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STRUCTURAL ANALYSIS FOR DIAGNOSIS - THE MATCHING PROBLEM REVISITED

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Abstract: Aiming at design of algorithms for fault diagnosis, structural analysis of systems offers concise yet easy overall analysis. Graph-based matching, which is the essential technique to obtain redundant information for diagnosis, is re-considered in this paper. Matching is re-formulated as a problem of relating faults to known parameters and measurements of a system. Using explicit fault modelling, minimal over-determined subsystems are shown to provide necessary redundancy relations from the matching. Details of the method are presented and a realistic example used to clearly describe individual steps.

Keywords: Structural analysis, fault diagnosis, FDI, autonomous systems.

1. INTRODUCTION

Timely diagnosis of faults are instrumental to enhance safety and reliability of technical systems. If traditional mathematical models are taken as the basis for diagnostic algorithms, analysis will require a major effort when the the object to diagnose have the complexity of a common industrial plant. There is a recognized need for simple but efficient methods for overall analysis before going to a detailed diagnostic algorithm design. Attributes of such methods should include that parameters and other exact information is sparse in industrial systems.

The structural analysis framework (Declerck and Staroswiecki, 1991), (Cassar *et al.*, 1994), (Staroswiecki *et al.*, 1999) (Izadi-Zamanabadi and Staroswiecki, 2000) offers graph-based approach to make rapid overall analysis and design. The salient feature is that detailed design can be spared to a few diagnostic algorithms that are guaranteed to have desired overall properties.

Despite its virtues, the structural analysis method has not yet become widely used. Presumably, the reason is lack of easily understood methods to conduct matching, which is the essential graph-technique to obtain analytic redundancy relations for diagnosis. Another obstacle could be lack of explicit inclusion of the faults we wish to diagnose into the structural analysis method.

This paper contributes by formulating the matching problem explicitly as a matching from known variables, by clarifying the individual steps of the matching and by formulating a straightforward but concise representation of the faults to be considered. The methodology is illustrated on a ship propulsion benchmark (Izadi-Zamanabadi and Blanke, 1999).

The paper concerns a structural model for the object to diagnose, matching to disclose inherent redundant information which can be used for diagnosis, representation of faults and techniques to examine the isolation of faults. Application to a propulsion system benchmark illustrates the techniques.

2. STRUCTURAL MODEL

Consider the system S as a set of components $\bigcup_{i=1}^{m} \mathsf{C}_i$, each imposing a relation f_i between a set of variables

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and parameters $z_j, j=1,..,n$ i.e.

$$f_i(z_1,..,z_p)=0, \qquad 1<p\le n \tag{1}$$

where f_i can represent a dynamic, static, linear, or non-linear relation. These relations are also called constraints as the value of an involved variable can not change independent of the other involved variables (Cassar *et al.*, 1994) (see also (Declerck and Staroswiecki, 1991) and (Blanke *et al.*, 2000)). The system's structural model is represented by the set of constraints $\mathscr{F}=\{f_1,f_2,\cdots,f_m\}$ and the set of variables $\mathscr{Z}=\mathscr{K}\cup\mathscr{X}=\{z_1,z_2,\cdots,z_n\}$. $\mathscr{X}$ is the set of unknown variables and $\mathscr{K}=\mathscr{U}\cup\mathscr{P}\cup\mathscr{Y}$ is the set of known variables/parameters i.e. input/reference signals ($\mathscr{U}$), known constant/parameters ($\mathscr{P}$), and measured signals ($\mathscr{Y}$).

The system's structural model can now be represented by a bipartite directed graph. This is a graph whose vertices (nodes) can be divided into two classes 'R' and 'Y' such that no edge (arc) of the graph runs between two 'R' vertices or between two 'Y' vertices.

