consider the quadratic function $V(\bar{e}) = \bar{e}^T\bar{S}(s,\hat{x})\bar{e}$, then

$$\begin{aligned}\dot{V} &= 2\bar{e}^T\bar{S}(s,\hat{x})\dot{\bar{e}} + 2\bar{e}^T\Lambda^T S\dot{\Lambda}\bar{e}\\ &= \theta\left(2\bar{e}^T\bar{S}(s,\hat{x})F(s,\hat{x})\bar{e} - 2\bar{e}^T\bar{C}^T\bar{C}\bar{e}\right)\\ &+ 2\bar{e}^T\bar{S}(s,\hat{x})\Delta_\theta^{-1}\left(F(s,\hat{x})z - F(s,x)x\right)\\ &+ 2\bar{e}^T\bar{S}(s,\hat{x})\Delta_\theta^{-1}\left(G(u,s,\hat{x}) - G(u,s,x)\right)\\ &+ 2\bar{e}^T\Lambda^T S\dot{\Lambda}\bar{e} - 2\bar{e}^T\bar{S}(s,\hat{x})\Delta_\theta^{-1}\varepsilon(t)\\ &\le -\theta V - \theta\|\bar{C}\bar{e}\|^2\\ &+ 2\|\bar{S}(s,\hat{x})\bar{e}\|\|\Delta_\theta^{-1}\left(F(s,\hat{x})x - F(s,x)x\right)\|\\ &+ 2\|\bar{S}(s,\hat{x})\bar{e}\|\|\Delta_\theta^{-1}\left(G(u,s,\hat{x}) - G(u,s,x)\right)\|\\ &+ 2\bar{e}^T\Lambda^T S\dot{\Lambda}\bar{e} + 2\|\bar{S}(s,\hat{x})\bar{e}\|\frac{\delta}{\theta^{q-1}}\end{aligned}$$

Now, assume that $\theta \ge 1$, then, because of the triangular structure and the Lipschitz assumption on G and F, one can show that :

$$\begin{aligned}\|\Delta_\theta^{-1}\left(G(u,s,\hat{x}) - G(u,s,x)\right)\| &\le \zeta_1\|\bar{e}\|\\ \text{and } \|\Delta_\theta^{-1}\left(F(s,\hat{x})x - F(s,x)x\right)\| &\le \zeta_2\|\bar{e}\|\end{aligned}$$

for some constants ζ_1 and ζ_2 which do not depend on θ (see (Gauthier *et al.*, 1992)).
Hence, $\dot{V} \le -\theta V + c_1 V + \frac{c_2\delta}{\theta^{k_0-1}}$
for some positive constants c_1 and c_2.
Now taking $\theta_0 = max\{1, c_1\}$ and $\theta > \theta_0$, we obtain :

$$\begin{aligned}\|\bar{e}(t)\| \le\ & \sigma(S)\frac{\breve{\kappa}}{\kappa}\exp\left[-\left(\frac{\theta - c_1}{2}\right)t\right]\|\bar{e}(0)\|\\ &+ \frac{c_2\delta}{\theta^{q-1}(\theta - c_1)\kappa\sqrt{\lambda_{min}(S)}}\end{aligned} \tag{14}$$

To end the proof, it suf£ces to see that for $\theta \ge 1$,

$$\text{we have: } \|e(t)\| \le \|\bar{e}(t)\| \le \theta^{q-1}\|e(t)\|$$