Definition 1. The structure graph of the system is a bipartite directed graph $G(\mathscr{F},\mathscr{Z},\mathscr{A})$ where the elements in the set of arcs $\mathscr{A}\subset(\mathscr{F}\times\mathscr{Z})$ are defined by the following mappings:

$$\begin{cases}\mathbf{A} & :\mathscr{F}\times\mathscr{X}\longrightarrow\{0,1\},\\ \mathbf{A}^* & :\mathscr{X}\times\mathscr{F}\longrightarrow\{0,1\},\\ \mathbf{KF} & :\mathscr{F}\times\mathscr{K}\longrightarrow\{0,1\}.\end{cases}$$

The elements $(f_i,x_j)=a_{ij}\in A$, $(x_i,f_j)=a^*_{ij}\in A^*$, and $(f_i,k_j)=kf_i\in KF$ are defined as:

$$a_{ij}=\begin{cases}1 & \text{iff } f_i \text{ applies to } x_j,\\ 0 & Otherwise\end{cases}$$

$$a^*_{ij}=\begin{cases}1 & \text{iff } x_i \text{ is calculable through } f_j\\ 0 & Otherwise\end{cases}$$

$$kf_i=\begin{cases}1 & \text{iff } f_i \text{ applies to a known var. } k_j\\ 0 & Otherwise\end{cases}$$

An element $a_{ij}=1$ means that there is a directed arc that connects the j^{th} relation with the i^{th} unknown variable; $a_{ij}=0$ means there is no arc.

An incidence matrix $\mathbf{I_{md}}$ can be used as a representation of the structure graph, in compact form:

$$\begin{array}{c|ccc} & \mathscr{K} & \mathscr{F} & \mathscr{X}\\ \hline \mathscr{K} & 0 & \mathbf{KF} & \mathbf{0}\\ \mathscr{F} & \mathbf{KF}^T & \mathbf{0} & \mathbf{A}\\ \mathscr{X} & \mathbf{0} & \mathbf{A}^* & \mathbf{0}\end{array}=\mathbf{I_{md}} \tag{2}$$

Notice that $\mathbf{A}^*$ is not necessarily the same as $\mathbf{A}^T$. $\mathbf{A}^*$ is defined to address a property called *calculability*:

Definition 2. **Calculability:** Let $z_j, j=1,\cdots,p,\cdots,n$ be variables that are related through a relation f_i, e.g. $f_i(z_1,\cdots,z_p,\cdots,z_n)=0$. The variable z_p is calculable if its value can be determined through the constraint f_i under the condition that the values of the other variables $z_j, j=1,\cdots,n, j\neq p$ are known.

Remark 3. By definition, calculability, def. 2, solves the problem: When $f(x_1,\cdots,x_n)=0$ be solved explicitly for x_i in terms of $x_j, \quad j=1,\cdots,n,\ j\neq i$? Conditions for the local solution is provided by the *implicit function theorem* ((Apostol, 1974)): for a given function $f:\mathbb{R}^n\to\mathbb{R}$ and a local point $\mathbf{x}_0$ (a possible operating point), where $f(\mathbf{x}_0)=0$ and for which $(\partial f/\partial x_i|_{\mathbf{x}_0})\neq 0, \quad 1\le i\le n$, then there exists a function g, defined on $\mathbb{R}$, such that $x_i=g(x_1,x_2,\cdots,x_{i-1},x_{i+1},\cdots,x_n)$.

Remark 4. A state may be observable despite it is not calculable.

Calculability versus observability is illustrated by following example:

Example 5. Consider the relation $f(\dot{x},u)=\dot{x}-u=0$. Which of the variables, x and u, can be explicitly calculated. The implicit function theorem we get:

$$\frac{\partial f}{\partial x}=\frac{\partial(\dot{x}-u)}{\partial x}=\frac{\partial\frac{\partial x}{\partial t}}{\partial x}=\frac{\partial\frac{\partial x}{\partial x}}{\partial t}=0$$

and

$$\frac{\partial f}{\partial u}=\frac{\partial(\dot{x}-u)}{\partial u}=-1\neq 0$$

Obviously, u can be explicitly computed knowing the instantaneous value of x through the relation f. The opposite is not true: x can not be reconstructed explicitly using u since x is given by $x=\int udt+x_0$ when the initial value x_0 is not known (since $x\in\mathscr{X}$ which is the set of *unknown* variables.

However, the variable x may anyway be observable since observability implies the ability to asymptotically reconstruct the initial state. If state observation is available, x_0 could be considered a *known* variable implying less restrictions on in the matching process.

Let $\mathscr{E}$ denote a set (such as $\mathscr{F}_{\mathscr{X}}$ or $\mathscr{Z}$) and $\mathbb{P}(\mathscr{E})$ denote the power set of $\mathscr{E}$. Then a subsystem $(F,Q(F)),\ F\in\mathbb{P}(\mathscr{F}_{\mathscr{X}})$ will be defined as

$$Q:\mathbb{P}(\mathscr{F}_{\mathscr{X}})\longrightarrow\mathbb{P}(\mathscr{Z}) \tag{3}$$
$$Q(F)=\{z_j\mid\exists f_i\in F \text{ such that } (f_i,z_j)\in\mathscr{A}\}.$$

3. MATCHING

Consider a graph $G(F_X,X,A_X)$ representing a selected part of the system's structured graph. Let $a=(F_X(a),X(a))$ be the arc that connects a constraint $F_X(a)$ with an unknown variable $X(a)$.

Definition 6. The (sub)graph $G(F_{X_M},X_M,A_{X_M})$ is a *matching* on $G(F_X,X,A_X)$, $F_{X_M}\subseteq F_X$ and $X_M\subseteq X$, iff:

1− $A_{X_M} \subseteq A_X$,
2− $\forall \ a1, a2 \in A_{X_M} \ \mid \ a1 \neq a2 \Updownarrow$
$F_{X_M}(a1) \neq F_{X_M}(a2) \ \wedge \ X_M(a1) \neq X_M(a2)$.

A complete matching w.r.t. F_X is obtained when $F_{X_M} = F_X$
A complete matching w.r.t. X is obtained when $X_M = X$. By applying matching one can decompose the system into three parts according to the following theorem:

Theorem 7. (Dulmage and Mendelsohn, 1958) Any bipartite graph of finite external dimension can be uniquely decomposed:

- $G^+ = (F^+, X^+, A^+)$ such that $Q(F^+) = X^+$and a complete matching exists on X^+ but not on F^+.
- $G^= = (F^=, X^=, A^=)$ such that $Q(F^=) = X^= \cup X^+$ and a complete matching exists on $X^=$ as well as on $F^=$.
- $G^- = (F^-, X^-, A^-)$ such that $Q(F^-) = X^- \cup X^= \cup X^+$ and a complete matching exists on F^- but not on X^-.

G^+ represents the part of the system with possible redundant information as $|F^+| > |X^+|$, where $|F|$ denotes the cardinality of F. The unknown variables in X^+ can be calculated in several ways by using the known variables. The subsystem(s) represented by G^+ is said to be *over-determined*, as the number of relations exceeds the number of unknown variables. That means a variable x in X^+ can be computed/calculated through different sets of relations (equations) in F^+, or seen from a graph-theoretical point of view, there are different paths from x to the known variables (see next section). This property can be used for FDI purposes: if a component, such as a sensor, fails the related variable can be computed/estimated via other sets of relations and be used in the control loop. $G^=$ and G^- represent the parts with no redundant information. Related issues are discussed in (Declerck and Staroswiecki, 1991) and (Blanke *et al.*, 2000).

3.1 *Matching procedure*

The main purpose of developing a matching algorithm is to identify the sub-graph G^+ that represents the subsystem(s) which contain redundant information. The idea is depicted in figure 1. The algorithm initiates the matching from the known variables. The figure illustrates the idea of making the unknown variable "known" by successively matching them to previously known variables. First, variables x_1 and x_2 are matched to constraints f_1 and f_2 (full line). These variables become “known” as all the other variables that enter f_1 and f_2 are known. Hence, the new set of known variables can be considered as $\mathcal{K}_{new} = \mathcal{K} \cup \{x_1, x_2\}$. Next, x_3 and x_4 are matched to f_3 and f_4 correspondingly (dotted line) etc. The matching procedure makes

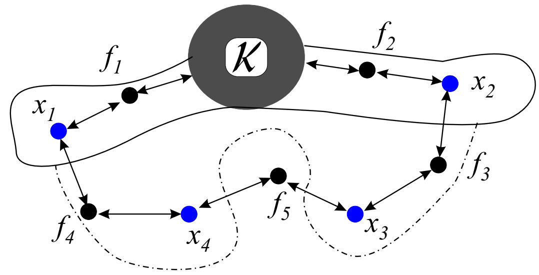

Fig. 1. The process of matching.

extensive use of the incidence matrix, $\mathbf{I_{md}}$, of the system’s bipartite directed graph model. The algorithm is repeated until a stop criteria is met. Since several matchings may exist, different over-determined subsystems can be obtained.

4. MODELLING OF FAULTS

A system fault occurs when one or more components $\mathsf{C}_i \in \mathsf{S}$ fail to operate properly. Correct operation is represented by $f_i(z_1, .., z_p) = 0, \quad 1 < p \leq n$. A failure implies in loose language that this constraint does not hold anymore. A concise analysis requires explicit ways to represent faults.

4.1 *Sensor fault*

A sensor measures a system variable, hence it can be represented by a relation of this form:

$$f(x_s, y) = 0,$$

where x_s is the unknown variable and y is the measured one. A sensor fault can be structurally represented in either of two ways:

(1) As the faulty sensor is not functioning properly, i.e.

$$f(x_s, y) \neq 0, \tag{4}$$

then this relation can simply be removed from $\mathcal{F}_X$.

(2) Another way of considering the faulty component is to say that the output of the sensor y is a function of x_s and an additional variable Δx_s which has its own dynamics. Hence the original relation is replaced by

$$f^{\Delta}(x_s, \Delta x_s, y) = 0, \tag{5}$$
$$f'^{\Delta}(\Delta x_s) = 0. \tag{6}$$

4.2 *Actuator fault*

Similar to the sensor case, the actuator failure can be represented in following two manners.

(1) The actuator can fail abruptly and loose completely the actuation possibility. Thus the non-functionality of actuator is represented by

$$f(u_c,u_a) \neq 0. \qquad (7)$$

(2) In the faulty situation,

$$f^{\Delta}(u_c,u_a,\Delta u_a) = 0, \qquad (8)$$
$$f'^{\Delta}(\Delta u_a) = 0. \qquad (9)$$

The additional variable Δu denotes the actuator fault and has its own dynamics.

4.3 *Parameter fault*

Parameter changes in the system's dynamic equations (Isermann, 1997) can be represented in following two (computationally) identical forms

(1) Introducing a parameter as an unknown variable, denoted p. So the affected relation will be changed from

$$f(z_i,\cdots,z_j) = 0 \qquad i,j \in \{1,\cdots,n\} \qquad (10)$$

to

$$f(z_i,\cdots,z_j,p) = 0 \qquad i,j \in \{1,\cdots,n\}, \qquad (11)$$

where $\mathscr{X}_{new} = \mathscr{X} \cup p$ and hence $|\mathscr{Z}_{new}| = n+1$.

(2) Denoting the (known) parameter, p_0, the parameter change will be represented as

$$p = p_0(1+\Delta p), \qquad (12)$$

where Δp has its own dynamics. The involved relation is represented by

$$f(z_i,\cdots,z_j,p_0) = 0 \qquad i,j \in \{1,\cdots,n-1\} \qquad (13)$$

where $\mathscr{K}_{new} = \mathscr{K} \cup p_0$. In faulty conditions this is replaced by the ensuing relations

$$f^{\Delta}(z_i,\cdots,z_j,p_0,\Delta p) = 0 \qquad (14)$$
$$f'^{\Delta}(\Delta_p) = 0 \qquad (15)$$

where $i,j \in \{1,\cdots,n-1\}$, $\mathscr{X}_{new} = \mathscr{X} \cup \Delta p$, $\mathscr{Z}_{new} = \mathscr{Z} \cup p_0 \cup \Delta p$, and $\mathscr{F}_{new} = \mathscr{F} \cup f'$.

5. FAULT DETECTION AND ISOLATION

FDI possibility can be examined by considering the obtainable *minimal over-determined subsystem*s in an over-determined subsystem. They are defined as

Definition 8. A minimal over-determined subsystem, $G^+_{min-f} = (F^+_{min-f},\ X^+_{min-f}, A^+_{min-f})$ is the smallest over-determined (hence observable) subsystem which is obtained by back-tracking the unknown variables in an unmatched relation f in F^+. For a minimal over-determined subsystem, the following statement is valid:

$$|F^+_{min-f}| = |X^+_{min-f}| + 1.$$

In (Izadi-Zamanabadi and Staroswiecki, 2000), it was shown that any involved unknown variable in a over-determined subsystem is (structurally) observable. Any minimal over-determined subsystem yield an expression of the following form

$$f(z_i,\cdots,z_j) = 0 \qquad z_i,\cdots,z_j \in \mathscr{K} \qquad (16)$$

where all involved variables are known. The expression can be directly used as an expression for a residual

$$r = f(z_i,\cdots,z_j) \qquad z_i,\cdots,z_j \in \mathscr{K}. \qquad (17)$$

This residual can be directly used for fault detection.

6. SHIP BENCHMARK

A ship propulsion system benchmark (Izadi-Zamanabadi and Blanke, 1999) provides a fairly realistic scenario. The main elements of a propulsion system are modelled in the benchmark and a command level gives set-points for shaft speed and propeller pitch. The structural analysis method is applied to the torque-thrust related part of the benchmark (See also (Izadi-Zamanabadi, 1999)). The outline of this part of the system is shown in Fig. 2. The components are: diesel engine dynamics C_4, shaft speed dynamics C_5, propeller's torque and thrust characteristics C_6 and C_7, ship speed dynamics C_8, hull characteristics C_9, and related sensors $\mathsf{C}_1,\mathsf{C}_2,\mathsf{C}_3$, and C_{10}. The related constraints are listed below:

$$\begin{array}{ll}
\mathsf{C}_1 : f_1(v,v_m) = 0 : & v = v_m \\
\mathsf{C}_2 : f_2(\omega,\omega_m) = 0 : & \omega = \omega_m \\
\mathsf{C}_3 : f_3(Y,Y_m) = 0 : & Y = Y_m \\
\mathsf{C}_4 : f_4(K_y,Y,Q_{\text{eng}}) = 0 : & Q_{\text{eng}} + \tau_c \dot{Q}_{eng} = K_y Y \\
\mathsf{C}_5 : f_5(Q_{\text{eng}},Q_{\text{prop}},\omega) = 0 : & I_m\dot{\omega} = Q_{\text{eng}} - Q_{\text{prop}} \\
\mathsf{C}_6 : f_6(\omega,v,U,Q_{\text{prop}}) = 0 : & EQ_Q \\
\mathsf{C}_7 : f_7(\omega,v,U,T_{\text{prop}}) = 0 : & EQ_T \\
\mathsf{C}_8 : f_8(U,R_u,T_{\text{prop}}) = 0 : & \dot{U} = \dfrac{1}{m_s}(T_p - R_u) \\
\mathsf{C}_9 : f_9(R_u,U) = 0 : & Table^* \\
\mathsf{C}_{10} : f_{10}(U,U_m) = 0 : & U = U_m
\end{array} \qquad (18)$$

where v is propeller pitch, ω and U denote shaft revolution and ship speed, Y is the fuel index, K_y is the engine gain, and Q_{eng} and Q_{prop} are engine and propeller torque, respectively. T_{prop} is the propeller thrust and $T_p = (1-t_T)T_{\text{prop}}$ where t_T is a constant term. The developed propeller thrust and torque are determined by (Blanke, 1981)

$$EQ_T : T_{\text{prop}} = T_{\omega\omega} v\omega^2 + T_{\omega U}\omega U \qquad (19)$$
$$EQ_Q : Q_{\text{prop}} = Q_{\omega\omega} v\omega^2 + Q_{\omega v} v\omega U,$$

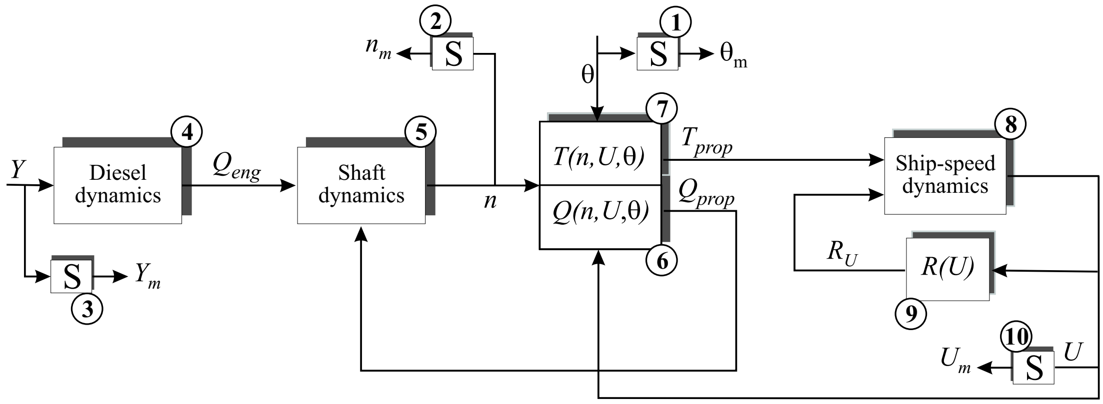

Fig. 2. Ship propulsion system with diesel, shaft dynamics with nonlinear trust and torque functions, and ship's hull resistance.

in the relevant region of operation $0 \leq v < 1$, $\omega > 0$, and $U \geq 0$. The nonlinear hull resistance is obtained by data interpolation in *Table**.

6.1 *Fault scenario*

Two faults are: fault in the shaft speed ($\Delta\omega$) measurement and engine gain fault (ΔK_y). Shaft speed is measured by a dual pulse pick-up. EMI disturbances on one pick-up can generate a too high signal $\Delta\omega_{\text{high}}$, while a minimum signal $\Delta\omega_{\text{low}}$ is produced due to loss of both pick-up signals. A drop in generated shaft torque, manifested by ΔK_y, is due to following causes: less (or hot) air inlet, less fuel oil inlet, or drop-out on one or more cylinders.

6.2 *System's structural model*

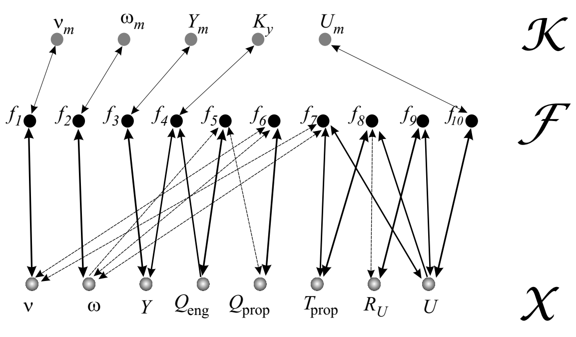

Fig. 3. Matching for the considered part of the propulsion system. Thick arcs illustrate the matched pairs (x, f), where $x \in \mathscr{X}$ and $f \in \mathscr{F}$.

The system structure is $\mathsf{S} = \bigcup_{i=1}^{10} \mathsf{C}_i$, $\mathscr{F} = F_X = \{f_1, f_2, \cdots, f_{10}\}$, $\mathscr{K} = \{v_m, \omega_m, Y_m, U_m, K_y\}$, $\mathscr{X} = \{U, v, \omega, Y, Q_{\text{prop}}, T_{\text{prop}}, Q_{\text{eng}}, R_U\}$, and $\mathscr{Z} = \mathscr{K} \bigcup \mathscr{X}$. The measurement noise is disregarded here, hence $v_m = v$ and $\omega = \omega_m$ and $\cdots$. A bipartite digraph representation is depicted in Fig. 3. Thick arcs on the figure show the matching.

6.3 *Fault detection*

The basic idea for the matching is as follows: the value of v, which is matched to f_1, can be computed when we know the value of all other variables related to f_1, in this case v_m. The same procedure is used to match the other variables. In the performed matching, two relations f_4 and f_7 are not matched. By backtracking the involved variables in each relation one can construct the related minimal over-determined subsystem: for instance, $G^+_{min-f_7}$ is determined by $F^+_{min-f_7} = \{f_1, f_2, f_{10}, f_9, f_7\}$ and $X^+_{min-f_7} = \{U, R_U, T_{\text{prop}}, v, \omega\}$. The two minimal over-determined subsystems give the following residual expressions:

$$r_Q = f_Q(v_m, \omega_m, Y_m, K_y, U_m) \tag{20}$$

$$r_T = f_T(v_m, \omega_m, U_m). \tag{21}$$

It is obvious that fault detection is possible since r_Q will be affected by both measurement and gain faults, while r_T will only be affected by the measurement fault.

6.4 *Gain fault isolation*

An essential problem in diagnosis is whether faults can be isolated. In this example, isolation of a gain fault from a shaft speed measurement fault is desired. The first step to take is to represent this situation in the structural model. According to Eq. 12

$$k_y = K_y(1 + \Delta K_y)$$

In faulty condition, the diesel engine dynamics are given by Eqs. 14 and 15,

$$f_4(\Delta K_y, K_y, Y, Q_{\text{eng}}) = 0$$
$$f_4'(\Delta K_y) = 0$$

The new set of unknowns is $\mathscr{X} = \{\Delta K_y, U, v, \omega, Y, Q_{\text{prop}}, T_{\text{prop}}, Q_{\text{eng}}, R_U\}$. The relation representing shaft revolution measurement, f_2, is considered to be non-valid acc. to Eq. 4, as the sensor is not functioning properly. This relation is thus removed from $\mathscr{F}_X$.

The performed matching on the new structural model is shown in table 1. Since $f'^4(\Delta K_y)$ is the only un-

Table 1. Matching for the nonlinear system.

$f_1 \longleftrightarrow v$	$f_3 \longleftrightarrow Y$	$f_{10} \longleftrightarrow U$
$f_9 \longleftrightarrow R_U$	$f_8 \longleftrightarrow T_{\text{prop}}$	$f_7 \longleftrightarrow \omega$
$f_6 \longleftrightarrow Q_{\text{prop}}$	$f_5 \longleftrightarrow Q_{\text{eng}}$	$f_4 \longleftrightarrow \Delta K_y$

matched relation and $|\mathscr{F}_X| - |\mathscr{X}| = 1$, the resulting system is a minimal over-determined system. A residual expression will involve all relations except f_2. The resulting equations are suitable for use in a diagnostic observer,

$$\begin{aligned}
\dot{Q}_{\text{eng}} &= \frac{1}{\tau_c}\big(-Q_{\text{eng}} + K_y \Delta K_y Y_m + K_y Y_m\big) \\
\dot{\omega} &= \frac{1}{I_m}\big(Q_{\text{eng}} - Q_{\omega\omega} v_m \omega^2 - Q_{\omega v} v_m \omega U\big) \\
\dot{U} &= \frac{1-t_T}{m_s}\big(T_{\omega\omega} v_m \omega^2 + T_{\omega U}\omega U - \frac{R_U}{1-t_T}\big) \\
\Delta \dot{K}_y &= 0
\end{aligned}$$

System causality shows that inputs are Y_m, v_m and the output is U_m. Obviously, the dynamics of this subsystem is not affected by the measurement ω_m and isolation can be achieved.

7. CONCLUSIONS

The graph based structural analysis approach was employed to examine the fault diagnosis possibilities in a dynamic system. The representation of the system was described and the matching concept used to identify the (sub)systems that contain redundant information. The original method of (Cassar *et al.*, 1994) and (Staroswiecki *et al.*, 1999) was extended by a unified fault model representation and minimal over-determined subsystems were defined and used to obtain residual expressions for fault diagnosis. Fault isolability was shown to be accessible by inspection. The techniques of the matching process were made clear by re-defining the matching problem as relation of faults to known parameters and measurements.

Salient features of the method were illustrated by application to a ship propulsion benchmark, emphasizing how residual generators are obtained to help detect and isolate particular faults.

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